

# Liquidity of Corporate Bonds

Jack Bao, Jun Pan and Jiang Wang\*

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## Abstract

This paper examines the liquidity of corporate bonds. Using transaction-level data for a broad cross-section of corporate bonds from 2003 through 2007, we construct a measure of illiquidity by estimating the magnitude of price reversals in corporate bonds. We find the illiquidity in corporate bonds to be significant and substantially more severe than what can be explained by bid-ask bounce. We establish a robust connection between our illiquidity measure and liquidity-related bond characteristics. In particular, it is higher for older and smaller bonds and bonds with smaller average trade sizes and higher idiosyncratic return volatility. Aggregating our illiquidity measure across bonds, we find strong commonality in the time variation of bond illiquidity, which rises sharply during market crises and reaches an all-time high during the recent sub-prime mortgage crisis. Moreover, monthly changes in aggregate illiquidity are strongly related to changes in the CBOE VIX Index. We also find a robust positive relation between our illiquidity measure and bond yield spreads that is economically significant.

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\*Bao is from MIT Sloan School of Management (jackbao@mit.edu); Pan is from MIT Sloan School of Management and NBER (junpan@mit.edu); and Wang is from MIT Sloan School of Management, CCFR and NBER (wangj@mit.edu). Support from the outreach program of J.P. Morgan is gratefully acknowledged.

# 1 Introduction

The liquidity of the corporate bond market has been of interest for researchers, practitioners and policy makers. Many studies have attributed deviations in corporate bond prices from their “theoretical values” to the influence of illiquidity in the market.<sup>1</sup> Yet, our understanding of how to quantify illiquidity remains limited. And without a credible measure of illiquidity, it is difficult to have a direct and serious examination of the asset-pricing influence of illiquidity and its implications on market efficiency. For this reason, we focus in this paper directly on the issue of illiquidity. In particular, we construct an empirical measure of illiquidity by extracting the transitory component in the price movement of corporate bonds. We find that the lack of liquidity in the corporate bond market is economically significant and is related to several bond characteristics that are known to be linked to liquidity issues. Moreover, we find that, in aggregate, the illiquidity in corporate bonds varies substantially over time along with the changing market conditions. We also find economically important implications of illiquidity on bond yield spreads.

Several measures of illiquidity have been considered in the literature for corporate bonds. A simple measure is the bid-ask spread, which is analyzed in detail by Edwards, Harris, and Piwowar (2007).<sup>2</sup> Although the bid-ask spread is a direct and potentially important indicator of illiquidity, it does not fully capture many important aspects of liquidity such as market depth and resilience. Relying on theoretical pricing models to gauge the impact of illiquidity has the advantage of directly measuring its influence on prices. But it suffers from potential mis-specifications of the pricing model. In this paper, we rely on a salient feature of illiquidity to measure its significance. It has been well recognized that the lack of liquidity in an asset gives rise to transitory components in its prices (see, e.g., Grossman and Miller (1988) and Huang and Wang (2007)). Since transitory price movements lead to negatively serially correlated price changes, the negative of the autocovariance in price changes, which we denote by  $\gamma$ , provides a simple empirical measure of illiquidity. In the simplest case when the transitory price movements arise purely from bid-ask bounce, as considered by Roll (1984),  $2\sqrt{\gamma}$  equals the bid-ask spread. But in more general cases,  $\gamma$  captures the broader impact of

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<sup>1</sup>For example, Huang and Huang (2003) find that yield spreads for corporate bonds are too high to be explained by credit risk and question the economic content of the unexplained portion of yield spreads (see also Colin-Dufresne, Goldstein, and Martin (2001) and Longstaff, Mithal, and Neis (2005)). Bao and Pan (2008) document a significant amount of transitory excess volatility in corporate bond returns and attribute this excess volatility to the illiquidity of corporate bonds.

<sup>2</sup>See also Bessembinder, Maxwell, and Venkataraman (2006) and Goldstein, Hotchkiss, and Sirri (2007).

illiquidity on prices, which we show goes beyond the effect of bid-ask spread, and it does so without relying on specific bond pricing models.

Using TRACE, a transaction-level dataset, we estimate  $\gamma$  for a broad cross-section of the most liquid corporate bonds in the U.S. market. Our results show that, using trade-by-trade data, the median estimate of  $\gamma$  is 0.3598 and the mean estimate is 0.5814 with a robust  $t$ -stat of 22.23; using daily data, the median  $\gamma$  is 0.5533 and the mean  $\gamma$  is 0.9080 with a robust  $t$ -stat of 29.13. To judge the economic significance of such magnitudes, we can use the quoted bid-ask spreads to calculate a bid-ask implied  $\gamma$ . For the same sample of bonds and for the same sample period, we find that the median  $\gamma$  implied by the quoted bid-ask spreads is 0.0313 and the mean is 0.0481, which are tiny fractions of our estimated  $\gamma$ . An alternative comparison is to use the Roll's model to calculate the  $\gamma$ -implied bid-ask spread, which is  $2\sqrt{\gamma}$ , and compare it with the quoted bid-ask spread.<sup>3</sup> Using our median estimates of  $\gamma$ , the  $\gamma$ -implied bid-ask spread is \$1.1996 using trade-by-trade data and \$1.4876 using daily data, significantly larger values than the median quoted bid-ask spread of \$0.3538 or the estimated bid-ask spread reported by Edwards, Harris, and Piwowar (2007) (see Section 8 for more details).

The difference in the magnitudes of  $\gamma$ , estimated using the trade-by-trade vs. daily data, is itself indicative that our illiquidity measure  $\gamma$  captures the price impact of illiquidity above and beyond the effect of simple bid-ask bounce. To further explore this point, we use the trade-by-trade data to estimate the magnitude of price reversals after skipping a trade and find it to be still significant both in economic magnitude and statistical significance. This implies that, at the transaction level, the mean-reversion in price changes lasts for more than one trade. Our  $\gamma$  measured at the daily level, capturing this persistent transaction-level mean-reversion cumulatively, yields a higher magnitude than its counterpart at the transaction level. Performing the same analysis for daily data, we find a much weaker price reversal after skipping a day, indicating that the half life of the transitory price component due to illiquidity is short.

We also find that autocovariance exhibits an asymmetry for positive and negative price changes. In particular, negative price changes, likely caused by excess selling pressure, are followed by stronger reversals than positive price changes. Such an asymmetry was described as a characteristic of the impact of illiquidity on prices by Huang and Wang (2007). Our results provide an interesting empirical test of this proposition.

We next examine the connection between our illiquidity measure  $\gamma$  and cross-sectional

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<sup>3</sup>Roll's model assumes that directions of trades are serially independent. For a given bid-ask spread, positive serial correlation in trade directions, which could be the case when liquidity is lacking and traders break up their trades, tends to increase the implied bid-ask spreads for a given  $\gamma$ .

bond characteristics, particularly those known to be relevant for liquidity. We find a strong positive relation between  $\gamma$  and the age of a bond, a variable widely used in the fixed-income market as a proxy of illiquidity. We also find that bonds with smaller issuance tend to have higher  $\gamma$ , and the same is true for bonds with higher idiosyncratic return volatility and smaller average trade sizes. In particular, including average trade sizes in the cross-sectional regression drives out the issuance effect and cuts the age effect by half. Finally, using quoted bid-ask spreads, we find a positive relation between our estimate of  $\gamma$  and that implied by the quoted bid-ask spread. But the result is weak statistically (with a  $t$ -stat of 1.57), indicating that the magnitude of illiquidity captured by our illiquidity measure  $\gamma$  is related to but goes beyond the information contained in the quoted bid-ask spreads.

The connection between  $\gamma$  and average trade sizes turns out to be more interesting than a simple cross-sectional effect. We find that price changes associated with large trades exhibit weaker reversals than those associated with small trades, and this effect is robust after controlling for the overall bond liquidity. Using trade-by-trade data, we are able to construct empirical measures of  $\gamma$  conditional on trade sizes, and we find that the conditional  $\gamma$  decreases monotonically as trade sizes increase. For example, for the group of least liquid bonds in our sample, as we move from trade sizes being less than \$5K to over \$500K, the median value of the conditional  $\gamma$  decreases monotonically from 1.8844 to 0.4835. This monotonic pattern of decreasing conditional  $\gamma$  with increasing trade sizes is present for all groups of bonds of varying degrees of illiquidity, and persists even after skipping a trade. Since both trade sizes and prices are endogenous, we cannot interpret the negative relation between  $\gamma$  and trade sizes simply as more liquidity for larger trades. But our result does suggest a strong link between liquidity and trade sizes.

One interesting aspect of our results emerges as we aggregate  $\gamma$  across bonds to examine its time-series properties. We find strong commonality in bond illiquidity that is closely related to market conditions, especially during credit-market crises. Over our sample period, there is an overall trend of decreasing  $\gamma$ , which was on average 1.0201 in 2003, dropped steadily from then on to 0.7618 in 2006, and then partially bounced back to 0.9222 in 2007. With the exception of the later half of 2007, there seems to be an overall improvement of liquidity in the corporate bond market.

Against this backdrop of an overall time trend, we find substantial monthly movements in the aggregate measure of illiquidity. During the periods that eventually lead to the downgrade of Ford and GM bonds to junk status, our aggregate illiquidity measure increases sharply from

0.87 in March 2005 to 1.08 in April and 1.03 in May 2005. This sharp increase in  $\gamma$ , however, is dwarfed by what happens during the sub-prime mortgage crisis in August 2007. In May 2007, our aggregate illiquidity measure  $\gamma$  hovers around 0.75, and then increases in a steady fashion all the way to 1.37 in August 2007. It relents somewhat during September and October, and then shoots back up to 1.38 in November, and an all-time high level of 1.39 in December. Moreover, the conditional  $\gamma$  for large trades increases more in percentage terms during crises than small trades, suggesting that illiquidity shocks are market-wide and affect all clienteles.

To link our aggregate illiquidity measure more closely to the overall market condition, we consider a list of market-level variables including the VIX index, term spread, and lagged aggregate stock and bond returns. Regressing changes in aggregate  $\gamma$  on changes in VIX, we find a positive and significant coefficient and the R-squared is close to 40%. We also find that aggregate  $\gamma$  increases when the default spread increases, and when the aggregate stock or bond market under performs in the previous month. Using these variables together to explain the monthly changes in aggregate  $\gamma$ , we find that both VIX and lagged aggregate stock returns remain significant. But the default spread and lagged aggregate bond returns — two variables that are measured from the credit market and are expected to be more closely related to our  $\gamma$  measure — fail to remain significant. Moreover, there is no significant relation between changes in our aggregate  $\gamma$  and changes in the volatility of the aggregate bond returns. The fact that the VIX index, measured from index options, is the most important variable in explaining changes in aggregate illiquidity of corporate bonds is rather intriguing. Indeed, from an aggregate perspective, this implies that a significant portion of our estimated bond market illiquidity is not contained just in the bond market. This raises the possibility of illiquidity being an additional source of systemic risk, as examined by Chordia, Roll, and Subrahmanyam (2000) and Pastor and Stambaugh (2003) for the equity market.

Finally, we examine the relation between our illiquidity measure  $\gamma$  and bond yield spreads. Controlling for bond rating categories, we perform monthly cross-sectional regressions of bond yield spread on bond  $\gamma$ . We find a coefficient of 0.4220 with a  $t$ -stat of 3.95 using Fama and MacBeth (1973) standard errors. Given that the cross-sectional standard deviation of  $\gamma$  is 0.9943, our result implies that for two bonds in the same rating category, a two standard deviation difference in their  $\gamma$  leads to a difference in their yield spreads as large as 84 bps. This is comparable to the difference in yield spreads between Baa and Aaa or Aa bonds, which is 77.21 bps in our sample. From this perspective, the economic significance of our illiquidity measure is important. Moreover, our result remains robust in its magnitude and statistical

significance after we control for a spectrum of variables related to the bond's fundamental information and bond characteristics. In particular, liquidity related variables such as bond age, issuance size, quoted bid-ask spread, and average trade size do not change our result in a significant way.

Our paper is related to the growing literature on the impact of liquidity on corporate bond yields. Using illiquidity proxies that include quoted bid-ask spreads and the percentage of zero returns, Chen, Lesmond, and Wei (2007) find that more illiquid bonds earn higher yield spreads. Using nine liquidity proxies including issuance size, age, missing prices, and yield volatility, Houweling, Mentink, and Vorst (2003) reach similar conclusions for euro corporate bonds. de Jong and Driessen (2005) find that systematic liquidity risk factors for the Treasury bond and equity markets are priced in corporate bonds, and Downing, Underwood, and Xing (2005) address a similar question. Using a proprietary dataset on institutional holdings of corporate bonds, Nashikkar, Mahanti, Subrahmanyam, Chacko, and Mallik (2008) and Mahanti, Nashikkar, and Subrahmanyam (2008) propose a measure of latent liquidity and examine its connection with the pricing of corporate bonds and credit default swaps.

We contribute to this growing body of literature by proposing a measure of illiquidity that is theoretically motivated and empirically more direct. We are able to establish a connection between our measure of illiquidity and the commonly used liquidity proxies such as age, issuance and trading activities. But more importantly, our illiquidity measure contains information above and beyond such proxies in explaining, for example, the average bond yield spreads across a broad cross-section of bonds. Moreover, the degree of illiquidity captured by our illiquidity measure is significantly higher in magnitude than that implied by the quoted or estimated bid-ask spreads. Finally, the close connection between our aggregate illiquidity measure and the overall market condition is a clear indication that our measure indeed extracts useful information about illiquidity from the transaction-level data. We hope that the properties we uncover in this paper about the illiquidity of corporate bonds can provide a basis to further analyze its importance to the efficiency of the bond market.

