How to Tell If a Money Manager Knows More?

Sergey Iskoz and Jiang Wang*

First Draft : April 20, 2001
This Draft : November 1, 2004

Abstract

In this paper, we develop a methodology to identify money managers who have private information about future asset returns. The methodology does not rely on a specific risk model, such as the Sharpe ratio, CAPM, or APT. Instead, it relies on the observation that returns generated by managers with private information cannot be replicated by those without it. Using managers' trading records, we develop distribution-free tests that can identify such managers. We show that our approach is general with regard to the nature of private information the managers may have, and with regard to the trading strategies they may follow.

*Iskoz is from the MIT Sloan School of Management, and Wang is from the MIT Sloan School of Management, CCFR and NBER. We thank Neal Stoughton and participants at the 2003 Western Finance Association Meetings for comments. Research support from the National Science Foundation (Grant No. SBR-9709976) is gratefully acknowledged.

Contents

1 Introduction

Money managers often claim that they can make better investment decisions than investors themselves. However, how to evaluate a money manager's ability to do so remains a challenging question. The traditional approach to evaluate a manager's ability is to measure the excess return of his portfolio, adjusted for risk according to a model (Sharpe (1966), Jensen (1969), Treynor and Black (1973)). This approach crucially relies on the specific risk model it uses (such as the CAPM or a linear multi-factor model). If the risk model is mis-specified, the ability measure becomes questionable.

In this paper, we pursue a different approach to evaluate money managers that does not rely on a specific risk model. We start by assuming that a money manager is valuable only if he brings additional information into the investment process.[1] Thus, a basic question in evaluating a manager is whether or not he has private information about future asset returns.[2] If the answer is affirmative, the returns he generates must reflect his private information. In particular, investment strategies without the benefit of this information cannot generate the same returns. Thus, by examining the managers' returns, we should be able to identify those that reflect private information from those that do not. We explore methods to do so based on the managers' trading records. In particular, by analyzing the joint distribution of asset returns, managers' profit and loss (P&L) and additional information on their trading strategy, we can identify those managers who have private information.

Conceptually, we can decompose the evaluation of money managers into two steps. The first step is to identify managers who bring new information into the investment process and the second step is to determine the "value" of their information. The first step need not rely on any specific risk model, but the second step does. The traditional approach combines the two steps by directly analyzing the value-added of a manager. Such a direct approach is desirable, and is possible if an appropriate risk model is available. However, if the appropriate risk model is absent or mis-specified in the analysis, this approach becomes

[1] A money manager can be valuable in other ways such as having the ability to implement simple investment strategies at lower costs. In this paper, we ignore these possibilities.

[2] It is conceivable that a money manager might possess valuable information about things other than future asset returns that are also valuable, such as investors' future consumption needs and financial constraints. Although we do not consider these possibilities in this paper, our methodology can be extended to include them.

ineffective and potentially misleading.

The contribution of a money manager is to generate returns that investors cannot achieve on their own. These returns fall into two categories. The first category contains returns that represent arbitrage profits and the second category contains the rest. Arbitrage profits are considered by Merton (1981) and Henriksson and Merton (1981), and they only arise under restrictive conditions.[3] For example, in the case considered by Merton, the manager has perfect information about the direction of future price movements.[4] When the market is complete with respect to the arbitrage profits considered, their evaluation is straightforward, at least conceptually. When the market is incomplete, the value of arbitrage profits is harder to determine, but is always positive.[5]

Short of being arbitrage profits, money manager's returns are difficult to value. The main difficulty comes from the risky nature of these returns. The traditional approach relies on a specific risk model to overcome this difficulty. But it has long been recognized that the possibility of using a wrong model can be devastating.[6] Actually, the difficulty goes beyond the search of the "correct" risk model. The very existence of such a risk valuation model is much in question. It is important to realize that based on his private information, an informed manager sees a different return distribution than that seen by an investor. Thus, he uses a different risk model (see, e.g., Grossman and Stiglitz (1980), Admati and Ross (1985) and Wang (1993)). Using the investor's own model to evaluate the manager is conceptually problematic, since it is based on inferior information. Yet, most traditional evaluation methods, such as those of Sharpe (1966), Jensen (1969) and Treynor and Black (1973), take this approach. In fact, the investor can reach wrong conclusions using his own

[3]See Jagannathan and Korajczyk (1986), Cumby and Modest (1987), Glosten and Jagannathan (1994), Ferson and Schadt (1996) and Goetzmann, Ingersoll and Ivkovič (2000) for applications and extensions of the Henriksson-Merton methodology. The stochastic discount factor (SDF) framework for performance evaluation, which is similar in spirit to Merton's original idea, is reviewed in Ferson (2002). See also Chen and Knez (1996) and Farnsworth, Ferson, Jackson and Todd (2002) for applications of the SDF methodology.

[4]Merton (1981) and Glosten and Jagannathan (1994) also considered extensions of their analysis when profits have the nature of "risky arbitrages". Using APT type of arguments (Ross, (1976)), they treat these as equivalent to arbitrage profits ("approximate arbitrages"). But, "risky arbitrages" belong to the second category, which we discuss below.

[5]The case of arbitrage profits raises the issue of how they can be consistent with an equilibrium. In the model Merton (1981) considered, the manager knows for sure when the stock will out- or under-perform the bond. Thus, he disagrees with investors on the support for future stock prices. Further constraints (e.g., Dybvig and Willard (1999)) or behavior assumptions (e.g. Kyle (1985)) on the manager are needed to obtain a sustainable equilibrium in this situation.

[6]See, for example, Jensen (1972), Roll (1978), Dybvig and Ross (1985), and Grinblatt and Titman (1989).

model to judge the manager (see, e.g., Dybvig and Ross (1985)). Using the manager's model is usually infeasible since it is based on his private information. Moreover, even knowing the money manager's model, an investor may not agree with it since with incomplete markets, his objective and valuation of returns would generally not coincide with those of the manager.[7]

This difficulty in establishing the appropriate risk valuation model justifies our approach, which focuses on the first question in evaluating a manager: Does he know something that investors do not? This question is less ambitious, but more direct and basic. Our approach has the advantage of being independent of the valuation model and the objectives of the manager and the investors. Its limitations are also obvious compared with the traditional approach. For example, if our methodology identifies a manager with useful information, it does not necessarily tell us how valuable that information is to a particular investor, nor does it allow us to rank the information across different managers.

