18.338 Eigenvalues of Random Matrices

Problem Set 1

Due Date: Wed Feb. 15, 2012

Instruction

Read chapter *1, 3.1-3.2, 8.1-8.3* of the class notes.

Comments on the readings are a *required* part of the homework! Please comments about what you have read. Comments should include any errors you catch, or stylistic comments. If the material is hard to follow, I want to know. Okay to let me know where the writing became hard to follow.

Please hand in hardcopy or email (including both the answers to the problems and comments to the readings) to Bernie Wang (ywang02@mit.edu).

Homeworks

Do at least four out of the following problems (Computational/Mathematical problems are denoted as C/M. The second problem can be both or either. Exercise with numbers and pages are from the class notes.)

1. (C) Exercise 1.3 (p13)
2. (MC) Show mathematically and/or with a Monte Carlo experiment that the Beta distribution (with the right parameters) generates random numbers from the semicircle. In other words, it is not necessary to do an eigenvalue computation as in Code 1.2 to obtain random numbers from the semicircle distribution. (Google `betarnd` not `betarand` for the MATLAB command)
3. (M) Exercise 3.2 (p32)
4. (C) Exercise 3.3 (p32)
5. (M) Exercise 8.1 (p81) Optional: Show off by solving the advanced exercise that follows!
6. (M) Exercise 8.2 (p84) Please tell us if the formula is correct!
7. (C) Experimentally confirm Equation (8.2) on p71. Try a range of ϵs and describe what you see. Write a function that computes the matrix of partial derivatives $\partial X^3/\partial X_{kl}$ given X, k, l using the following formula:
$$\frac{\partial X^3}{\partial x_{kl}} = X^2(E_{kl}) + X(E_{kl})X + (E_{kl})X^2.$$
8. (C) Take any code from the notes and translate to Julia (ask Jeff (bezanson@mit.edu for help)). Next translate to parallel Julia and compare the performance to serial Julia (part of this exercise is to perhaps patiently wait for an account or use one of ours).
9. (M) The Cumulative Density Function and its inverse. Let x be a continuous random variable with probability density $f(t)$. Let
$$F(s) = \int_{-\infty}^{s} f(t)\mathrm{d}t$$
(the "cdf") and $F^{-1}(y) = s$ (the "inverse cdf") is the function inverse of F. Notice that $F : \mathbb{R} \to [0,1]$ and $F^{-1} : [0,1] \to \mathbb{R}$. Then, if ω is uniform in $[0,1]$, show that $F^{-1}(\omega)$ has probability density $f(t)$ and mean
$$\int_0^1 F^{-1}(y)dy.$$