The paper is organized as follows. Section 2 describes the data we use in our analysis and provides some simple summary statistics. In Section 3, we report the estimates of our illiquidity measure and its basic properties. We analyze the cross-sectional properties of illiquidity in Section 4 and its time-series properties in Section 5. We further examine illiquidity and trade sizes in Section 6. Section 7 is devoted the connection between illiquidity and bond yield spreads. In Section 8, we compare our illiquidity measure with the effect of bid-ask spreads.

Section 9 concludes.

## 2 Data Description and Summary

The main data set used for this paper is FINRA’s TRACE (Transaction Reporting and Compliance Engine). This data set is a result of recent regulatory initiatives to increase the price transparency in secondary corporate bond markets. FINRA, formerly the NASD, is responsible for operating the reporting and dissemination facility for over-the-counter corporate bond trades. On July 1, 2002, the NASD began Phase I of bond transaction reporting, requiring that transaction information be disseminated for investment grade securities with an initial issue size of \$1 billion or greater. Phase II, implemented on April 14, 2003, expanded reporting requirements, bringing the number of bonds to approximately 4,650. Phase III, implemented completely on February 7, 2005, required reporting on approximately 99% of all public transactions. Trade reports are time-stamped and include information on the clean price and par value traded, although the par value traded is truncated at \$1 million for speculative grade bonds and at \$5 million for investment grade bonds.

In our study, we drop the early sample period with only Phase I coverage. We also drop all of the Phase III only bonds. We sacrifice in these two dimensions in order to maintain a balanced sample of Phase I and II bonds from April 14, 2003 to December 2007. Of course, new issuances and retired bonds generate some time variations in the cross-section of bonds in our sample. After cleaning up the data, we also take out the repeated inter-dealer trades by deleting trades with the same bond, date, time, price, and volume as the previous trade.<sup>4</sup> We further require the bonds in our sample to have frequent enough trading so that the illiquidity measure can be constructed from the trading data. Specifically, during its existence in the TRACE data, a bond must trade on at least 75% of business days to be included in our sample. Finally, to avoid bonds that show up just for several months and then disappear from TRACE, we require that the bonds in our sample be in existence in the TRACE data for at least one full year.

Table 1 summarizes our sample, which consists of frequently traded Phase I and II bonds from April 2003 to December 2007. There are 1,249 bonds in our full sample, although the total number of bonds do vary from year to year. The increase in the number of bonds from 2003 to 2004 could be a result of how NASD starts its coverage of Phase III bonds, while

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<sup>4</sup>This includes cleaning up withdrawn or corrected trades, dropping trades with special sell conditions or special prices, and correcting for obvious mis-reported prices.

Table 1: Summary Statistics

	2003			2004			2005			2006			2007			Full		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std
#Bonds	775			1,216			1,166			1,075			944			1,249		
Issuance	1,017	1,000	727	858	700	676	853	700	683	833	650	662	827	650	665	867	700	680
Rating	5.60	5.67	2.55	6.91	6.00	3.93	7.20	6.00	4.15	7.61	6.00	4.65	7.56	6.00	4.60	7.27	6.00	4.25
Maturity	7.35	5.23	6.83	7.92	5.71	7.40	7.40	5.20	7.39	6.84	4.59	7.37	6.62	4.21	7.41	6.84	4.43	7.14
Coupon	5.88	6.00	1.66	5.88	6.10	1.89	5.86	6.00	1.89	5.80	6.00	1.91	5.83	6.00	1.92	5.88	6.03	1.90
Age	2.68	1.94	2.62	3.18	2.41	2.94	3.91	3.13	2.95	4.77	4.03	2.94	5.67	4.77	3.02	4.15	3.24	2.85
Turnover	11.60	8.34	9.43	9.36	7.08	7.49	8.26	6.16	6.79	6.30	5.10	4.91	5.08	4.09	3.93	7.83	6.61	5.16
Trd Size	586	467	464	528	405	474	437	344	391	391	300	360	347	268	322	448	366	368
#Trades	244	148	359	176	118	187	195	119	284	152	104	141	136	96	126	174	121	185
Avg Ret	0.64	0.42	0.90	0.73	0.37	1.92	0.03	0.18	0.90	0.72	0.40	1.39	0.39	0.45	1.01	0.43	0.35	0.54
Volatility	2.48	2.23	1.56	2.05	1.62	2.53	2.20	1.47	2.61	1.92	1.23	2.48	1.99	1.33	2.45	2.24	1.64	2.37
Price	108	109	10	106	106	11	103	103	11	100	101	12	102	101	14	103	103	11

#Bonds is the average number of bonds. Issuance is the bond's amount outstanding in millions of dollars. Rating is a numerical translation of Moody's rating: 1=Aaa and 21=C. Maturity is the bond's time to maturity in years. Coupon, reported only for fixed coupon bonds, is the bond's coupon payment in percentage. Age is the time since issuance in years. Turnover is the bond's monthly trading volume as a percentage of its issuance. Trd Size is the average trade size of the bond in thousands of dollars of face value. #Trades is the bond's total number of trades in a month. Med and std are the time-series averages of the cross-sectional medians and standard deviations. For each bond, we also calculate the time-series mean and standard deviation of its monthly returns, whose cross-sectional mean, median and standard deviation are reported under Avg Ret and Volatility. Price is the average market value of the bond in dollars.

the gradual reduction of number of bonds from 2004 through 2007 is a result of matured or retired bonds.

The bonds in our sample are typically large, with a median issuance size of \$700 million, and the representative bonds in our sample are investment grade, with a median rating of 6, which translates to Moody's A2. The average maturity is close to 7 years and the average age is about 4 years. Over time, we see a gradual reduction in maturity and increase in age. This can be attributed to our sample selection which excludes bonds issued after February 7, 2005, the beginning of Phase III.<sup>5</sup> Given our selection criteria, the bonds in our sample are more frequently traded than a typical bond. The average monthly turnover — the the bond's monthly trading volume as a percentage of its issuance size — is 7.83%, the average number of trades in a month is 174. The average trade size is \$448,000.

In addition to the TRACE data, we use CRSP to obtain stock returns for the market and the respective bond issuers. We use FISD to obtain bond-level information such as issue date, issuance size, coupon rate, and credit rating, as well as to identify callable, convertible and puttable bonds. We use Bloomberg to collect the quoted bid-ask spreads for the bonds in our sample, from which we have data only up to 2006. We use Datastream to collect Lehman Bond indices to calculate the default spread and returns on the aggregate corporate bond market. To calculate yield spreads for individual corporate bonds, we obtain Treasury bond yields from the Federal Reserve, which publishes constant maturity Treasury rates for a range of maturities. Finally, we obtain the VIX index from CBOE.

### 3 Measure of Illiquidity

In the absence of a theory, a definition of illiquidity and its quantification remain imprecise. But two properties of illiquidity are clear. First, it arises from market frictions, such as costs and constraints for trading and capital flows; second, its impact to the market is transitory.<sup>6</sup> Our empirical measure of illiquidity is motivated by these two properties.

Let  $P_t$  denote the clean price of a bond at time  $t$ . We start by assuming that  $P_t$  consists

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<sup>5</sup>We will discuss later the effect, if any, of this sample selection on our results. An alternative treatment is to include in our sample those newly issued bonds that meet the Phase II criteria, but this is difficult to implement since the Phase II criteria are not precisely specified by NASD.

<sup>6</sup>In a recent paper, Vayanos and Wang (2008) provide a unified theoretical model for liquidity. Huang and Wang (2007) consider a model in which trading costs give rise to illiquidity in the market endogenously and show that it leads to transitory deviations in prices from fundamentals.

of two components:

$$P_t = F_t + u_t. \quad (1)$$

The first component  $F_t$  is its fundamental value — the price in the absence of frictions, which follows a random walk. The second component  $u_t$  comes from the impact of illiquidity, which is transitory. In such a framework, the magnitude of the transitory price component  $u_t$  characterizes the level of illiquidity in the market. Our measure of illiquidity is aimed at extracting the transitory component in the observed price  $P_t$ . Specifically, let  $\Delta P_t = P_t - P_{t-1}$  be the price change from  $t - 1$  to  $t$ . We define the measure of illiquidity  $\gamma$  by

$$\gamma = -\text{Cov}(\Delta P_t, \Delta P_{t+1}). \quad (2)$$

With the assumption that the fundamental component  $F_t$  follows a random walk,  $\gamma$  depends only on the transitory component  $u_t$ , and it increases with the magnitude of  $u_t$ .

Several comments are in order before our analysis of  $\gamma$ . First, other than being transitory, we know little about the dynamic properties of  $u_t$ . Even though  $\gamma$  provides a simple gauge of the magnitude of  $u_t$ , it also depends on other properties of  $u_t$ . For example, both the instantaneous volatility of  $u_t$  and its persistence will affect  $\gamma$ . Second, in terms of measuring illiquidity, other aspects of  $u_t$  that are not captured by  $\gamma$  may also matter. In this sense  $\gamma$  itself gives only a partial measure of illiquidity. Third, given the potential richness in the dynamics of  $u_t$ ,  $\gamma$  will in general depend on the horizon over which we measure price changes. The  $\gamma$  for different horizons may capture different aspects of  $u_t$  or illiquidity. For most of our analysis, we will use either trade-by-trade prices or end of the day prices in estimating  $\gamma$ . Thus, our  $\gamma$  estimate captures more of the high frequency components in the transitory prices.

### 3.1 Empirical Estimation of $\gamma$

Table 2 summarizes the illiquidity measure  $\gamma$  for the bonds in our sample.<sup>7</sup> Focusing first on Panel A, in which  $\gamma$  is estimated using trade-by-trade data, we see an illiquidity measure of  $\gamma$  that is important both economically and statistically. In terms of magnitude,  $\gamma$  has a cross-sectional average of 0.5814 using the full time-series sample. By comparison, the quoted bid-ask spreads for the same cross-section of bonds and for the same sample period, would have generated an average negative autocovariance in the neighborhood of 0.048, which is one order of magnitude smaller than the empirically observed autocovariance. This illiquidity

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<sup>7</sup>To be included in our sample, the bond must trade on at least 75% of business days and at least 10 observations of  $(\Delta P_t, \Delta P_{t-1})$  are required to calculate  $\gamma$ .

Table 2: Measure of Illiquidity:  $\gamma_\tau = -\text{Cov}(P_t - P_{t-1}, P_{t+\tau} - P_{t+\tau-1})$ 

		2003	2004	2005	2006	2007	Full
Panel A: Using trade-by-trade data							
$\tau = 1$	Mean $\gamma$	0.6546	0.6714	0.5717	0.4677	0.4976	0.5814
	Median $\gamma$	0.4520	0.3928	0.3170	0.2588	0.2830	0.3598
	Per t-stat $\geq 1.96$	99.74	97.53	99.31	98.69	97.45	100.00
	Robust t-stat	16.87	16.01	19.10	20.56	19.51	22.23
$\tau = 2$	Mean $\gamma$	0.0808	0.0679	0.0824	0.0598	0.1012	0.0805
	Median $\gamma$	0.0373	0.0236	0.0320	0.0261	0.0554	0.0395
	Per t-stat $\geq 1.96$	27.87	19.77	38.03	39.78	52.87	67.41
	Robust t-stat	10.24	7.42	13.22	11.02	13.97	13.81
$\tau = 3$	Mean $\gamma$	0.0105	0.0239	0.0221	0.0280	0.0277	0.0233
	Median $\gamma$	0.0054	0.0048	0.0049	0.0049	0.0067	0.0065
	Per t-stat $\geq 1.96$	5.16	5.52	6.27	8.68	6.69	11.93
	Robust t-stat	2.71	4.30	7.87	7.26	7.72	10.70
Panel B: Using daily data							
		2003	2004	2005	2006	2007	Full
$\tau = 1$	Mean $\gamma$	1.0201	0.9842	0.9047	0.7618	0.9222	0.9080
	Median $\gamma$	0.6949	0.5328	0.4558	0.4149	0.5590	0.5533
	Per t-stat $\geq 1.96$	95.35	90.64	96.04	95.50	92.63	99.36
	Robust t-stat	22.03	17.22	26.81	26.13	24.92	29.13
$\tau = 2$	Mean $\gamma$	0.0205	0.0194	0.0037	0.0021	0.0043	0.0038
	Median $\gamma$	0.0160	0.0084	0.0038	0.0029	0.0040	0.0044
	Per t-stat $\geq 1.96$	4.52	4.73	3.96	3.84	4.91	4.00
	Robust t-stat	1.25	1.19	0.42	0.18	0.34	0.66
$\tau = 3$	Mean $\gamma$	-0.0082	0.0012	0.0068	0.0249	0.0094	0.0035
	Median $\gamma$	-0.0036	-0.0028	0.0010	0.0009	0.0026	-0.0006
	Per t-stat $\geq 1.96$	2.20	2.74	2.67	2.81	2.56	2.72
	Robust t-stat	-0.54	0.09	0.73	2.23	0.84	0.73
Panel C: Implied by quoted bid-ask spreads							
		2003	2004	2005	2006	2007	Full
$\tau = 1$	Mean $\gamma$	0.0455	0.0414	0.0527	0.0519		0.0481
	Median $\gamma$	0.0363	0.0312	0.0293	0.0250		0.0313

For each bond, its  $\gamma$  is calculated for the year or for the full sample, using either trade-by-trade or daily data. Each  $\gamma$  has its own t-stat, and Per t-stat  $\geq 1.96$  reports the percentage of bond with statistically significant  $\gamma$ . Robust t-stat is a test on the mean of  $\gamma$  with standard errors clustered by bond and day. Monthly quoted bid-ask spreads are used to calculate the implied  $\gamma$  for  $\tau = 1$ . We have quoted bid-ask data for only 890 out of 1,249 bonds in our sample.

measure  $\gamma$  is also found to be statistically significant. The cross-sectional mean of  $\gamma$  has a robust t-stat of 22.24.<sup>8</sup> Moreover, the significant mean estimate of  $\gamma$  is not generated by just a few highly illiquid bonds. The cross-sectional median of  $\gamma$  is 0.3598, and at the individual bond level, 100% of the bonds have a statistically significant  $\gamma$ . Breaking our full sample by year also shows that the illiquidity measure  $\gamma$  is important and stable across years.