Our approach is closely related to the method considered by Cornell (1979), Copeland and Mayers (1982) and Grinblatt and Titman (1993) for performance evaluation of money managers. These authors use the predictive power of a manager's asset holdings for average future returns as a measure of their ability.[8] This measure is intuitive and does not rely on a risk model and can detect performance if the manager can forecast average future returns. But it has several drawbacks. First, it makes implicit assumptions about the nature of managers' private information. In particular, it can detect the ability in forecasting means, but not the ability in forecasting other aspects of future returns, such as volatility. Second, it is sensitive to the managers' trading strategies. If there are two managers with the same information on expected future returns, the measure gives a higher score to the manager who trades more aggressively on the information. Third, it applies mainly when the returns are IID from investors' point of view. If the returns are non-IID, the measure is not directly usable.[9] Fourth and more importantly, it relies on observations of managers' asset holdings, which is incomplete at best in practice. Our approach is in the same spirit in the sense that

[7]It should be pointed out that the assumption of complete markets, in addition to being unrealistic, is inappropriate in dealing with situations when market participants have different information. See, for example, Grossman and Stiglitz (1980), and Milgrom and Stokey (1982).

[8]See also Cumby and Modest (1987), Graham and Harvey (1996), and Chance and Hemler (2001) for applications involving evaluation of direct forecasts.

[9]Ferson and Khang (2002) have considered the situation of non-IID returns by making additional assumptions about the return dynamics. We return to this issue in more detail in Section 4.

it also develops an evaluation method that does not rely on a risk model. But it is more general and does not have these drawbacks. Moreover, it can be applied even when we do not observe managers' asset holdings.[10]

We proceed as follows. In Section 2, we use a simple example to illustrate the basic idea of our approach. In Section 3, we develop a general method to distinguish informed managers from uninformed managers based on their trading records. In Section 4, we apply the general methodology to several settings when the managers' positions are observable. Section 5 shows how our procedure can be used when the information about managers' positions is unavailable. In Section 6, we apply our methodology to the situation when asset returns are non-IID, to the comparison of two managers, and to the situation where managers can trade in-between observations. Section 7 contains several robustness checks of our methodology. Section 8 concludes.

2 An Example

We start our discussion with the example considered by Dybvig and Ross (1985) to demonstrate the drawback of the traditional approach. We use this example to illustrate how to identify a manager with superior private information without relying on any risk model. We further use this example to raise and to clarify several issues related to our approach.

Setup

Suppose that a money manager can invest in cash and a risky asset. Cash earns zero interest, and change in the price of the risky asset in period $t+1$, denoted by Q_{t+1}, follows the process

$$Q_{t+1} = \mu + \sigma_z Z_t + \sigma_u u_{t+1} \tag{1}$$

where μ, σ_z and σ_u are positive constants, Z_t and u_{t+1} are IID normal with zero mean and variance of 1. Throughout this paper, we model and, later, simulate price changes of the risky asset, which we also refer to as "dollar returns" or simply "returns". For convenience, we refer to the risky asset as "stock" and express a manager's holdings of it as number of shares. We compute the Sharpe ratio of a manager by dividing the mean of her P&L by its

[10] A methodology similar to ours was used by Lo, Mamaysky and Wang (2000) to evaluate the merit of technical analysis.

standard deviation. These departures from a more traditional setup using rate of return and portfolio weights are merely for convenience.

We consider two money managers, an uninformed manager, denoted by 0, and an informed manager, denoted by 1. The uninformed manager has no private information about future returns. To him, stock return is IID over time with a mean of μ and a variance of $\sigma_Q^2 \equiv \sigma_Z^2 + \sigma_u^2$. The informed manager has private information about future stock returns. In particular, she observes Z_t at t ($t = 0, 1, \cdots$). To her, returns are no longer IID, but predictable given her information.

Both managers form their investment strategies based on their information about future returns. Let N_t^i denote manager i's stock position (expressed in number of shares) at t, where $i = 0, 1$. We assume that the two managers follow simple investment strategies of the following form:

$$N_t^0 = a^0, \quad N_t^1 = a^1 + b^1 \frac{Z_t}{\sigma_Z} = a^1 \left(1 + \lambda Z_t\right) \tag{2}$$

where a^0, a^1 and b^1 are all positive constants, presumably depending on the managers' objective functions, especially their risk tolerances and $\lambda \equiv a^1/b^1$. It can be shown that these strategies are optimal given certain form of the managers' objective functions.[11] From (2), the portfolio of the uninformed manager is constant over time, reflecting the fact that he possesses no information about future stock returns. The portfolio of the informed manager, however, does change over time as she receives new information about future returns. The coefficient λ measures the informed manager's intensity to trade on her information.

In the remainder of the paper, the following benchmark values are used for the parameters when needed in numerical illustrations: $\mu = 0.12$, $\sigma_Q = 0.15$. Both a^0 and a^1 are set to 1. Obviously, these values are chosen merely for concreteness. Our results do not depend on any particular choice.

[11] For example, under a similar return process and constant absolute risk aversion (CARA) preferences, Wang (1994) shows that the informed manager's optimal stock holdings are linear in her signal. The linear form assumed here, however, is for illustrative purposes only. The methodology we develop works as long as manager's holdings are monotone in her signal.

Sharpe Ratio as An Information Identifier

Can we distinguish the informed manager from the uninformed manager, given certain information on their trading records? The traditional approach to performance evaluation is to look at the (excess) return of each strategy, adjusted for its risk according to a specific risk model. A well-known risk model is the mean-variance model, in which the risk-adjusted measure of excess return is the Sharpe ratio. Given the strategies of the two managers, we can easily compute their Sharpe ratios, denoted by SR^0 and SR^1, respectively:

$$SR^0 = \frac{\mu}{\sigma_Q} \equiv SR^*, \quad SR^1 = SR^* \frac{1 + \lambda R / SR^*}{\sqrt{1+2\lambda\, R\, SR^* + \lambda^2 (1+R^2+SR^{*2})}} \tag{3}$$

where $R^2 \equiv \sigma_Z^2 / \sigma_Q^2$ measures the amount of private information the informed manager has about future returns. It is worth pointing out that the Sharpe ratio of the uninformed manager is independent of the number of shares he holds. This reflects a desirable property of the Sharpe ratio in measuring performance: By simply taking on more or less risk, a manager cannot improve his performance measure. This also implies that the Sharpe ratio of any uninformed manager can not exceed SR^*.[12]

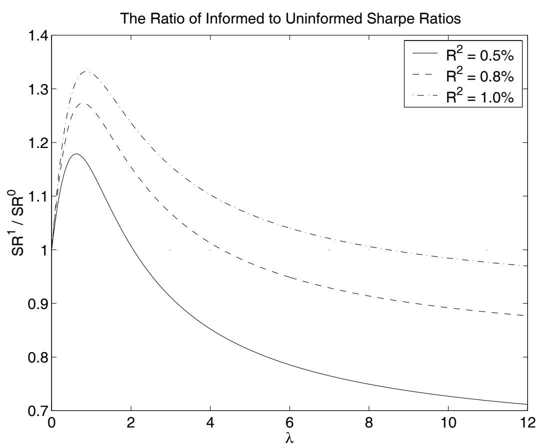

Figure 1: Relative Sharpe ratio of the informed and the uninformed manager.