To further examine the dynamic properties of this transitory component, we measure the autocovariance of price changes that are separated by a few days or a few trades:

$$\gamma_\tau = -\text{Cov}(\Delta P_t, \Delta P_{t+\tau}). \quad (3)$$

For  $\tau > 1$ ,  $\gamma_\tau$  measures the extent to which the mean-reversion persists after the initial price reversal at  $\tau = 1$ . As shown in Panel A of Table 2, the initial bounce back is the strongest while the mean-reversion still persists after skipping a trade. In particular,  $\gamma_2$  is on average 0.10 with a robust t-stat of 13.81. At the individual bond level, 67% of the bonds have a statistically significant  $\gamma_2$ . After skipping two trades, the amount of residual mean-reversion dissipates further in magnitude. The cross-sectional average of  $\gamma_3$  is only 0.028, although it is still statistically significant with a robust t-stat of 10.70. At the individual bond level, fewer than 7% of the bonds have a statistically significant  $\gamma_3$ . This persistent mean-reversion at the transaction level is interesting in its own right, and will show up again as we next examine mean-reversion at the daily level.

At the daily frequency, the magnitude of the illiquidity measure  $\gamma$  is stronger. As shown in Panel B of Table 2, the cross-sectional average of  $\gamma$  is 0.9080 with a robust  $t$ -stat of 29.13. This is expected since our trade-by-trade results show that the mean-reversion persists for a few trades before fully dissipating, and the autocovariance at the daily level captures this effect cumulatively. At the daily level, however, the mean-reversion dissipates rather quickly, with an insignificant  $\gamma_2$ . This, of course, would have a direct impact on any trading strategies devised to take advantage of the large negative autocovariance, which we will examine more carefully later in this section.

Although the focus of this paper is on extracting the transitory component at the trade-by-trade and daily frequencies, it is nevertheless interesting to provide a general picture of  $\gamma$  over longer horizons. For example, moving to the weekly frequency, the magnitude of our illiquidity measure  $\gamma$  increases to 1.0899, although its statistical significance decreases somewhat to a

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<sup>8</sup>The moment condition is  $\hat{\gamma} + \Delta P_t^i \Delta P_{t-1}^i = 0$  for all bond  $i$  and time  $t$ , where  $\Delta P$  is demeaned. We can then correct for cross-sectional and time-series correlations in  $\Delta P_t^i \Delta P_{t-1}^i$  using standard errors clustered by bond and day.

robust  $t$ -stat of 16.81. At the individual bond level, 82.79% of the bonds in our sample have a positive and statistically significant  $\gamma$  at this frequency. Extending further to the bi-weekly and monthly frequencies,  $\gamma$  starts to decline in both magnitudes and statistical significance, equaling 0.9199 with a robust  $t$ -stat of 8.04 for bi-weekly, and 0.5076 with a robust  $t$ -stat of 2.18 for monthly horizons. At the individual bond level, the fraction of bonds that have positive and statistically significant  $\gamma$  is 42.88% for bi-weekly, and only 16.5% for monthly. At the six-week horizon, the magnitude of the estimate inches up a little from its monthly counterpart, but there is no longer any statistical significance.

As mentioned earlier in the section, the transitory component  $u_t$  might have richer dynamics than what can be offered by a simple AR(1) structure for  $\Delta u_t$ . By extending  $\gamma$  over various horizons, we are able to uncover some of the rich dynamics. For example, our results show that at the trade-by-trade level,  $\Delta u_t$  is by no means a simple AR(1). Likewise, in addition to the mean-reversion at the daily horizon that is captured in this paper, the transitory component  $u_t$  may also have a slow moving mean-reversion component at a longer horizon. To examine this issue more thoroughly is certainly an interesting topic, but requires time-series data for a longer sample period than ours.<sup>9</sup>

### 3.2 Asymmetry in Price Reversals

One interesting question regarding the mean-reversion captured in our result is whether or not the magnitude of mean-reversion is symmetric in the sign of the initial price change. Specifically, with  $\Delta P$  properly demeaned, let  $\gamma^- = E(\Delta P_t \Delta P_{t+1} | \Delta P_t < 0)$  be a measure of mean-reversion conditioning on an initial price change that is negative, and let  $\gamma^+$  be the counterpart conditioning on a positive price change. In a simple theory of liquidity based on costly market participation, Huang and Wang (2007) show that the bounce-back effect is more severe conditioning on an initial price movement that is negative, predicting a positive difference between  $\gamma^-$  and  $\gamma^+$ .

We test this hypothesis in Table 3, which shows that indeed there is a positive difference between  $\gamma^-$  and  $\gamma^+$ . Using trade-by-trade data, the cross-sectional average of  $\gamma^- - \gamma^+$  is 0.0802 with a robust  $t$ -stat of 5.59. Skipping a trade, the asymmetry in  $\gamma_2$  is on average 0.0457 with

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<sup>9</sup>By using monthly bid prices from 1978 to 1998, Khang and King (2004) report contrarian patterns in corporate bond returns over horizons of one to six months. Instead of examining autocovariance in bond returns, their focus is on the cross-sectional effect. Sorting bonds by their past monthly (or bi-monthly up to 6 months) returns, they find that past winners under perform past losers in the next month (or 2-month up to 6 months). Their result, however, is relatively weak and is significant only in the early half of their sample and goes away in the second half of their sample (1988–1998).

Table 3: Asymmetry in  $\gamma$ 

		Panel A: Using trade-by-trade data					
		2003	2004	2005	2006	2007	Full
$\tau = 1$	Mean	0.1442	0.0674	0.0120	0.0455	0.0689	0.0802
	Median	0.1347	0.0292	-0.0030	0.0257	0.0574	0.0347
	CS t-stat	7.92	3.71	0.92	3.93	5.87	5.98
	Robust t-stat	6.53	3.44	0.88	3.71	5.55	5.59
$\tau = 2$	Mean	0.0351	0.0328	0.0444	0.0411	0.0508	0.0457
	Median	0.0146	0.0077	0.0104	0.0160	0.0228	0.0145
	CS t-stat	5.01	4.34	9.47	9.63	8.14	9.29
	Robust t-stat	4.94	4.11	8.20	8.17	7.61	8.59

		Panel B: Using daily data					
		2003	2004	2005	2006	2007	Full
$\tau = 1$	Mean	0.2759	0.1628	0.1090	0.1232	0.1529	0.1753
	Median	0.1948	0.0449	0.0173	0.0469	0.0952	0.0708
	CS t-stat	9.92	5.50	4.82	5.77	6.22	9.63
	Robust t-stat	8.92	4.85	4.40	5.01	5.65	8.89
$\tau = 2$	Mean	-0.0036	0.0026	0.0091	-0.0021	0.0154	0.0059
	Median	0.0003	-0.0011	-0.0003	0.0012	0.0012	0.0009
	CS t-stat	-0.33	0.18	1.01	-0.26	1.26	0.96
	Robust t-stat	-0.28	0.18	0.86	-0.24	1.07	0.87

Asymmetry in  $\gamma$  is measured by the difference between  $\gamma^-$  and  $\gamma^+$ , where  $\gamma^- = E(\Delta P_{t+1} \Delta P_t | \Delta P_t < 0)$ , with  $\Delta P$  properly demeaned, measures the price reversal conditioning on a negative price movement. Likewise,  $\gamma^+$  measures the price reversal conditioning on a positive price movement. Robust t-stat is a pooled test on the mean of  $\gamma^- - \gamma^+$  with standard errors clustered by bond and day. CS t-stat is the cross-sectional t-stat.

a robust t-stat of 8.59. Compared with how  $\gamma_\tau$  dissipates across  $\tau$ , this measure of asymmetry does not exhibit the same dissipating pattern. In fact, in the later sample period, the level of asymmetry for  $\tau = 2$  is almost as important for the first-order mean-reversion, with an even higher statistical significance. Using daily data, the asymmetry is stronger, incorporating the cumulative effect from the transaction level. The cross-sectional average of  $\gamma^- - \gamma^+$  is 0.18, which is close to 20% of the observed level of mean reversion. Skipping a day, however, produces no evidence of asymmetry, which is expected since there is very little evidence of mean-reversion at this level in the first place.

### 3.3 Profiting from Illiquidity

Given the large magnitude of negative autocovariance documented in this section, it is natural to ask whether or not there is a feasible trading strategy to profit from this severe illiquidity

Table 4: Trading Profitability of  $\gamma$ -Based Strategies

year	Buy if $\Delta P < 0$ and Sell if $\Delta P > 0$						Buy if $\Delta P \leq -1$ and Sell if $\Delta P \geq 1$						
	No Skip			Skip a Trade			No Skip			Skip a Trade			
	mean	t-stat	med	mean	t-stat	med	mean	t-stat	med	mean	t-stat	med	
2003	Overall	4.52	10.92	1.44	0.28	12.57	0.16	5.39	10.32	2.02	0.28	12.13	0.12
	Buy Signal	2.57	11.05	0.90	0.23	11.56	0.11	3.34	10.98	1.52	0.23	11.56	0.06
	Sell Signal	2.32	10.99	0.78	0.07	5.02	0.07	2.93	9.97	1.12	0.09	6.41	0.03
	Trades	10.05		5.66	10.05		5.65	5.64		2.61	5.66		2.63
2004	Overall	3.09	15.48	1.07	0.16	10.42	0.10	4.00	14.75	1.81	0.17	8.59	0.08
	Buy Signal	1.79	15.62	0.67	0.14	10.65	0.07	2.39	15.33	1.25	0.15	9.09	0.03
	Sell Signal	1.58	15.72	0.58	0.03	3.06	0.04	2.31	14.72	1.18	0.06	4.96	0.02
	Trades	8.03		5.20	8.04		5.21	4.23		2.24	4.25		2.27
2005	Overall	3.25	12.49	0.93	0.25	13.39	0.10	4.46	12.09	1.67	0.32	11.85	0.08
	Buy Signal	1.88	12.64	0.57	0.19	12.95	0.06	2.67	12.57	1.16	0.25	12.35	0.03
	Sell Signal	1.70	12.56	0.52	0.08	8.97	0.05	2.68	12.14	1.20	0.13	8.59	0.02
	Trades	7.94		4.71	7.95		4.72	4.34		2.06	4.36		2.08
2006	Overall	2.11	19.56	0.76	0.16	6.52	0.09	2.94	21.12	1.52	0.19	8.53	0.07
	Buy Signal	1.27	20.59	0.48	0.15	12.50	0.06	1.93	24.71	1.14	0.17	10.36	0.02
	Sell Signal	1.07	18.30	0.39	0.03	1.24	0.04	1.66	17.81	1.02	0.07	5.14	0.01
	Trades	6.54		4.40	6.54		4.40	3.10		1.98	3.12		1.99
2007	Overall	2.03	23.61	0.86	0.27	11.14	0.17	2.61	26.92	1.41	0.28	12.37	0.14
	Buy Signal	1.26	23.65	0.60	0.19	16.35	0.10	1.75	29.29	1.12	0.21	13.02	0.06
	Sell Signal	1.03	24.56	0.44	0.11	5.25	0.08	1.48	25.61	0.83	0.14	9.10	0.04
	Trades	6.05		4.04	6.04		4.05	2.97		1.98	2.99		1.99
Full	Overall	2.88	16.90	0.99	0.22	17.16	0.12	3.84	16.64	1.67	0.25	16.12	0.10
	Buy Signal	1.69	17.36	0.63	0.18	18.91	0.08	2.39	17.89	1.22	0.20	17.25	0.04
	Sell Signal	1.48	16.86	0.53	0.06	7.80	0.05	2.21	16.35	1.07	0.10	11.73	0.02
	Trades	7.53		4.75	7.53		4.76	3.99		2.15	4.01		2.17

The trading strategy is to buy when  $\Delta P_t < 0$  and sell when  $\Delta P_t > 0$  or to buy when  $\Delta P_t \leq -1$  and sell when  $\Delta P_t \geq 1$ . The buy and sell happens either at the signal time (“No Skip”) or one trade after the signal time (“Skip a Trade”). Each bond is allocated with \$100, and the reported mean profit is in dollars, per bond and day. The t-stat’s are clustered by bond and day. #Trades is the average number of trades, buy and sell, per bond and day. The median profit is the time-series average of the cross-sectional median.

in corporate bonds. To address this question, we devise the simple contrarian strategy that takes a long position in a bond when its price moves downward by more than a threshold, and takes a short position when the price moves upward by more than the threshold. This strategy entails supplying liquidity in the market. For comparison, we consider two values for the threshold, zero and one dollar. Given our asymmetry result for  $\gamma$ , as well as the differing implications of taking long or short positions in corporate bonds, we also report the profits for the short and long positions separately. Table 4 reports the trading profits using trade-by-trade data. For the full sample and for the trading strategy with a zero threshold in price changes, the average daily profit per bond is \$2.88 for a \$100 notional position. The robust  $t$ -stat (clustered by bond and day) for this profit is 16.90. On average, the bond is traded 7.53 times a day, indicating that on average there are four buy or sell signals for a bond on any given day. Separating the signal to buy and sell separately, the buy signal yields a slightly higher profit, which is consistent with our asymmetry result on price reversals.