The informed manager's Sharpe ratio depends on the specific strategy she follows. In

[12]In fact, SR^* gives the upper bound for the Sharpe ratio the uninformed manager can achieve. To see this, let us suppose that the uninformed manager introduces noise into his investment strategy. In particular, assume his stock position is $N_t^0 = a^0 + \tilde{n}_t$, where $\tilde{n}_t$ is a random variable independent of Z_t and u_{t+1}, with a zero mean. It then follows that his Sharpe ratio becomes $SR = SR^* \frac{a^0}{\sqrt{(a^0)^2 + (1+SR^{*2})\sigma_{\tilde{n}}^2}}$, which is less than SR^*.

particular, it may not always exceed that of the uninformed manager (SR^*). Figure 1 shows the ratio of SR^1 to SR^0 as a function of λ, where μ and σ take their benchmark values: $\mu = 0.12$ and $\sigma_Q = 0.15$. Thus, the Sharpe ratio of the stock is $SR^* = \mu/\sigma_Q = 0.8$ on an annual basis.[13] We assume that the managers are trading on a weekly basis. The R^2 for weekly stock return takes values ranging from 0.5% to 1%.

It is obvious that the informed manager can generate Sharpe ratios below SR^*, especially when her private information is limited (low R^2) and she trades aggressively on it. As Dybvig and Ross (1985) have pointed out, the reason for the failure of Sharpe ratio in detecting the informed manager's superior ability is that volatility as a measure of risk becomes inappropriate for her P&L profile, which is no longer normal. In particular, the P&L profile of the informed manager has a χ^2 component in it, due to the positive correlation between her position in the stock and its return.

This points to a fundamental drawback of the traditional approach, its reliance on a specific risk model. Besides the possibility of being mis-specified, a particular risk model is usually based on the perspective of investors who have no private information. It does not necessarily incorporate the potential private information that better informed investors/managers might have, and thus may not be the right risk model from their perspective.

Information Identification without A Risk Model

However, in this example, we do not need a risk model to identify the informed manager. In fact, simply by looking at the distribution of P&L, we should be able to tell who the informed manager is. Figure 2 shows the unconditional distribution of the P&L for both the informed and the uninformed manager. Thus, the uninformed manager follows a passive strategy and holds one share of stock. The informed manager has predictive power of $R^2 = 1\%$ for future stock return and she actively trades on her private information. For comparison, we consider two values for λ, the intensity at which the manager trades on her information: $\lambda = \lambda^*$ and $\lambda = 5\lambda^*$, where $\lambda^* = 0.901$.[14]

[13] A Sharpe ratio of 0.8 on annual basis is higher than the historic average for passive strategies. It is less clear what the reasonable range for active strategies should be. We choose this number merely for illustrative purposes.

[14] In the portfolio choice problem with CARA preferences and returns given by (1), the optimal policies of the uninformed and the informed managers take the form of (2). At the specified parameter values ($\mu = 0.12$, $\sigma_Q = 0.15$ and $R^2 = 1\%$), $a^0 = a^1 = 1$ implies a risk aversion coefficient, denoted by γ, of

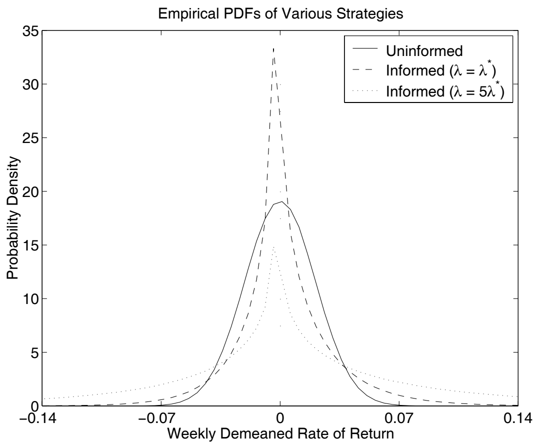

Figure 2: Distribution of the managers' P&L.

It is obvious that the P&L of the informed manager is positively skewed while the P&L of the uninformed manager is always symmetric (around its mean). Table 1.A reports the moments of the P&L for the two managers. Since the uninformed manager follows a passive strategy, his P&L has the same moments as the stock return, with a weekly average gain of 0.231% and volatility of 2.080%. The corresponding Sharpe ratio is 0.111 (weekly). The informed manager, trading on her private information about future returns, generates large average gains, but with higher volatility. For $\lambda = \lambda^*$, the average weekly P&L is 0.418% with a volatility of 2.830%, giving a Sharpe ratio of 0.148, which is significantly higher than that of the uninformed manager. However, when she trades more aggressively with $\lambda = 5\lambda^*$, her Sharpe ratio falls to the same range as that of the uninformed manager.

What is worth noting is the symmetric nature of the uninformed manager's return profile. Given that conditional on public information (which is summarized in the prior distribution of Z_t and u_{t+1}), the stock dollar return is symmetrically distributed, any strategy that the uninformed manager adopts based on the same public information can only generate symmetric P&L. This implies that any strategy that can generate an asymmetric P&L profile must be relying on additional information about stock returns.

Thus, in this example, information revealed by the unconditional distribution of the managers' P&L is sufficient to identify the informed manager. However, as we will see

$5\frac{1}{3}$ for both managers. Moreover, for the informed manager, $\lambda^* = 0.901$ for the optimal policy. Although our analysis does require any optimality on the manager's part, we use values implied by optimality under CARA preferences in our numerical illustrations.

later, relying only on the unconditional P&L distribution is restrictive in general. For example, using option-like strategies, the uninformed investor can closely mimic the important characteristics of the unconditional distribution of the informed investor's P&L. Robust identification of informed managers has to rely on additional information about their trading records. For example, when the trading record also contains the history of manager's positions, we can determine if she has private information by looking at the distribution of asset returns conditioned on her positions. In particular, future returns conditioned on her current stock position being above its average must have a different distribution than the returns conditioned on her current position being lower than its average. In other words, her current position can predict future returns, which is the point exploited by Cornell (1979) and Grinblatt and Titman (1993), among others.

(a) (b)

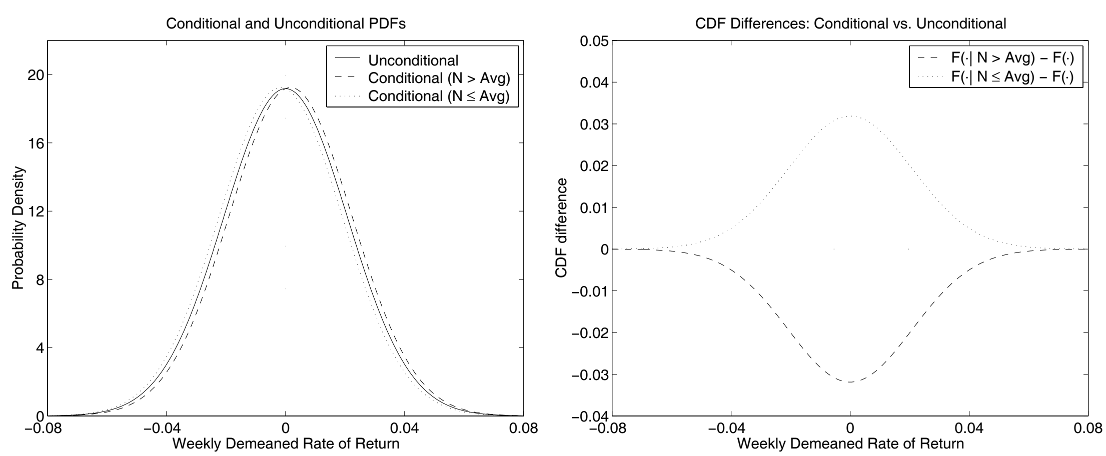

Figure 3: Comparison of conditional and unconditional distributions of stock dollar return: PDFs (Panel A) and CDF differences (Panel B). Distributions conditioned on the manager's stock position being above and below average are denoted by the dashed and dotted lines, respectively. The solid line in Figure 3(a) gives the unconditional distribution.

Table 1.B reports the moments of stock returns conditioned on the informed manager's stock position being above and below average. Clearly, conditioned on the informed manager's stock position being above average, the future stock returns have a positive mean and skewness, and conditioned on her stock position being below average, the future stock returns have a negative mean and skewness.

Figure 3(a) plots the stock return distribution conditional on the manager's position being above and below average. It is obvious that for the uninformed investor, the two distributions should be identical and equal to the unconditional distribution. They are shown in the figure by the solid line. For the informed manager, the dollar return distribution conditional on her position being below average is shown by the dotted line, and the dollar return distribution conditional on her position being above average is shown by the dashed line. Clearly, the two distributions are different and the former lies to the left of the latter. The difference between the two conditional distributions unambiguously demonstrates that manager's position contains information about future returns.

Given that predictive power of the informed manager's signal about future stock return has an R^2 of only 1%, the differences between the conditional distributions (when her position is above and below average, respectively) and the unconditional distribution, as shown in Figure 3(a), may seem small. But they may well be detectable statistically, depending on the methods used. In particular, the difference between the cumulative density function of the conditional distribution and the unconditional distribution is quite significant as Figure 3(b) shows. The test we consider later is related to this observation.

The above example illustrates our basic idea in this paper, which is to identify managers who have private information about future returns. We explore methods to achieve this by relying solely on the managers' trading records, not on any particular notion of risk.

Obviously, this example is special in many ways, such as the return generating process, the nature of the informed manager's private information, the information we have on managers' trading records, etc. This raises many questions about how far we can push our idea in more general settings. For example, the asymmetry in the informed manager's P&L profile is related to the fact that her information is about the mean of future stock return. If her information is about the volatility of future stock return, can we still identify any special features of her P&L profile? What if her information is about some aspect of future stock returns, which we do not even know? What if we do not directly observe the manager's holdings? Also, we have assumed that we have information about managers' portfolio P&L at the same horizon as their trading period. What if we only observe P&L at a horizon that is longer than their trading period? In this case, dynamic trading strategies within the horizon over which we record P&L can generate a rich set of P&L profiles even when they are only based on public information. Can we still tell if a strategy contains new information?

We will address these questions in the following sections.

3 Methodology

We now consider a methodology that allows us to separate an informed manager from an uninformed manager. The effectiveness of any such methodology depends on two factors: (1) the underlying model, including the return generating process, the nature of private information, and the trading strategy, and (2) the amount of data available, including the length of managers' trading records. Of course, less reliance on both of these factors is more desirable. For most of our discussion, we assume that sufficient data is available, and focus primarily on the first factor. We return to the data issue in Section 7.

Our methodology is based on the availability of managers' trading records. Let N_t^i and Q_t denote manager i's stock position and the realized stock return over time. Define

$$G_t^i \equiv N_{t-1}^i Q_t \tag{4}$$

as the P&L of manager i in period t where $t = 1, \dots T$. Let $(\cdot)_{[1,\,T]}$ denote the history of a variable from time 1 to T. We assume that a manager's trading record always includes $G_{[1,\,T]}$, his P&L. It also includes public information such as past returns on the stock and realizations of certain events or state variables (e.g., realized earnings or success/failure of mergers). In addition, it may contain more detailed information, such as the manager's positions. Formally, the trading record (history) of manager i is given by

$$H_T^i \equiv \left(G^i, Q, S^i\right)_{[0,\,T]} \equiv \left\{\left(G_1^i, Q_1, S_0^i\right), \left(G_2^i, Q_2, S_1^i\right), \dots, \left(G_T^i, Q_T, S_{T-1}^i\right)\right\}. \tag{5}$$

Here, S represents the information on the manager's trading history in addition to his P&L and stock returns. We consider two possible forms of S in this paper. In the first instance, $S_t^i = N_t^i$ is the manager's stock position. In the second instance, S_t^i is the ex-post realization of an event or a state variable X that affects the distribution of future stock returns and on which the manager might have private information. For example, if the manager trades on information about earnings, the actual earnings realizations are observable and constitute an important part of his trading record. In the first case, we assume that the manager's position is a monotonic function of X. In the second case, S^i is monotonic in X by construction.