It is important to note that only the market makers can trade at the price for which the signal is observed. A realistic trading strategy is therefore to skip a trade after the signal is observed and then buy and sell accordingly. As shown in the right panel of Table 4, the average profit of this trading strategy is markedly lower. For the full sample and for the trading strategy with the threshold of \$1, the average profit is 25 cents on a \$100 notional, and it carries a robust  $t$ -stat of 16.12. The buy signal generates a profit that is twice as large as the sell signal, consistent with the fact that the asymmetry remains important after skipping a trade.

## 4 Cross-Sectional Properties of Illiquidity

Our sample includes a broad cross-section of bonds, which allows us to examine the connection between our illiquidity measure  $\gamma$  and various bond characteristics, some of which are known to be linked to bond liquidity. The cross-sectional variation in our illiquidity measure  $\gamma$  and bond characteristics are reported in Table 5. We use daily data to construct yearly estimates for  $\gamma$  for each bond and perform yearly cross-sectional regressions on various bond characteristics. Reported in square brackets are the  $t$ -stat's calculated using the Fama and MacBeth (1973) standard errors.

We find that older bonds on average have higher  $\gamma$ , and the results are robust regardless of which control variables are used in the regression. On average, a bond that is one-year older is associated with an increase of 0.0726 in its  $\gamma$ , which accounts for 8% of the full-sample

Table 5: Cross-Sectional Variation in  $\gamma$  and Bond Characteristics

Cons	0.8795 [21.93]	0.8775 [23.28]	0.8671 [14.97]	0.8763 [23.03]	0.8830 [22.83]	0.8786 [22.66]	0.8908 [13.65]
Age	0.0726 [4.37]	0.0523 [6.18]	0.0517 [4.24]	0.0464 [4.97]	0.0326 [3.95]	0.0571 [5.98]	0.0811 [3.74]
Maturity	0.0708 [11.05]	0.0424 [19.59]	0.0401 [3.12]	0.0461 [11.04]	0.0481 [10.96]	0.0450 [9.80]	0.0672 [17.76]
ln(Issuance)	-0.1951 [-5.87]	-0.1373 [-3.23]	-0.1294 [-5.31]	-0.1368 [-3.57]	-0.0257 [-1.05]	-0.1551 [-3.81]	-0.2914 [-8.09]
Rating	0.0415 [8.05]	0.0164 [3.95]	0.0105 [1.58]	0.0232 [3.03]	0.0314 [3.35]	0.0190 [2.40]	0.0419 [4.32]
beta (stock)	0.4389 [4.34]	0.1536 [0.70]	0.24 [1.13]				
beta (bond)	-0.0237 [-0.90]	0.0351 [0.69]	0.0307 [0.59]				
sig(e)		0.4730 [4.37]		0.4581 [4.04]	0.4120 [3.82]	0.4397 [3.79]	
sig( $e^{\text{firm}}$ )			-0.0357 [-0.42]				
sig( $e^{\text{firm res}}$ )			0.6570 [11.31]				
Turnover				-0.0165 [-2.60]			
ln(Trd Size)					-0.2350 [-10.15]		
ln(#Trades)						0.0571 [1.66]	
Quoted BA $\gamma$							2.0645 [1.57]
R-sqd (%)	49.11	62.68	74.46	61.79	63.86	61.46	48.16

Yearly Fama-MacBeth regression with  $\gamma$  as the dependent variable. T-stats are reported in square brackets using Fama-MacBeth standard errors with serial correlations corrected using Newey-West. *Issuance* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Maturity* is the bond's time to maturity in years. *Turnover* is the bond's monthly trading volume as a percentage of its issuance. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *#Trades* is the bond's total number of trades in a month. *beta(stock)* and *beta(bond)* are obtained by regressing weekly bond returns on weekly returns on the CRSP value-weighted index and the Lehman US bond index, and *sig(e)* is the standard deviation of the residual. For firms with more than 10 bonds, *sig(e)* is further decomposed into a firm-level *sig( $e^{\text{firm}}$ )* and the residual *sig( $e^{\text{firm res}}$ )*. *Quoted BA  $\gamma$*  is the  $\gamma$  implied by the quoted bid-ask spreads. The sample size varies across specifications due to data availability.

average of  $\gamma$ . Given that the age of a bond has been widely used in the fixed-income market as a proxy for illiquidity, it is important that we establish this connection between our illiquidity measure  $\gamma$  and age. Similarly, we find that small bonds tend to have larger  $\gamma$ . We also find that bonds with longer time to maturity and lower credit rating typically have higher  $\gamma$ .

Using weekly bond returns, we also estimate, for each bond, its beta's on the aggregate stock- and bond-market returns, using the CRSP value-weighted index as a proxy for the stock market and the Lehman US bond index as a proxy for the bond market. We find that while  $\gamma$  cannot be explained by the cross-sectional variation in the bond beta, it is positively related to the stock beta. But this result goes away after adding the volatility,  $\text{sig}(e)$ , of the idiosyncratic component of the bond returns. Specifically, our results show that a bond with a higher idiosyncratic volatility has higher  $\gamma$ . For a sub-sample of our bonds whose issuer issues more than 10 bonds, we can further decompose the idiosyncratic volatility into a firm-level component and a bond-specific component. We find that the firm-specific component is not related to our illiquidity measure  $\gamma$ , while the bond-specific component exhibits a strong connection to our illiquidity measure. Interestingly, bond ratings are not significantly related to  $\gamma$  in this regression, although this could be because of the specific sub-sample.

Given that we have transaction-level data, we can also examine the connection between our illiquidity measure and bond trading activities. We find that, by far, the most interesting variable is the average trade size of a bond. In particular, bonds with smaller trade sizes have higher illiquidity measure  $\gamma$ . We will examine this issue more directly later in Section 6, where we break down our illiquidity measure by trades of different sizes.

Finally, we use the quoted bid-ask spreads for each bond in our sample to calculate the bid-ask spread implied autocovariance, or bid-ask implied  $\gamma$ . We find a positive relation between our  $\gamma$  measure and the  $\gamma$  measure implied by the quoted bid-ask spread. The regression coefficient is on average close to 2, which implies that one unit difference in  $\gamma$  implied by quoted bid-ask spreads gets amplified to twice the difference in our measure of  $\gamma$ . This coefficient, however, has a t-stat of 1.57, indicating that the magnitude of illiquidity captured by our  $\gamma$  measure is related but goes beyond the information contained in the quoted bid-ask spreads.

## 5 Time-Series Properties of Illiquidity

We next examine the time variation of illiquidity in the bond market. From Table 2, we see a steady reduction in the annual  $\gamma$  averaged over all bonds in our sample from 2003

through 2006. For example, the average  $\gamma$  using daily data is 1.0204 in 2003, which decreases monotonically to 0.7818 in 2006, suggesting an overall improvement of liquidity in the bond market from 2003 through 2006. During 2007, however, the average  $\gamma$  jumped back to 0.9222, reflecting worsening liquidity in the market. Our focus in this section is on the time variation beyond this simple time trend and its association with the conditions in the credit market. For this, we turn our attention to monthly fluctuations in the illiquidity measure  $\gamma$ .

## 5.1 Fluctuations in Market Illiquidity and Market Conditions

Monthly illiquidity measures  $\gamma$  are calculated for each bond using daily data within that month. Aggregating  $\gamma$  across all bonds, we plot in Figure 1 the time-series of the monthly aggregate illiquidity measure  $\gamma$  and the lower and upper bounds of its 95% confidence interval calculated using robust standard errors that take into account both time-series and cross-sectional correlations. It is clear that the aggregate  $\gamma$  exhibits significant time variation. After decreasing markedly but relatively smoothly during 2003 and the first half of 2004, it reversed its trend and started to climb up in late 2004 and then spiked in April/May 2005. This rise in  $\gamma$  coincides with the downgrade of Ford and GM to junk status in early May 2005, which rattled the credit market. The illiquidity measure  $\gamma$  quieted down somewhat through 2006, and then in August 2007, it rose sharply to an unprecedented level of  $\gamma$  since the beginning of in our sample. August 2007 is when the sub-prime mortgage crisis hit the market and the credit conditions in the U.S. worsened in a precipitous fashion. Compared with its value in late 2006, which was below 0.8, the quick rise to a level of 1.37 in August 2007 was quite dramatic. Even relative to July 2007, when the aggregate  $\gamma$  was at a level of 0.9727, the upward jump was an extreme event. For our sample, the standard deviation of monthly changes in aggregate  $\gamma$  is 0.1084, making the monthly jump from July to August a close to four-standard-deviation event. In September and October, the illiquidity measure  $\gamma$  came down somewhat. But then, on October 24, Merrill Lynch reported the biggest quarterly loss in its 93-year history after taking \$8.4 billion of write-downs, almost double the firm's forecast three weeks before. Less than a week later, the CEO of Merrill resigned. This was followed by Citigroup's announcement of write-downs of even larger magnitudes and the resignation of its CEO in early November. Not surprisingly, our illiquidity measure  $\gamma$  quickly jumped up again in November and December 2007 to an all time high level of 1.39.

The fact that  $\gamma$  increased drastically during the two periods of credit market turmoil indicates that not only does bond market illiquidity vary over time, but, more importantly,

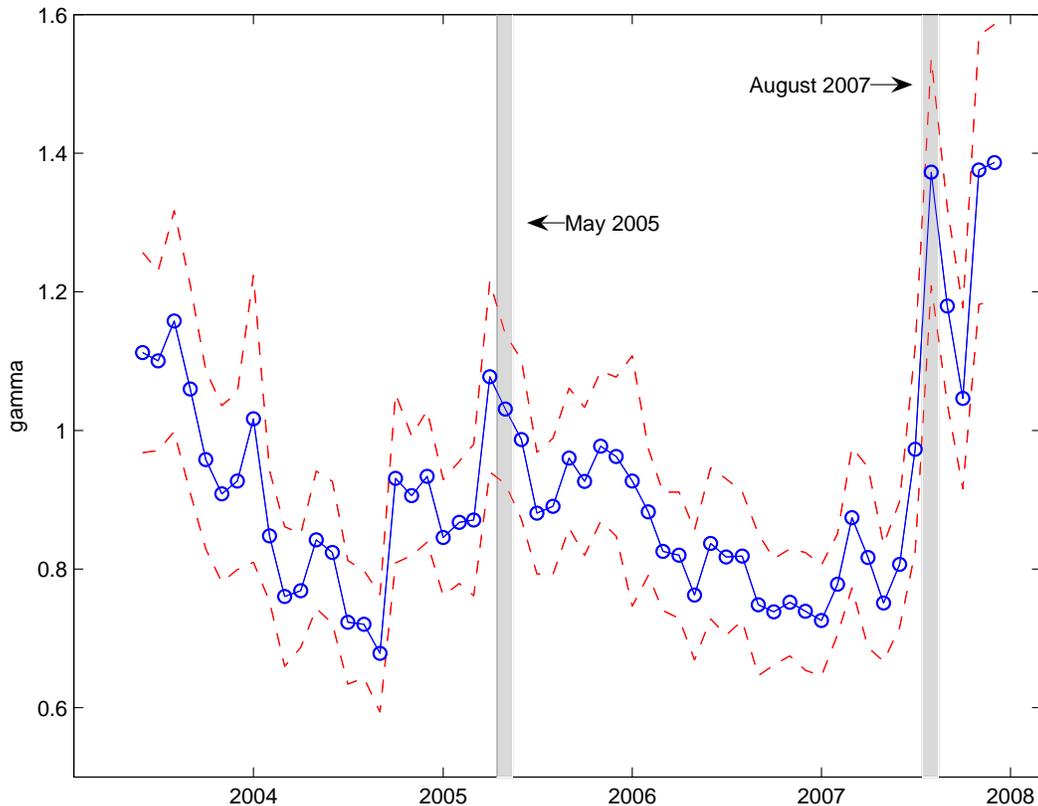


Figure 1: Monthly time-series of  $\gamma$ , averaged across all bonds. For each bond and month, daily data is used to estimate  $\gamma$ . The dashed lines are the upper and lower bounds of the 95% confidence interval, using robust standard errors clustered by bond and day.

it also varies together with the changing conditions of the market. In Figure 2, we plot the average  $\gamma$  along with several variables that are known to be linked to market conditions. To capture the credit market condition, we use default spread, measured as the difference in yields between AAA- and BBB-rated corporate bonds, using the Lehman US Corporate Intermediate indices. To capture the overall market condition, we use the CBOE VIX index, which is also known as the “fear gauge” of the market. To capture the overall volatility of the corporate bond market, we construct monthly estimates of annualized bond return volatility using daily returns to the Lehman US Investment Grade Corporate Index. Comparing the time variation in these variables with that of our aggregate  $\gamma$ , we have several observations.

First, there does not seem to be an obvious link between  $\gamma$  and the volatility of bond returns. In fact, regressing changes in  $\gamma$  on contemporaneous changes in the bond volatility, the  $t$ -stat of the slope coefficient is 0.71 and the R-squared of the regression is 0.45%. This is somewhat surprising. To the extent that volatility affects the risks in market making, one might expect a positive relation between illiquidity and return volatility. Second, contrasting

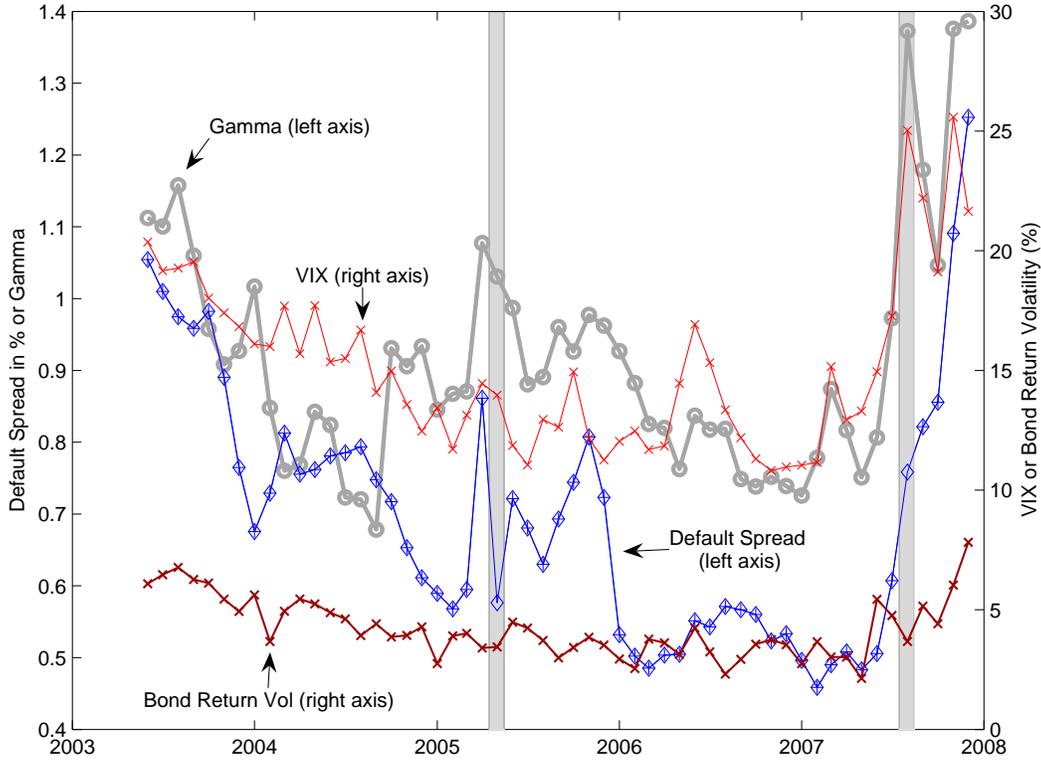


Figure 2: Monthly time-series of  $\gamma$  along with CBOE VIX index, default spread, and bond return volatility.

its lack of comovement with bond volatility, the aggregate  $\gamma$  comoves with VIX in a rather significant way. As shown in Table 6, regressing changes in  $\gamma$  on contemporaneous changes in VIX, we obtain a slope coefficient of 0.0312 with a  $t$ -stat of 3.45 (adjusted for serial correlation using Newey-West). The R-squared of the OLS regression is 39.53%, and the adjusted R-squared is 37.96%. Third, the aggregate  $\gamma$  also comoves with the default spread in a positive way. Regressing changes in  $\gamma$  on contemporaneous changes in the default spread, the slope coefficient is 0.4757 with a  $t$ -stat of 2.31 and the adjusted R-squared is 13.93%. By far, the connection between  $\gamma$  and the CBOE VIX index seems to be the strongest, which is quite interesting given that one variable is constructed using transaction-level corporate bond data and the other using index options.

We further examine in Table 6 the relation between monthly changes of our aggregate  $\gamma$  and the performance of the aggregate stock and bond markets in the previous month. We find that our aggregate  $\gamma$  typically increases after a poor performance in the aggregate bond or stock market. The slope coefficient is -0.0125 with a  $t$ -stat of -2.31 for the lagged stock return,

Table 6: Time Variation in  $\gamma$  and Market Variables

Cons	0.0035 [0.30]	0.0029 [0.33]	0.0066 [0.53]	0.0027 [0.33]	0.0159 [1.11]	0.0060 [0.48]	0.0126 [1.51]
Bond Volatility	0.0079 [0.71]						0.0063 [0.72]
$\Delta$ VIX		0.0312 [3.46]					0.0270 [3.02]
$\Delta$ Term Spread			0.1010 [1.57]				0.0210 [0.37]
$\Delta$ Default Spread				0.4757 [2.31]			0.2100 [1.57]
Lagged Stock Return					-0.0125 [-2.31]		-0.0087 [-3.07]
Lagged Bond Return						-0.0215 [-3.52]	-0.0102 [-1.26]
Adj R-sqrd (%)	-1.43	37.96	0.44	13.92	7.15	2.74	43.51

Monthly changes in  $\gamma$  regressed on monthly changes in bond index volatility, VIX, term spread, default spread, and lagged stock and bond returns. The Newey-West  $t$ -stats are reported in square brackets.

and is -0.0215 with a  $t$ -stat of -3.52 for the lagged bond return.<sup>10</sup> These results are consistent with the observation that liquidity is more likely to worsen following a down market.

The various market condition variables considered so far are closely inter-connected. To evaluate their relative importance, Table 6 reports the result of the multivariate regression using all variables simultaneously to explain the monthly changes in aggregate  $\gamma$ . Both VIX and lagged stock returns remain significant, but the default spread and lagged bond returns fail to remain significant. It is quite intriguing that two variables measured from the same market fail to explain our aggregate  $\gamma$ , while two other variables, one from index options and the other from the stock market, remain important.

## 5.2 Commonality in Illiquidity: Principal Component Analysis

Our analysis above reveals two important properties of  $\gamma$  as a measure of illiquidity for corporate bonds. First, there exists commonality in the illiquidity of individual bonds, which is reflected in the significant time variation in aggregate  $\gamma$ . Second, such common movements in

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<sup>10</sup>We use monthly excess stock and bond returns, with the one-month T-bill rate as the riskfree rate. It might also be interesting to observe that in the univariate regression, changes in VIX and lagged bond return have similar magnitudes of  $t$ -stat but very different R-squareds. This is because our  $t$ -stats are corrected for serial correlation using Newey-West. Our results imply that the regression residuals are positively autocorrelated in the regression involving changes in VIX, and negatively autocorrelated in the regression involving lagged bond return.

bond market illiquidity are closely connected with overall market conditions in an important way.

Table 7: Principal Component Analysis of  $\gamma$

Panel A: The Relative Importance of the PC's					
		PC1	PC2	PC3	PC4
% Explained		30.32	21.05	17.68	11.01
Cumulative %		30.32	51.37	69.05	80.06

Panel B: Factor Loadings on the First Four PC's					
size	age	PC1	PC2	PC3	PC4
1=small	1=young	0.2817	-0.0494	0.2421	0.3232
1	2	-0.0778	0.7943	0.5572	0.1330
1	3	0.3659	-0.0147	-0.2269	0.5218
2	1	0.1979	-0.1125	0.2104	0.1809
2	2	0.2930	-0.0135	-0.0905	0.4271
2	3	0.5682	-0.2876	0.4969	-0.4674
3	1	0.1130	-0.0228	0.0823	0.2106
3	2	0.1621	-0.0459	0.1455	0.0165
3=large	3=old	0.5420	0.5180	-0.5021	-0.3568

The principal component analysis is performed on 9 portfolios of bonds sorted by age and issuance size.

In order to further explore the commonalities in bond market illiquidity, we conduct a principal component analysis for the changes in the  $\gamma$  of individual bonds. In particular, we sort bonds by their age and issuance size into nine portfolios. We choose these two bond characteristics because they are known to be linked to bond liquidity. For each portfolio, we compute its aggregate  $\gamma$  by averaging the bond level  $\gamma$  (estimated monthly using daily data) across all bonds in the portfolio. Using monthly changes in the  $\gamma$ 's for the nine age and size sorted portfolios, we estimate the variance-covariance matrix and compute its eigenvalues. The results are summarized in Table 7.

The first principal component explains over 30% of the changes in the portfolio  $\gamma$ 's, while the next three principal components explain 21%, 18% and 11% of the variation, respectively. The first two principal components collectively explain over 51% of the variation in portfolio  $\gamma$ 's, and the first four principal components explain over 80%. Examining the factor loadings of the first four principal components, we find it difficult to link them to any economically meaningful variables. The first principal component, however, resembles our aggregate  $\gamma$ , with the exception of small-size and medium-age bonds whose factor loading is slightly negative.

## 6 Trade Size and Illiquidity

Since our illiquidity measure is based on transaction prices, a natural question is how it is related to the sizes of these transactions. In particular, are reversals in price changes stronger for trades of larger or smaller sizes? In order to answer this question, we consider the autocovariance of price changes conditional on different trade sizes.

### 6.1 Illiquidity Measure $\gamma$ Conditional on Trade Size

For a change in price  $P_t - P_{t-1}$ , let  $V_t$  denote the size of the trade associated with price  $P_t$ . The autocovariance of price changes conditional on trade size being in a particular range, say,  $R$ , is defined as

$$\text{Cov}(P_t - P_{t-1}, P_{t+1} - P_t, | V_t \in R), \quad (4)$$

where six brackets of trade sizes are considered in our estimation: ( $\$0$ ,  $\$5\text{K}$ ], ( $\$5\text{K}$ ,  $\$15\text{K}$ ], ( $\$15\text{K}$ ,  $\$25\text{K}$ ], ( $\$25\text{K}$ ,  $\$75\text{K}$ ], ( $\$75\text{K}$ ,  $\$500\text{K}$ ], and ( $\$500\text{K}$ ,  $\infty$ ), respectively. Our choice of the number of brackets and their respective cutoffs is influenced by the sample distribution of trade sizes. In particular, to facilitate the estimation of  $\gamma$  conditional on trade size, we need to have enough transactions within each bracket for each bond to obtain a reliable conditional  $\gamma$ .

For the same reason, we construct our conditional  $\gamma$  using trade-by-trade data. Otherwise, the data would be cut too thin at the daily level to provide reliable estimates of conditional  $\gamma$ . For each bond, we categorize transactions by their time- $t$  trade sizes into their respective bracket  $s$ , with  $s = 1, 2, \dots, 6$ , and collect the corresponding pairs of price changes,  $P_t - P_{t-1}$  and  $P_{t+1} - P_t$ . Grouping such pairs of price changes for each size bracket  $s$  and for each bond, we can estimate the autocovariance of the price changes, the negative of which is our conditional  $\gamma(s)$ .

Equipped with the conditional  $\gamma$ , we can now explore the link between trade size and illiquidity. In particular, does  $\gamma(s)$  vary with  $s$  and how? We answer this question by first controlling for the overall liquidity of the bond. This control is important as we find in Section 4 the average trade size of a bond is an important determinant of the cross-sectional variation of  $\gamma$ . So we first sort all bonds by their unconditional  $\gamma$  into quintiles and then examine the connection between  $\gamma(s)$  and  $s$  within each quintile.