The basic idea behind our methodology is that the joint distribution of (G^i, Q, S^i) for an informed manager must be different from that for an uninformed manager. In other words, the statistical properties of (G^i, Q, S^i) alone should allow us to distinguish an informed manager from an uninformed manager, independent of the mangers' trading strategies and a specific risk model. We next develop our methodology for the two cases defined above with different information on managers' trading records.

3.1 Positions As Part of Trading Record

We first consider the situation where $S^i_t = N^i_t$, that is, the manager i's trading positions are available for his evaluation. In this case, $H^i_T = (G^i, Q, N^i)_{[1,T]}$ (notice that since $G^i_t = N^i_{t-1}Q_t$, we can drop G^i from H^i_T in this setting). If manager i trades on private information about future returns, his stock position at any point in time must reflect this information. In particular, conditional on his position in the stock, the distribution of future returns must be different from the unconditional distribution of returns.

Let

$$F_A(x) \equiv F\left(Q_{t+1} \le x \,|\, N^i_t \in A\right) \tag{6}$$

denote the cumulative density function of the stock dollar returns next period conditioned on manager i's current stock position N^i_t being in set A, where A is a Lebesgue-measurable set on $\Re$. If N^i_t contains no information about Q_{t+1}, we have

$$F_A(\cdot) = F_{A'}(\cdot) \quad \forall \;\; A,\; A'. \tag{7}$$

If, however, N^i_t contains information about Q_{t+1}, then we have

$$F_A(\cdot) \neq F_{A'}(\cdot) \quad \forall \;\; A \neq A'. \tag{8}$$

Thus, the problem of identifying the informed manager reduces to the task of distinguishing the two conditional return distributions. The null hypothesis to be tested is simply (7). Instead of making assumptions about the return generating process and the nature of the manager's private information, we rely on the Kolmogorov-Smirnov test, which is distribution-free.

For manager i, $F_A(x)$ and $F_{A'}(x)$ denote the cumulative return distribution functions conditioned on his stock position being in A and A', respectively. The null hypothesis is that $F_A = F_{A'}$. The estimated cumulative distribution function $\widehat{F}_S$ $(S = A, A')$ based on the trading record of manager i with sample size n_s is given by

$$\widehat{F}_S(x) \equiv \frac{1}{n_S} \sum_{t=1}^{n_S} 1_{\{Q_t \leq x,\, N_{t-1}^i \in S\}} \tag{9}$$

where $1_{\{.\}}$ is the indicator function. The Kolmogorov-Smirnov statistic based on the estimated distributions is

$$\delta(n_A, n_{A'}) \equiv \left(\frac{n_A n_{A'}}{n_A + n_{A'}} \right)^{1/2} \sup_{-\infty < x < \infty} \left| \, \widehat{F}_A(x) - \widehat{F}_{A'}(x) \, \right|. \tag{10}$$

The distribution of $\delta(n_A, n_{A'})$ under the null can be readily computed (see, for example, Hollander and Wolfe (1999), Table A.10). We can perform an approximate α-level test of the null hypothesis by computing the statistic and rejecting the null if it exceeds the 100α-th percentile of its distribution under the null.

When n_A and $n_{A'}$ are large, the statistic should be small under the null. In particular, when $\min(n_A,\, n_{A'}) \to \infty$, it has the following limiting distribution:

$$\lim_{\min(n_A, n_{A'}) \to \infty} \mathrm{Prob}\big(\delta(n_A, n_{A'}) < \delta\big) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2\delta^2}. \tag{11}$$

Assuming, without loss of generality, that $n_A > n_{A'}$, this approximation is valid for $n_A \geq 100$, $n_{A'} \geq [0.10 n_A]$.[15] An approximate α-level test of the null hypothesis can be performed based on the limiting distribution of the test statistic under the null, given by (11).

To implement the test proposed above, we need to choose the sets A and A'. The choice depends on several factors. In order to increase the power of the test, we want to choose A and A' such that the difference between the two conditional distributions, as measured by the Kolmogorov-Smirnov statistic, is maximized. Also, we want to use the data most efficiently. Any particular choice of A and A' involves a trade off between these two considerations. Suppose that on average, the managers' stock position is zero. Then, one choice is to let $A = (-\infty,\, \infty)$ and $A' = (-\infty,\, 0]$ or $A' = (0,\, \infty)$. Another choice is to let $A = (-\infty,\, 0]$ and $A' = (0,\, \infty)$.[16] Both choices use all observations. But the second choice gives a larger

[15] See Kim and Jennrich (1973). The numerical results presented later in the paper also use the continuity correction suggested by these authors, which amounts to adding $1/(2\sqrt{n_A})$ to the computed value of $\delta(n_A, n_{A'})$.

[16] If the manager's average stock position is not zero, but is $\bar{N}$, then we can simply choose $A = (\infty,\, \bar{N}]$

difference between F_A and $F_{A'}$. Therefore, setting $A = (-\infty, 0]$ and $A' = (0, \infty)$, our test becomes the test of the hypothesis that $F_+ = F_-$, where

$$F_+(x) \equiv F\big(Q_{t+1} \leq x \,|\, N_t \leq 0\big) \quad \text{and} \quad F_-(x) \equiv F\big(Q_{t+1} \leq x \,|\, N_t > 0\big). \tag{12}$$

Since the Kolmogorov-Smirnov test makes no assumptions about the actual return distribution, it can detect any difference in the distributions conditioned on a manager's position. In particular, the private information that gives rise to this difference does not have to be limited to expected returns; it can involve better forecasts about other moments of returns such as volatility. We consider this case in Section 4.2.

In the above discussion, we assume that the uninformed manager as well as the evaluator have no information about future returns. In other words, conditioned on their information, stock returns are IID over time. This is a restrictive assumption.[17] Conditioned on public information, returns can be non-IID. Our method can be easily extended to the case of non-IID returns. Suppose that the distribution of future stock return Q_{t+1} depends on public information Y_t. Then, for an uninformed manager who nonetheless observes Y_t at time t, we have

$$F_A\big(\,\cdot\,|\, Y\big) = F_{A'}\big(\,\cdot\,|\, Y\big) \quad \forall \;\; A,\; A' \text{ and } Y \tag{13}$$

where $F_S\big(x \,|\, Y\big) \equiv F\big(Q_{t+1} \leq x \,|\, N_t \in S,\, Y_t = Y\big)$ is the cumulative distribution of future stock return conditioned on the current public information being $Y_t = Y$ and the stock position being in the set $S = A, A'$. In this case, the identification of an informed manager is equivalent to rejecting the null hypothesis (13).