As shown in Panel A of Table 8, for each  $\gamma$  quintile, there is a pattern of decreasing conditional  $\gamma$  with increasing trade size and the relation is monotonic for all  $\gamma$  quintiles. For example, quintile 1 consists of bonds with the highest  $\gamma$  and therefore the least liquid in our sample. The mean  $\gamma$  is 2.1129 for trade-size bracket 1 (less than  $\$5\text{K}$ ) but it decreases to

Table 8: Variation of  $\gamma$  with Trade Size

Panel A: Lag=1								
$\gamma$ Quint	trade size=	1	2	3	4	5	6	1 - 6
1	Mean	2.1129	1.6404	1.4614	1.2703	0.8477	0.6171	1.4292
	Median	1.8844	1.4902	1.3459	1.2088	0.7812	0.4835	1.3132
	Robust t-stat	13.55	10.09	9.18	9.20	8.44	6.27	10.72
2	Mean	1.0974	0.9468	0.8440	0.6748	0.3330	0.1906	0.9064
	Median	0.9990	0.8773	0.7962	0.6274	0.3138	0.1716	0.8272
	Robust t-stat	10.49	9.42	9.53	10.44	13.35	11.54	8.92
3	Mean	0.6282	0.5545	0.4882	0.3544	0.1726	0.0804	0.5493
	Median	0.5423	0.4989	0.4577	0.3327	0.1646	0.0723	0.4656
	Robust t-stat	8.43	12.98	13.46	14.00	15.71	12.15	7.49
4	Mean	0.3881	0.3217	0.2662	0.1814	0.0971	0.0424	0.3472
	Median	0.3242	0.2831	0.2308	0.1673	0.0893	0.0394	0.2879
	Robust t-stat	8.25	12.77	12.98	14.47	16.70	12.52	7.46
5	Mean	0.2172	0.1652	0.1327	0.0895	0.0469	0.0202	0.1976
	Median	0.1957	0.1490	0.1167	0.0833	0.0430	0.0175	0.1755
	Robust t-stat	10.19	13.72	11.73	15.34	17.53	15.35	9.39

Panel B: Lag=2								
$\gamma$ Quint	trade size=	1	2	3	4	5	6	1 - 6
1	Mean	0.3652	0.1774	0.1784	0.1622	0.1164	0.0936	0.3497
	Median	0.3418	0.1995	0.1754	0.1341	0.1016	0.0495	0.2688
	Robust t-stat	7.57	6.72	6.19	6.11	4.50	3.52	7.70
2	Mean	0.1997	0.1416	0.1043	0.0842	0.0566	0.0195	0.1806
	Median	0.1503	0.0927	0.0865	0.0644	0.0410	0.0155	0.1275
	Robust t-stat	8.37	6.06	7.49	7.12	8.19	3.84	7.70
3	Mean	0.0961	0.0721	0.0509	0.0420	0.0226	0.0086	0.0878
	Median	0.0782	0.0542	0.0358	0.0285	0.0183	0.0060	0.0702
	Robust t-stat	7.32	7.92	7.39	5.78	6.45	2.92	6.66
4	Mean	0.0647	0.0484	0.0341	0.0257	0.0083	0.0052	0.0599
	Median	0.0474	0.0318	0.0191	0.0160	0.0066	0.0027	0.0432
	Robust t-stat	6.75	7.88	6.88	8.08	5.50	2.85	6.20
5	Mean	0.0317	0.0219	0.0126	0.0122	0.0065	0.0016	0.0301
	Median	0.0254	0.0146	0.0103	0.0084	0.0043	0.0014	0.0231
	Robust t-stat	7.48	7.11	4.87	5.77	6.91	2.21	7.05

Trade size is categorized into 6 groups with cutoffs of \$5K, \$15K, \$25K, \$75K, and \$500K. For Lag=1,  $\gamma = -\text{Cov}(P_t - P_{t-1}, P_{t+1} - P_t)$ , and for Lag=2,  $\gamma = -\text{Cov}(P_t - P_{t-1}, P_{t+2} - P_{t+1})$ . In both cases,  $\gamma$  is calculated conditioning on the trade size associated with  $P_t$ . Bonds are sorted by their “unconditional”  $\gamma$  into quintiles, and the variation of  $\gamma$  by trade size is reported for each quintile group. The trade-by-trade data is used in the calculation. For the daily data, the results are similar but stronger for Lag=1, and weaker and statistically insignificant for Lag=2.

0.6171 for trade-size bracket 6 (greater than \$500K). The mean difference in  $\gamma$  between the trade-size bracket 1 and 6 is 1.4292 and has a robust  $t$ -stat of 10.72. Likewise, for quintile 5, which consists of bonds with the lowest  $\gamma$  measure and therefore are the most liquid, the same pattern emerges. The average value of  $\gamma$  is 0.2172 for the smallest trades and then decreases monotonically to 0.0202 for the largest trades. The difference between the two is 0.1976, with a robust  $t$ -stat of 9.39, indicating that the conditional  $\gamma$  between small and large size trades remains significant even for the most liquid bonds. To check the potential impact of outliers, we also report the median  $\gamma$  for different trade sizes. Although the magnitudes are slightly smaller, the general pattern remains the same.

We next examine the connection between trade sizes and conditional  $\gamma_\tau$  for  $\tau = 2$ . As introduced in equation (3), we use  $\gamma_2$  to estimate the persistence of mean-reversion using price changes after skipping a trade. The conditional version of  $\gamma_2$  can be calculated as the negative of  $\text{Cov}(P_t - P_{t-1}, P_{t+2} - P_{t+1} \mid V_t \in \text{bracket } s)$ , for  $s = 1, 2, \dots, 6$ . The empirical estimates are reported in Panel B of Table 8. Again, we see a quite robust pattern of decreasing  $\gamma_2(s)$  with increasing trade size bracket  $s$ , indicating that even skipping a trade, there are weaker reversals after large-size trades and stronger reversals after small-size trades.

Overall, our results demonstrate a clear negative relation between trade sizes and our illiquidity measure. The interpretation of this result, however, requires caution. It would be simplistic to infer from this pattern that larger trades face less illiquidity or have less impact on prices. It is important to realize that both trade sizes and prices are endogenous variables. Their relation arises from an equilibrium outcome in which traders optimally choose their trading strategies, taking into account the price dynamics and the impact of their trades. For example, when liquidity varies over time, traders may optimally break up their trades when liquidity is low. Consequently, during less liquid times, we see more small trades and a larger illiquidity measure  $\gamma$ .

## 6.2 Time Variation of Trade-Size Distribution and Illiquidity

The connection between illiquidity and trade size can be further investigated from a time-series perspective. In particular, fluctuations in the distributions of large and small trades can in principle be associated with fluctuations in  $\gamma$ . In Figure 3, we plot the fractions of trades belonging to the smallest (less than \$5K) and the largest (greater than \$500K) size brackets, respectively. Also plotted in Figure 3, is the total number of trades in each month aggregated across all bonds in our sample. Surprisingly, the total number of trades has been steadily

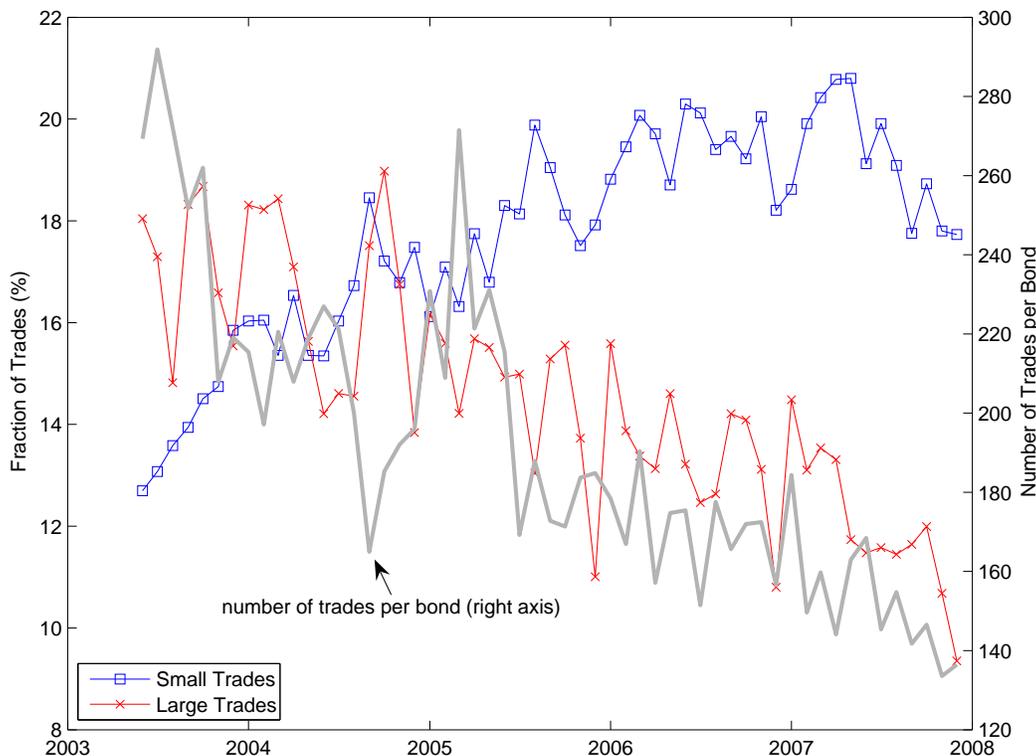


Figure 3: Monthly time-series of the numbers small and large trades as percentages of total numbers of trades. Trade size is categorized into six groups with cutoffs of \$5K, \$15K, \$25K, \$75K, and \$500K. In this figure, small trades belong to group 1, and large trades belong to group 6. For comparison, the total number of trades per month and per bond is also plotted.

decreasing over time during our sample period. There are also fluctuations in the fractions of small and large trades.

Before proceeding, we make several observations about our sample selection and its possible influence in our analysis. We focus only on Phase I and II bonds in TRACE to maintain a reasonably balanced sample. We did not include bonds that were included only after Phase III, which was fully implemented on February 7, 2005. Consequently, new bonds issued after that date were excluded from our sample, even though some of them would have been eligible for Phase II had they been issued earlier. As a result, we have a pool of aging bonds. Given that larger trades are more common for younger bonds, the aging population itself would produce a decreasing trend for the fraction of large trades and an increasing trend for small trades. For our sample, this impact kicks in after February 7, 2005 and accumulates gradually over time, which leads to the time trend in the fraction of large- and small-size trades seen

in Figure 3.<sup>11</sup> However, when we allow new bonds that are similar to Phase II bonds but were issued too late to be included in Phase II, the sample of bonds becomes more balanced in age distribution. Using this more balanced sample, we find that the downward trends in the fractions of large and small size trades are no longer present. Nonetheless, the decreasing trend of the total number of trades remains similar to that in Figure 3.

Table 9: Time Variations of  $\gamma$  and Fractions of Large and Small Trades

s=1 (small)	0.8339						0.6422
	[5.23]						[3.99]
s=2		0.1616					
		[1.57]					
s=3			0.2109				
			[1.54]				
s=4				-0.0068			
				[-0.06]			
s=5					-0.1001		
					[-0.94]		
s=6 (large)						-0.7540	-0.6171
						[-6.96]	[-5.73]
R-sqd (%)	0.18	0.01	0.01	0.00	0.00	0.21	0.31

OLS regressions of  $\Delta\gamma_{it}$  on  $\Delta\pi_{it}^s$ , where, for bond  $i$  and month  $t$ ,  $\gamma_{it}$  is estimated using daily data, and  $\pi_{it}^s$  is the fraction of trades in size group  $s$ . The six size groups are categorized by trade size with cutoffs of \$5K, \$15K, \$25K, \$75K, and \$500K. The t-stat's are reported in square brackets with robust standard errors clustered by bond and month. The R-sqd is a pool R-squared including cross-sectional and time-series variation.

Aside from the time trends, we now investigate the connection between changes in  $\gamma$  and changes in the fractions of varying trade sizes. For this, the impact of the aging population should be quite limited. Specifically, we regress monthly changes in unconditional  $\gamma$  on monthly changes in the fractions of trades in different size brackets. This regression is pooled across all bonds and months, with  $t$ -stat's calculated using robust standard errors clustered by bond and month. The results are give in Table 9. We find that changes in the fractions of large and small trades are significantly connected to changes in the bond illiquidity measure  $\gamma$ , while changes in the fraction of trades in the other size brackets do not have a significant impact. Specifically, increasing the fraction of small trades by 0.10 is associated with an in-

<sup>11</sup>In our time-series analysis, we observe an overall trend of decreasing  $\gamma$ . Given that the overall population of our sample is aging and  $\gamma$  is positively related to age, the overall downward trend would have been more pronounced had we been able to maintain a more balanced pool. The other aspects of our time-series analysis are not affected since the sudden increases in aggregate  $\gamma$  during crises are too large to be explained by the slow aging process, and our regression results are based on regressing changes on changes.

crease of 0.083 in the illiquidity measure, while increasing the fraction of large trades by 0.10 is associated with a reduction of 0.075 in  $\gamma$ . Both effects are statistically significant, although, with an R-squared of 0.31%, they can only explain a very small fraction of the time-series and cross-sectional variations of monthly changes in  $\gamma$ .

### 6.3 The Time-Series Properties of Conditional $\gamma$

As an extension to the time-series analysis of the aggregate  $\gamma$  in Section 5, we examine how the conditional  $\gamma$ 's for various trade sizes vary over time. In particular, it is interesting to examine whether or not crises affect the illiquidity measure for small and large trades alike.

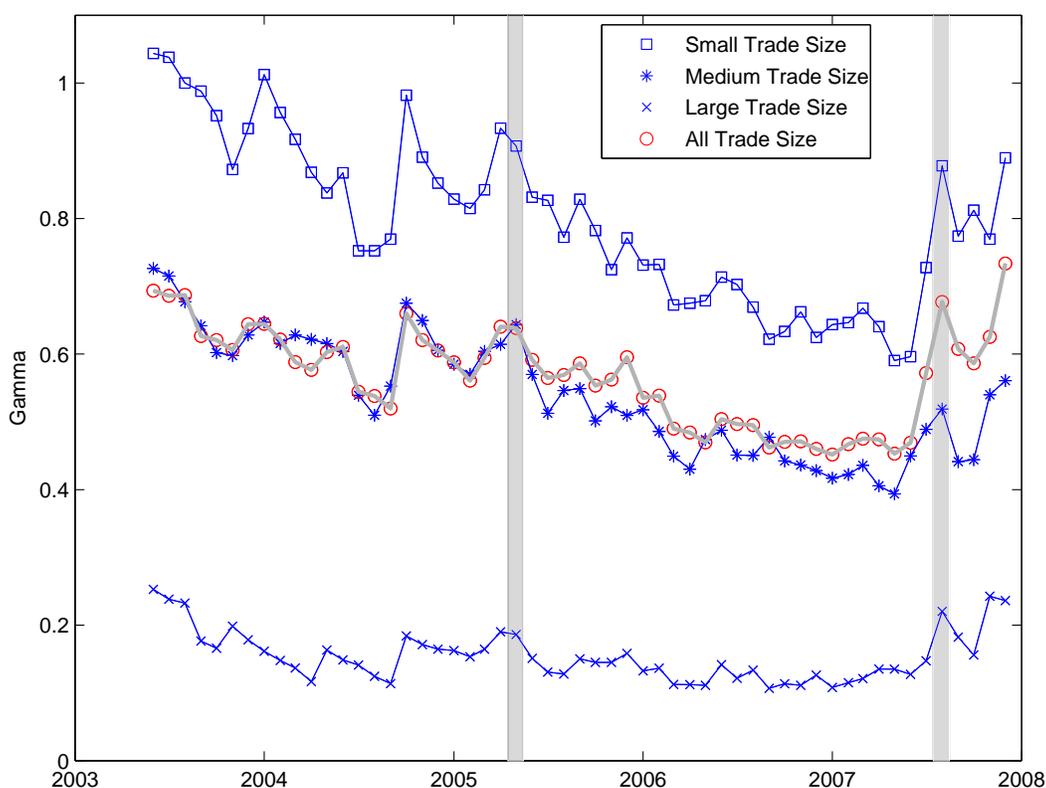


Figure 4: Monthly time-series of cross-sectional averages of  $\gamma$ , estimated using small, medium, and large trades. Trade size is categorized into six groups with cutoffs of \$5K, \$15K, \$25K, \$75K, and \$500K. Small trades in this figure are those in groups 1 and 2, medium trades are in groups 3 and 4, and large trades are in group 5 and 6. For each bond and month, the trade-by-trade data is used to estimate  $\gamma$ .

Figure 4 plots the conditional  $\gamma$  for three trade-size brackets, the smallest ( $s = 1, 2$ ), the medium ( $s = 3, 4$ ) and the largest ( $s = 5, 6$ ). It also plots the unconditional  $\gamma$ , using trade-by-trade data, which is lower in magnitude than the unconditional  $\gamma$  plotted in Figure 1

using daily data. We find that the conditional  $\gamma$ 's for different trade sizes tend to show the same fluctuations, rising together quite significantly during periods of market turbulence in 2005 and 2007. While the  $\gamma$  for smaller trades tend to rise more in magnitude during market crises, the  $\gamma$  for the larger trades increases more sharply as a percentage of its overall level. Economically, the liquidity for large trades can be more important. Thus, its drying up during bad times can have a larger impact on the market.

## 7 Bond Yield Spreads and Illiquidity

In this section, we examine the pricing implications of bond illiquidity. We focus on the bond yield spread, which is the difference between the corporate bond yield and the Treasury bond yield of the same maturity. For Treasury yields, we use the constant maturity rate published by the Federal Reserve and use linear interpolation whenever necessary. We perform monthly cross-sectional regressions of the yield spreads on the illiquidity measure  $\gamma$ , along with a set of control variables. We first report our results for our full sample of bonds, including both investment-grade and junk bonds, and then for only investment-grade bonds. Given that the Phase I and II bonds in TRACE are predominantly investment grade, this sub-sample analysis is important for us to rule out the possibility that our result is driven just by a handful of unrepresentative junk bonds.

### 7.1 The Full Sample with Both Investment-Grade and Junk Bonds

The results are reported in Table 10, where the  $t$ -stat's are calculated using the Fama-MacBeth standard errors with serial correlation corrected using Newey and West (1987). To include callable bonds in our analysis, which constitute a large portion of our sample, we use a callable dummy, which is one if a bond is callable and zero otherwise. We exclude all convertible and puttable bonds from our analysis. In addition, we also include three rating dummies for A, Baa, and junk ratings, respectively. The first column in Table 10 shows that the average yield spread of the Aaa and Aa bonds in our sample is 70.62 bps, relative to which the A bonds are 18.71 bps higher, Baa bonds are 77.21 bps higher, and junk bonds are 466.84 bps higher.

As reported in the second column of Table 10, adding  $\gamma$  to the regression does not bring much change to the relative yield spreads across ratings. This is to be expected since  $\gamma$  should capture more of a liquidity effect, and less of a fundamental risk effect, which is reflected in the differences in ratings. More importantly, we find that the coefficient on  $\gamma$  is 0.4220 with a  $t$ -stat of 3.95. This implies that for two bonds in the same rating category, if one

Table 10: Bond Yield Spread and Illiquidity Measure  $\gamma$ 

Intercept	0.7062 [11.10]	0.4585 [5.11]	-0.3343 [-1.12]	-0.4736 [-1.44]	-0.6808 [-1.81]	-0.9394 [-2.46]	-0.5688 [-1.54]	-1.2853 [-2.76]	0.8224 [5.94]	-0.5988 [-1.73]
$\gamma$	0.4220 [3.95]		0.4264 [5.85]	0.3595 [3.78]	0.3863 [3.96]	0.3475 [3.65]	0.3319 [3.79]	0.4684 [3.90]		
Equity Vol		0.0652 [3.34]	0.0593 [3.03]	0.0588 [3.04]	0.0549 [2.96]	0.0591 [3.05]	0.0558 [2.94]	0.0613 [2.71]		
Age			0.0357 [6.57]	0.0329 [6.39]	0.0329 [6.39]	0.0387 [5.79]				
Maturity			0.0049 [0.65]	0.0007 [0.09]	0.0056 [0.75]	0.0056 [0.80]				
ln(Issuance)			0.0124 [0.56]	0.0289 [1.48]	0.0257 [0.99]	-0.0822 [-3.58]				
Turnover			0.0298 [5.51]							
ln(Trd Size)										
ln(#Trades)								0.2582 [5.52]		
Quoted B/A Spread										
Call Dummy	0.0090 [0.10]	0.0050 [0.06]	-0.0630 [-0.99]	-0.0845 [-1.49]	-0.0442 [-1.13]	-0.0174 [-0.47]	-0.0447 [-1.13]	-0.0027 [-0.08]	-0.1655 [-2.94]	-0.1001 [-2.39]
A Dummy	0.1871 [3.70]	0.1710 [3.98]	0.0371 [0.60]	0.0316 [0.54]	0.0214 [0.35]	0.0037 [0.06]	0.0296 [0.49]	0.0639 [1.16]	0.2716 [9.36]	0.0866 [1.20]
Baa Dummy	0.7721 [5.62]	0.6061 [6.54]	0.5056 [5.22]	0.3757 [3.34]	0.3113 [2.83]	0.2374 [1.99]	0.3314 [2.96]	0.3750 [3.61]	1.0846 [8.38]	0.5246 [5.06]
Junk Dummy	4.6684 [10.19]	4.0201 [9.91]	2.7492 [13.29]	2.2182 [9.73]	2.2279 [9.64]	2.1747 [9.10]	2.2386 [9.81]	2.2797 [10.20]	4.7913 [6.52]	2.2005 [10.62]
R-sqd (%)	27.41	31.37	50.60	55.34	56.59	57.50	56.81	57.94	29.25	55.48

Monthly Fama-MacBeth cross-sectional regression with the bond yield spread as the dependent variable. The t-stats are reported in square brackets calculated using Fama-MacBeth standard errors with serial correlation corrected using Newey-West. The reported R-squareds are the time-series averages of the cross-sectional R-squareds.  $\gamma$  is the monthly estimate of illiquidity measure using daily data. *Equity Vol* is estimated using daily equity returns of the bond issuer. *Issuance*, *Turnover*, *Trd Size*, and *#Trades* are as defined in Table 5. *Call Dummy* is one if the bond is callable and zero otherwise. Convertible and puttable bonds are excluded from the regression. The sample period is from April 2003 through December 2007, except for the regressions involving quoted bid-ask spreads, which we have data only through December 2006.

bond, presumably less liquid, has a  $\gamma$  that is higher than the other by 1, the yield spread of this bond is on average 42.20 bps higher than the other. To put an increase of 1 in  $\gamma$  in context, the cross-sectional standard deviation of  $\gamma$  is on average 0.9943 in our sample. From this perspective, our illiquidity measure  $\gamma$  is economically important in explaining the cross-sectional variation in average bond yields.

To control for the fundamental risk of a bond above and beyond what is captured by the rating dummies, we use equity volatility estimated using daily equity returns of the bond issuer. Effectively, this variable is a combination of the issuer's asset volatility and leverage. We find this variable to be important in explaining yield spreads. As shown in the third column of Table 10, the slope coefficient on equity volatility is 0.0652 with a  $t$ -stat of 3.34. That is, a ten percentage point increase in the equity volatility of a bond issuer is associated with a 65.2 bps increase in the bond yield. (Later in this section when we focus only on investment-grade bonds, we will find that this control becomes less important.) While adding  $\gamma$  improves the cross-sectional R-squared from a time-series average of 27.41% to 31.37%, adding equity volatility improves the R-squared to 50.6%. Such R-squared's, however, should be interpreted with caution since it is a time-series average of cross-sectional R-squared, and does not take into account the cross-sectional correlations in the regression residuals. By contrast, our reported Fama-MacBeth  $t$ -stat's do and both variables are comparable in terms of their statistical significance. It is also interesting to observe that by adding equity volatility, the magnitudes of the rating dummies decrease significantly. This is to be expected since both equity volatility and rating dummies are designed to control for the bond's fundamental risk.

When used simultaneously to explain the cross-sectional variation in bond yield spreads, both  $\gamma$  and equity volatility are significant, with the slope coefficients for both remaining more or less the same as before. This implies a limited interaction between the two variables, which is to be expected since the equity volatility is designed to pick up the fundamental information about a bond while  $\gamma$  is to capture its liquidity information. Moreover, the statistical significance of our illiquidity measure  $\gamma$  increases to a  $t$ -stat of 5.85, indicating a closer connection between yield spreads and  $\gamma$  after controlling for the fundamental risk.

Adding three bond characteristics — age, maturity and issuance — to compete with  $\gamma$ , we find that the positive connection between  $\gamma$  and average bond yield spreads remains robust. Both bond age and bond issuance are known to be linked to liquidity.<sup>12</sup> Our results show that bond age remains an important liquidity variable above and beyond our  $\gamma$  measure. In

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<sup>12</sup>See, for example, Houweling, Mentink, and Vorst (2003) and additional references therein.

particular, a bond that is one year older is associated with an increase of 3.57 bps in average yield spreads.

Including the bond trading variables reveals that bonds with higher turnover and a large number of trades have higher average yield spreads. The slope coefficients for both variables are statistically significant. If one believes that more frequently traded bonds are more liquid, then this result would be puzzling. It is, however, arguable whether this variable actually captures the liquidity of a bond. We also find that bonds with higher average trade size have lower yield spreads, although the statistical significance is relatively weak (the  $t$ -stat is -1.95). This result seems to be consistent with a liquidity explanation. Overall, these variables are important control variables for us, since they are shown in Table 5 to be connected with our illiquidity measure  $\gamma$ . Our results show that these variables do not have a strong impact on the positive relation between our illiquidity measure  $\gamma$  and average yield spreads.

Finally, we examine the relative importance of the quoted bid-ask spreads and our illiquidity measure  $\gamma$ . This portion of our result encompasses a shorter time period, since we only have quoted bid-ask spread data through 2006. As shown in the last two columns of Table 10, the quoted bid-ask spreads are not significantly related to average yield spreads. Using both the quoted bid-ask spreads and our illiquidity measure  $\gamma$ , we find a robust result for  $\gamma$  and still statistically insignificant result for the quoted bid-ask spread. This aspect of our result is curious since Chen, Lesmond, and Wei (2007) report a positive relation between the quoted bid-ask spreads and yield spreads. We find that this discrepancy is due to the junk bonds in our sample. This is not surprising given that the Phase I and II bonds in TRACE are predominantly investment grades, and the junk bonds covered by TRACE could be an unrepresentative pool. To make sure that our result is not driven by a handful of unrepresentative junk bonds, we next repeat our analysis focusing only on investment grade bonds.

## 7.2 The Sub-Sample with Investment-Grade Bonds Only

The results for investment-grade bonds only are reported in Table 11. We find that our illiquidity measure  $\gamma$  remains important. Compared with the full sample result, the magnitude of the slope coefficient decreases from 0.4420 to 0.3538, which is to be expected since having junk bonds in the sample creates a larger spread among yields. But given how large an average spread junk bonds have relative to investment-grade bonds, this reduction in the slope coefficient seems rather small. This new slope coefficient implies that for two investment-grade bonds in the same rating category, if one bond has a  $\gamma$  that is higher than the other by 1, the

yield spread of this bond is on average 35.38 bps higher than the other. Within the investment grades, this difference in yields is quite large. Moreover, we also find that the  $t$ -stat of the slope coefficient is now 7.02, which is considerably higher than the full sample result. This implies a sharper connection between  $\gamma$  and the average yield spreads for investment-grade bonds.

Table 11: Yield Spread and  $\gamma$ , for Investment Grades Only

Intercept	0.7095	0.5172	0.5128	0.3217	0.2631	0.3895	0.1069
	[9.94]	[6.43]	[6.53]	[3.43]	[1.28]	[7.83]	[0.92]
$\gamma$		0.3548		0.3464	0.2271		0.3241
		[7.02]		[7.02]	[4.01]		[6.00]
Equity Vol			0.0101	0.0107	0.0103		0.0130
			[2.61]	[2.99]	[2.74]		[2.56]
Age					0.0186		
					[3.96]		
Maturity					0.0156		
					[3.86]		
ln(Issuance)					-0.0030		
					[-0.15]		
Quoted B/A Spread						0.8533	0.3160
						[4.93]	[3.53]
Call Dummy	-0.0070	-0.0787	0.0317	-0.0514	-0.0668	-0.1501	-0.1113
	[-0.14]	[-1.99]	[0.70]	[-1.47]	[-2.49]	[-3.43]	[-3.72]
A Dummy	0.1971	0.2029	0.1586	0.1628	0.1634	0.2499	0.2147
	[4.47]	[5.36]	[4.07]	[4.97]	[5.28]	[7.88]	[7.16]
Baa Dummy	0.7875	0.6971	0.7059	0.6264	0.5831	0.8680	0.7137
	[6.84]	[7.71]	[7.82]	[8.68]	[9.11]	[8.22]	[10.46]
R-sqd (%)	15.40	30.26	18.41	32.63	36.31	27.66	40.77

Monthly cross-sectional regressions with bond yield spread as the dependent variable. Only investment-grade bonds are included in the regression. The reported estimates are the time-series averages of the cross-sectional regression coefficients. The  $t$ -stats, reported in square brackets, are calculated using Fama-MacBeth standard errors with serial correlation corrected using Newey-West. The reported R-squareds are the time-series averages of the cross-sectional R-squareds. See Table 10 for definitions of independent variable.