3.2 State Variable(s) As Part of Trading Record

We have developed the methodology to identify informed managers under the assumption that their positions are available to us for evaluation. Now we consider the case when such data is not observable. Suppose, instead, that we have ex-post realizations of a state variable

and $A' = (\bar{N}, \infty)$.

[17] There are many papers documenting the serial dependence in stock returns. For example, Fama and French (1988b), Lo and MacKinlay (1988), and Poterba and Summers (1988) have shown serial correlation in stock index returns at different horizons. Campbell and Shiller (1988), Fama and French (1988a), and Kothari and Shanken (1997) show that dividend yield can predict future returns. Bollerslev, Chou and Kroner (1992), among others, survey evidence on predictability in stock return volatility.

X about which managers may have private information. Our procedure can be modified to detect those informed managers. In this case, $H_T = (G^i, Q, X)_{[1,T]}$.

The intuition behind our methodology in this case is as follows. Controlling for stock return, the only source of variation in manager's P&L is his position in the stock. If he has information about the state variable X, the variation in his position is related to the variation in X. Therefore, conditioned on the stock return, the distribution of his P&L when X is "low" should be different from the conditional distribution of his P&L when X is "high". Thus, identifying an informed manager again reduces to comparing two conditional distributions, except that in this setting the test is applied to the distribution of manager's P&L rather than to the distribution of asset returns.

Let

$$F_S^G\big(x\,|\,Q\big) \equiv F\big(G_t \leq x\,|\,X_{t-1} \in S,\, Q_t = Q\big) \tag{14}$$

denote the cumulative distribution of a manager's P&L conditioned on the concurrent stock return being $Q_t = Q$ and the state variable X being in the set $S = A,\, A'$. Then, under the null hypothesis that a manager has no information about X, we must have

$$F_A^G\big(\,\cdot\,|\,Q\big) = F_{A'}^G\big(\,\cdot\,|\,Q\big) \quad \forall \;\; A,\;\, A' \text{ and } Q. \tag{15}$$

Rejection of this null hypothesis implies that a manager has private information about X.

The test of the null on the distribution of P&L conditioned on the realization of state variables can be constructed in the same way as the test on the distribution of stock returns conditioned on the manager's positions and public information, which is described in Section 3.1. Here, the return on the stock plays the same role as the public signal does when returns are non-IID.

4 Analysis When Positions Are Observable

In this section, we apply the methodology proposed in the previous section to simulated trading records of managers under the assumption that their stock position is observable. We focus on the situation when asset returns are IID over time and consider the setting with non-IID returns in Section 6.1. We first analyze the case with a single risky asset when the informed manager has private information about the mean of future returns. We then consider the situation where the manager's private signal is about the volatility of future returns. Finally, we extend our analysis to a setting where managers can invest in multiple risky assets.

Relying on managers' positions to identify an informed manager was considered by Cornell (1979), Copeland and Mayers (1982), Grinblatt and Titman (1993), and Ferson and Khang (2002). In particular, they use the covariance between a manager's asset positions and subsequent returns as a performance measure. This measure is intuitive and has simple economic interpretations. But it is applicable only when the manager's information is on the mean of asset returns. It is also sensitive to how aggressively a manager trades on his information. The analysis in this section shows that our approach avoids these limitations. Its application in the case when managers' positions are observable is a natural generalization of the existing methodology.

4.1 Information on Mean Return

Consider the simple case in which stock return is given by (1) and the informed manager observes Z_t at time t. R^2, which measures the predictive power of the informed manager's private information, is set at 5%. The portfolio strategies of the uninformed and informed managers are given by (2). In particular, the stock position of the informed manager is linear in Z_t. The intensity of her trading on the private information is measured by λ.

Weekly stock returns are generated by simulating (1) for 400 weeks. The stock price is initialized to \$1, which allows us to interpret stock returns as (simple) returns per dollar invested at time 0. Each simulation gives us a time series of Q_t, Z_{t-1} and the managers' stock positions N^i_{t-1} with $i = 0, 1$. The simulated trading record of manager i is $(Q, N^i)_{[0,\,400]}$. The simulation is repeated 500 times to obtain various statistics and their standard errors.

The uninformed manager (manager 0) follows a passive strategy with $N^0_t = a^0 = 1$. We also consider an uninformed manager, denoted as manager $0'$, who adds noise to his stock holdings. This manager has no private information about future stock returns, but thinks that he does. He treats noise as signals about future returns, and trades on it. His stock position is $N^{0'}_t = a^0 + \tilde{n}_t = 1 + \tilde{n}_t$ where $\tilde{n}_t$ is normal, independent of Q_t, with a mean of zero and a volatility of 0.5. The informed manager follows an active strategy $N^1_t = a^1(1 + \lambda Z_t)$.

Her average position in the stock is a^1, which is also set to 1. The benchmark value of λ for this case is $\lambda^* = 2.016$.[18] To assess how manager's trading intensity affects our methodology, we consider two values of λ: $\lambda = \lambda^*$ and $\lambda = 5\lambda^*$.

Panel A of Table 2 reports the summary statistics of the P&L of the uninformed managers with and without noise, and of the informed manager with different intensities of her informed trading. The strategy of the uninformed manager without noise is passive, buy-and-hold. The moments of his P&L are equal to those of stock return, which has a weekly average of 0.231%, a volatility of 2.084%, and a Sharpe ratio of 0.111. The skewness is zero and kurtosis is 3. The strategy of the uninformed manager with noise is active but not rewarding. His average weekly P&L of 0.234% is comparable to the passive strategy, but the volatility of 2.331% is higher than that for the passive strategy. The skewness of his P&L distribution is still approximately zero, but the kurtosis becomes 5.038 (with a standard deviation of 1.138). The noise in the uninformed manager's strategy reduces his Sharpe ratio to 0.101 (with a standard deviation of 0.049). The P&L of the informed manager gives a different pattern. It tends to have higher average gain and volatility than the P&L of the buy-and-hold uninformed manager. But in general, the informed manager achieves larger Sharpe ratios. For $\lambda = \lambda^*$, her Sharpe ratio is 0.242, which is considerably higher than the Sharpe ratio of the uninformed manager. However, as λ increases, i.e. when the informed manager trades more aggressively on her information, the Sharpe ratio may decline, as shown in Figure 1. More importantly, the distribution of the informed manager's P&L is very different from those of the uninformed managers. In particular, it shows significant positive skewness, as discussed earlier. For $\lambda = \lambda^*$, the skewness is 1.28 (with a standard deviation of 0.55).

In the particular case we consider here, the unconditional distribution of the P&L of the informed manager provides strong evidence of her having private information about future stock returns. However, this situation is special. As we show later, the evidence from the unconditional distribution of P&L is insufficient and can be misleading. For this reason, we utilize the methodology discussed in Section 3 and construct tests based on the stock return distribution conditioned on the managers' positions.

Let $\bar{N}$ denote the average of a manager's stock position over the simulated sample. We

[18] The benchmark value of 2.016 for λ is optimal if the manager has a CARA preferences with a risk aversion of $5\frac{1}{3}$, which gives $a^1 = 1$. See footnote 14.

partition the sample into two sets, those with $N_t \leq \bar{N}$ and those with $N_t > \bar{N}$. We have n_- and n_+ sample points in the two sets. For each set, we use (9) to estimate conditional cumulative distribution of stock dollar return, $\widehat{F}_-$ and $\widehat{F}_+$, respectively. We also use the whole sample to compute the unconditional cumulative stock return distribution $\widehat{F}$. Table 2.B reports the summary statistics for the unconditional distribution and for the two conditional distributions. The unconditional distribution of stock return is normal with mean of 0.231% and standard deviation of 2.084%. Its skewness is zero and kurtosis is 3, as expected. The average return conditioned on the informed manager's position being above average is 0.604%, significantly higher than the unconditional mean. Conditioned on the manager's stock position being below average, the mean return is negative, -0.141%, which is lower than the unconditional mean and a lot smaller than the mean conditioned on her position being above average. The difference between the means of the conditional distributions reflects the private information the manager has about average future returns, as we assumed. The other moments of the conditional distributions are very similar to those of the unconditional distribution. The standard deviation of the conditional distributions is slightly lower than that of the unconditional distribution on average, but the difference is not statistically significant because the R^2 of the manager's signal is only 5%. The third and fourth moments of the conditional distributions are practically the same as those of the unconditional distribution. It is also worth noting that the moments of the conditional distribution do not depend on the intensity at which the manager trades on her private information (as long as it is not zero). The moments are essentially the same for different values of λ. This is not surprising. When the manager's stock position is linear in her signal about future stock return (Q_t), as we assume here, conditioning on her position being above average is equivalent to conditioning on the signal being positive, i.e., $Z_{t-1} > 0$. The magnitude of λ has no impact on the information revealed by the relative size of her position.

We apply the Kolmogorov-Smirnov test to each pair of the estimated distributions: $\widehat{F}_+$ versus $\widehat{F}$, $\widehat{F}_-$ versus $\widehat{F}$, and $\widehat{F}_+$ versus $\widehat{F}_-$, which allows us to accept or reject the null hypothesis (7) at a given confidence level. For each simulation, we compute the Kolmogorov-Smirnov test statistic and the corresponding p-value. The rejection rate at the 1% (5%) level is a fraction of simulations for which the p-value does not exceed .01 (.05). The standard deviation of the test statistic is shown for information purposes only.

Table 3 reports the results of the test. In the first sub-panel of Panel A, we have the Kolmogorov-Smirnov test statistic δ for the uninformed manager who trades on noise. It is clear that δ is close to zero. The rejection rate of the null hypothesis that the two conditional P&L distributions and the unconditional P&L distribution are the same is close to zero at both the 1% level and the 5% level. For the informed manager, the test statistic becomes larger and more significant. However, the rejection rate of the null that $\widehat{F}_+ = \widehat{F}$ or $\widehat{F}_- = \widehat{F}$ is still low, about 2% at the 1% level and 13-16% at the 5% level. But for the null that $\widehat{F}_+ = \widehat{F}_-$, the rejection rate becomes very high, 69% at the 1% level and 88% at the 5% level. This is not surprising, since the difference between $\widehat{F}_+$ and $\widehat{F}_-$ is larger than the difference between either $\widehat{F}$ and $\widehat{F}_+$ or $\widehat{F}$ and $\widehat{F}_-$. It should also be pointed out that the results of the test are independent of the intensity at which the informed manager trades on her private information. This is clearly a desirable property of our evaluation method.

For comparison, in Panel B of Table 3 we report the performance measure used by Grinblatt and Titman (1993) when managers' asset holdings are observable. In particular, they use the covariance between the manager's stock holding and the stock return. Clearly, the covariance is positive when the manager trades on private information about mean returns. It is worth noting that when trading more aggressively on her information ($\lambda = 5\lambda^*$), the manager attains larger covariance. This, however, does not imply that she has better information. For the purpose of identifying informed managers, the correlation between stock holding and stock return is a better measure, which is also reported in the table. It does not depend on the intensity of the informed manager's trading, *if* the strategy is linear in the signal. It is easy to see that for non-linear but monotonic strategies, even the correlation between holding and return can be problematic. In general, the magnitude of Grinblatt-Titman measure is not informative about the precision of manager's signal.

The above analysis of a simple case demonstrates the applicability of our methodology in identifying managers with private information. In the remainder of this section, we apply the same methodology to more general cases.

4.2 Information on Volatility

In the preceding analysis, we have considered the case where the informed manager has private information about the mean of future returns. As we discussed earlier, our procedure does not rely on assumptions on the nature of manager's private information. In order to demonstrate this, we consider a situation where the informed manager has private information about the volatility of future returns rather than the mean. Suppose that stock price increments are generated by the following process:

$$Q_{t+1} = \mu + \sigma_t \, u_{t+1} \tag{16}$$

where σ_t follows an IID two-point process and u_{t+1} is IID standard normal. We assume that σ_t takes two values, σ_h and σ_l, with equal probability. The informed manager observes σ_t at time t, while the uninformed manager does not.

For our test, we again let $\mu = 0.12$ and $\sigma_Q = 0.15$. We set $\sigma_h = 0.20$. The unconditional volatility of stock return then implies that $\sigma_l = 0.07$. In addition to the passive strategy defined in Section 4.1, we consider three active strategies given by $N_t = \mu/(\gamma v_t^2)$, where γ reflects the risk aversion, and $v_t = \sigma_t + \tilde{e}_t$ is a manager's signal about volatility. We use three different distributions of $\tilde{e}_t$ to simulate an uninformed manager with noise, and partially- and fully-informed managers. In particular, when $\sigma_t = \sigma_h$, $\tilde{e}_t$ takes on two values, 0 and $\sigma_l - \sigma_h$, with probability p and $1 - p$, respectively. Similarly, when $\sigma_t = \sigma_l$, $\tilde{e}_t$ is either 0 or $\sigma_h - \sigma_l$, with respective probabilities of p and $1 - p$. Thus, the signal v_t has the same unconditional distribution as σ_t. Straightforward calculations show that the informativeness of the signal depends on the value of p. For instance, if $p = 0.5$, the signal is worthless because $F(\sigma_t|v_t) = F(\sigma_t)$. We set $p = 0.5$, $p = 0.75$, and $p = 1.0$ to simulate an active uninformed manager, and partially- and fully-informed managers, respectively.