The result for equity volatility is much weaker now that we focus just on the investment-grade bonds. It also has a smaller cross-sectional explanatory power than our  $\gamma$  measure, as well as a weaker statistical significance. When it is used together with our  $\gamma$  measure, there is hardly any change in the slope coefficient for  $\gamma$ . Adding age, maturity and issuance to the regression, however, the slope coefficient for  $\gamma$  is now at 0.2271 with a  $t$ -stat of 4.01. The result is weaker, but remains important both economically and statistically. It is interesting to note that age remains an important variable. Bond maturity becomes important for this

sub-sample but not earlier for the full sample.

Turning to the quoted bid-ask spreads, we find a positive and significant relation between the quoted bid-ask spreads and yield spreads, a result that has been documented by Chen, Lesmond, and Wei (2007). When the quoted bid-ask spreads and our  $\gamma$  measure are used together, both variables remain important. The slope coefficient on  $\gamma$  is 0.3241 with a  $t$ -stat of 6, and the slope coefficient on the quoted bid-ask spread is 0.3160 with a  $t$ -stat of 3.53. The fact that our  $\gamma$  measure remains important is another indication that it captures information about illiquidity above and beyond the information contained in the quoted bid-ask spread. Moreover, in explaining the cross-sectional variation in average yield spreads, our measure of illiquidity is more important both statistically and economically. To be more specific, for the investment grade bonds in our sample, the cross-sectional standard deviation of our  $\gamma$  measure is on average 0.8792, and the cross-sectional standard deviation of the quoted bid-ask spreads is on average 0.1730. So a one-standard-deviation difference in  $\gamma$  generates a difference of 28.50 bps in average yields, while a one-standard-deviation difference in the quoted bid-ask spread generates a difference of only 5.5 bps in average yields.

## 8 Illiquidity and Bid-Ask Spread

It is well known that the bid-ask spread can lead to negative autocovariance in price changes. For example, using a simple specification, Roll (1984) shows that when transactions prices bounce between bid and ask prices, depending on whether they are sell or buy orders from customers, their changes exhibit negative autocovariance even when the “underlying value” follows a random walk. Thus, it is important to ask whether or not the negative autocovariances documented in this paper are simply a reflection of bid-ask bounce. Using quoted bid-ask spreads, we show in Table 2 that the associated bid-ask bounce can only generate a tiny fraction of the empirically observed autocovariance in corporate bonds. Quoted spreads, however, are mostly indicative rather than binding. Moreover, the structure of the corporate bond market is mostly over-the-counter, making it even more difficult to estimate the actual bid-ask spreads.<sup>13</sup> Thus, a direct examination of how bid-ask spreads contribute to our illiquidity measure  $\gamma$  is challenging.

We can, however, address this question to certain extent by taking advantage of the results

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<sup>13</sup>The corporate bond market actually involves different trading platforms, which provide liquidity to different clienteles. In such a market, a single bid-ask spread can be too simplistic in capturing the actual spreads in the market.

by Edwards, Harris, and Piwowar (2007) (EHP hereafter). Using a more detailed version of the TRACE data that includes the side on which the dealer participated, they provide estimates of effective bid-ask spreads for corporate bonds. To examine the extent to which our illiquidity measure  $\gamma$  can be explained by the estimated bid-ask spread, we use our illiquidity measure  $\gamma$  to compute the implied bid-ask spreads, and compare them with the estimated bid-ask spreads reported by EHP. The actual comparison will not be exact, since our sample of bonds is different from theirs. Later in the section, we will discuss how this could affect our analysis.

It is first instructive to understand the theoretical underpinning of how our estimate of  $\gamma$  relates to the estimate of bid-ask spreads in EHP. In the Roll (1984) model, the transaction price  $P_t$  takes the form of equation (1), in which  $P$  is the sum of the fundamental value and a transitory component. Moreover, the transitory component equals to  $\frac{1}{2} S q_t$  in the Roll model, with  $S$  being the bid-ask spread and  $q_t$  indicating the direction of trade. Specifically,  $q$  is  $+1$  if the transaction is buyer initiated and  $-1$  if it is seller initiated, assuming that the dealer takes the other side. More specifically, in the Roll model, we have

$$P_t = F_t + \frac{1}{2} S q_t. \quad (5)$$

If we further assume that  $q_t$  is *i.i.d.* over time, the autocovariance in price change then becomes  $-(S/2)^2$ , or  $\gamma = (S/2)^2$ . Conversely, we have

$$S_{\text{Roll}} = 2 \sqrt{\gamma}, \quad (6)$$

where we call  $S_{\text{Roll}}$  the implied bid-ask spread.

EHP use an enriched Roll model, which allows the spreads to depend on trade sizes. In particular, they assume

$$P_t = F_t + \frac{1}{2} S(V_t) q_t, \quad (7)$$

where  $V_t$  is the size of the trade at time  $t$ .<sup>14</sup> Since the dataset used by EHP also contains information about  $q_t$ , they directly estimate the first difference of equation (7), assuming a factor model for the increments of  $F_t$ .

Table 12 reproduces the results of EHP, who estimate percentage bid-ask spreads for average trade sizes of \$5K, \$10K, \$20K, \$50K, \$100K, \$200K, \$500K and \$1M. The cross-sectional medians of the percentage bid-ask spreads are 1.20%, 1.12%, 96 bps, 66 bps, 48

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<sup>14</sup>The model EHP use has an additional feature. It distinguishes customer-dealer trades from dealer-dealer trades. The spread they estimate is for the customer-dealer trades. Thus, in (7), we simply do not identify dealer-dealer trades. This decreases our estimate of  $\gamma$  relative to EHP since we are including inter-dealer trades which have a smaller spread than customer-dealer trades.

Table 12: Implied and Estimated Bid-Ask Spreads

trade size	Our period			EHP subperiod			EHP		
	#bonds	mean	med	#bonds	mean	med	EHP size	mean	med
$\leq 7,500$	1,201	2.05	1.76	956	2.06	1.81	5K	1.50	1.20
(7500, 15K]	1,209	1.82	1.61	1,069	1.98	1.79	10K	1.42	1.12
(15K, 35K]	1,211	1.69	1.41	1,067	1.81	1.60	20K	1.24	0.96
(35K, 75K]	1,210	1.43	1.15	925	1.39	1.20	50K	0.92	0.66
(75K, 150K]	1,183	1.13	0.90	831	1.00	0.89	100K	0.68	0.48
(150K, 350K]	1,088	0.82	0.70	701	0.67	0.66	200K	0.48	0.34
(350K, 750K]	1,126	0.69	0.59	801	0.60	0.57	500K	0.28	0.20
$> 750K$	1,144	0.64	0.55	982	0.52	0.54	1,000K	0.18	0.12

The bid-ask spreads are calculated as a percentage of the market value of the bond and are reported in percentages. The EHP bid-ask spread estimates are from Table 4 of Edwards, Harris, and Piwowar (2007), and the EHP subperiod is Jan. 2003 to Jan. 2005. Our bid-ask spreads are obtained using Roll's measure:  $2\sqrt{\gamma}$  divided by the average market value of the bond. The sample of bonds differs from that in EHP, and our selection criteria bias us toward more liquid bonds with smaller bid-ask spreads.

bps, 34 bps, 20 bps and 12 bps, respectively. To compare with their results, we form trade size brackets that center around their reported trade sizes. For example, to compare with their trade size \$10K, we calculate our illiquidity measure  $\gamma$  conditional on trade sizes falling between \$7.5K and \$15K, and then calculate the implied bid-ask spread. Using the average price for the respective bond, we further convert the spread to percentage spread so as to compare with the EHP result. The results are reported in Table 12, where to correct for the difference in our respective sample periods, we also report our implied bid-ask spreads for the period used by EHP. For the EHP sample period, the cross-sectional medians of our implied percentage bid-ask spreads are 1.81%, 1.79%, 1.60%, 1.20%, 89 bps, 66 bps, 57 bps, and 54 bps, respectively. As we move on to compare our median estimates to those in EHP, it should be mentioned that this is a simple comparison by magnitudes, not a formal statistical test.

Overall, our implied spreads are much higher than those estimated by EHP. For small trades, our median estimates of implied spreads are over 50% higher than those by EHP. Moving to larger trades, the difference becomes even more substantial. Our median estimates are close to doubling theirs for the average sizes of \$100K and \$200K, close to tripling theirs for the average size of \$500K, and more than quadrupling theirs for the average size of \$1,000K. In fact, our estimates are biased downward for the trade size group around \$1,000K, since our estimated bid-ask spreads include all trade sizes above \$750K, including trade sizes of \$2M, \$5M, and \$10M, whose median bid-ask spreads are estimated by EHP to be 6 bps, 2 bps, and 2 bps, respectively. We have to group such trade sizes because in the publicly available

TRACE data, the reported trade size is truncated at \$1M for speculative grade bonds and at \$5M for investment grade bonds.

In addition to differing in sample periods, which is easy to correct, our sample is also different from that used in EHP in the composition of the bonds that are used to estimate the bid-ask spreads. In particular, our selection criteria bias our sample towards highly liquid bonds. For example, to be included in our sample, the bond has to trade at least 75% of business days, while the median frequency of days with a trade is only 48% for the bonds used in EHP. The median average trade sizes is \$467K in 2003 and \$405K in 2004 for the bonds used in our sample, compared with \$240K for the bonds used in EHP; the median average number of trades per month is 148 in 2003 and 118 in 2004 for the bonds in our sample, while the median average number of trades per day is 1.1 for the bonds used in EHP. Given that more liquid bonds typically have smaller bid-ask spreads, the difference between our implied bid-ask spreads and EHP's estimates would have been even more drastic had we been able to match our sample of bonds to theirs. It is therefore our conclusion that the negative autocovariance in price changes observed in the bond market is much more substantial than merely the bid-ask effect. And our measure of illiquidity captures more broadly the impact of illiquidity in the market.

## 9 Conclusions

Our analysis attempts to gauge the level of illiquidity in the corporate bond market and to examine its basic properties. Using a theory motivated measure of illiquidity, i.e., the amount of price reversals or the negative of autocovariance of prices changes, we show that the illiquidity measure is both statistically and economically significant for a broad cross-section of corporate bonds examined in this paper. We demonstrate that the magnitude of the reversals is beyond what can be explained by bid-ask bounce. We also show that the reversals exhibit significant asymmetry: price reversals are on average stronger after a price reduction than a price increase. Simple contrarian strategies that take advantage of these price reversals yield substantial profits.

We find that a bond's illiquidity is related to several bond characteristics. In particular, illiquidity increases with a bond's age and maturity, but decreases with its rating and issue size. While a bond's illiquidity shows little relation with its market risk exposures, as measured by its beta with respect to the stock and bond market indices, it is positively related to its idiosyncratic return volatility. We also find that price reversals are inversely related to trade

sizes. That is, prices changes accompanied by small trades exhibit stronger reversals than those accompanied by large trades.

Furthermore, the illiquidity of individual bonds fluctuates substantially over time. More interestingly, these time fluctuations display important commonalities. For example, the average illiquidity over all bonds, which represents a market-wide illiquidity, increases sharply during the periods of market turmoil such as the downgrade of Ford and GM to junk status around May of 2005 and the sub-prime market crisis starting in August 2007. Exploring the relation between changes in the market-wide illiquidity and other market variables, we find that changes in illiquidity are positively related to changes in VIX while negatively related to lagged returns of the aggregate stock market. Surprisingly, there is only a weak relation with changes in the default spread and lagged returns of the aggregate bond market. Using principal component analysis, we further show that changes in illiquidity of individual bonds share four principal components, which explain over 80% of the variations in the liquidity of bond portfolios sorted on their characteristics.

We also find important pricing implications associated with bond illiquidity. Our result shows that for two bonds in the same rating category, a one-standard-deviation difference in their illiquidity measure would set their yield spreads apart by over 40 bps. This result remains robust in magnitude and statistical significance, after controlling for bond fundamental information and bond characteristics including those commonly related to bond liquidity.

Our results raise several questions concerning the liquidity of corporate bonds. First, what are the underlying factors giving rise to the high level of illiquidity? This question is particularly pressing when we contrast the measure of the illiquidity in the corporate bond market against that in the equity market. Second, what causes the fluctuations in the overall level of the illiquidity in the market? Are these fluctuations merely another manifestation of more fundamental risks or a reflection of new sources of risks such as a liquidity risk? Third, does the high level of illiquidity for the corporate bonds indicate any inefficiencies in the market? If so, what would be the policy remedies? We leave these questions for future work.

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