Based on simulated trading records of the four strategies over a sample of 400 weeks, we report the summary statistics of their P&L in Panel A of Table 4. With private information only on volatility, the informed manager can generate a Sharpe ratio of 0.171, which is higher than that of the passive strategy (0.107). Panel B of Table 4 reports the return moments conditional on the managers' position being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. Apparently, conditioned on the fully-informed manager's stock position being below average, volatility of stock return is significantly higher than when her position is above average. The stock position of the partially-informed manager is also informative. Panel C gives the results of the Kolmogorov-Smirnov test applied to the conditional return distributions. For the fully-informed manager, the test strongly rejects the null that the two distributions are the same. Rejection rates for the partially-informed manager are high as well, even though her Sharpe Ratio of 0.109 is almost the same as that of the passive strategy.

In Panel C, we also report the revised performance measure of Grinblatt and Titman (1993), the correlation between the manager's stock holding and the stock return in this case. Apparently, it fails to detect any abnormal performance. This is not surprising because in the current setting the manager's information is reflected in the higher moments (volatility) instead of the mean. The example clearly demonstrates that the Grinblatt-Titman measure is restrictive with respect to the nature of the private information a manager may have. In contrast, our methodology, which is based on non-parametric tests, is not. As long as sufficient data are available, our methodology can detect managers with any type of private information on future returns.

4.3 Multiple Assets

So far, we have only considered cases where managers can invest in a single risky asset. In this section, we show that our methodology easily applies to the case of multiple assets.

We assume that managers can invest in 50 stocks. Their price processes are given by

$$Q_{s,t+1} = \mu_s + \sigma_{s,Z} Z_{s,t} + \sigma_s u_{s,t+1}\ , \quad s = 1, \ldots, 25 \tag{17a}$$

$$Q_{s,t+1} = \mu_s + \sigma_{s,t}\ u_{s,t+1}\ , \quad s = 26, \ldots, 50. \tag{17b}$$

the same processes as those considered in Sections 4.1 and 4.2. The informed manager has private information on the mean returns of the first 25 stocks and on the volatility of the remaining 25 stocks. Expected annual stock returns (μ_s) range from 6% to 20%, while unconditional volatility of annual returns ranges from 10% to 19%. Here, $\sigma_{s,t}$ takes two values, σ_h and σ_l, with equal probability. We set $\sigma_l = 0.07$ for all stocks. The value of σ_h for each stock is then determined by its unconditional volatility. For simplicity, we assume that individual price processes are mutually independent. Imposing a factor structure - for example, adding a market factor - does not change the nature of the results.

We simulate trading records of two managers. The first manager, manager 0, is active, but uninformed. His position in stock s is given by $N^0_{s,t} = a^0 + \tilde{n}_{s,t}$, where $a^0 = 1$ and $\tilde{n}_{s,t}$ is normal with a mean of zero and a volatility of 0.5. The second manager, manager

1, has information of varying precision about the mean and volatility of future returns of individual stocks. For the stocks with price process given by (17a), her position is given by $N^1_{s,t} = a^1(1 + \lambda Z_{s,t}/\sigma(Z_s))$. We set $a^1 = 1$ and $\lambda = 2$. R^2, the ratio of variance of the manager's signal to total variance, ranges from 0 to 5%. Thus, the manager does not have superior information for some of the stocks, and she holds a constant number of shares in that case. For the stocks with price process (17b), her position is given by $N^1_{s,t} = \mu_s/(\gamma\, v^2_{s,t})$. As in Section 4.2, $v_{s,t} = \sigma_{s,t} + \tilde{e}_{s,t}$, where the distribution of $\tilde{e}_{s,t}$ is also described. The probability that the manager's signal about the volatility state is correct, p, ranges from 50% to 75%. As pointed out in Section 4.2, when the probability of correct forecast is 50%, the manager essentially trades on noise. This represents the non-informational component of the informed manager's trading. The varying accuracy of informed manager's signals about future stock price movements also makes the setting more realistic.

Simulations in this section use (17a) and (17b) to generate *monthly* stock returns for 36 months. Thus, the data requirements in the multi-asset setting are similar to those used in actual studies of mutual fund investment performance (e.g., Grinblatt and Titman (1993)) .

The reason we can significantly reduce the required number of time-series observations is because managers' trading records for individual securities are aggregated for use in a single test. Specifically, for each stock s, we follow the procedure described in Section 4.1 to partition the sample of its monthly returns into the two subsamples, Q^+_s and Q^-_s, based on the manager's position in s being above and below average. We then pool observations from the two subsamples across individual stocks to form $Q^+ = \bigcup_{s=1}^{50} Q^+_s$ and $Q^- = \bigcup_{s=1}^{50} Q^-_s$. The Kolmogorov-Smirnov test is applied to the distribution functions $\widehat{F}_+$ and $\widehat{F}_-$ estimated from these two samples.

The test results, based on 500 simulations, are in Table 5. Panel A reports the moments of managers' P&L distributions. In addition to the two active strategies, we also show the results for the passive strategy that holds one share of each stock. Compared to the uninformed manager, the P&L distribution of the informed manager has higher mean and variance, as well as larger skewness and kurtosis. Notably, the three strategies produce almost identical Sharpe ratios. This result once again demonstrates that this performance measure is inadequate in detecting an informed manager.

Panel B shows the results of the Kolmogorov-Smirnov test that compares conditional distributions estimated from the Q^+ and Q^- samples. It is clear that our procedure can identify both an informed and an uninformed managers. The rejection rate for the uninformed manager is very close to the theoretical size of the test. For the informed manager, the null hypothesis of no information is strongly rejected. Similar results are obtained when stock price process (17a, 17b) is modified to include a market factor. Thus, the analysis in this section shows that our methodology is applicable in multi-asset settings.

5 Analysis When Positions are Unobservable

The analysis in Section 4 demonstrates how to identify managers with information about asset returns under the assumption that we observe managers' positions. In this section, we show how our methodology can be used when this data is not available, but instead observations of the state variable are available ex post. Such a situation arises if we know the state variable(s) managers claim to have information on, such as earnings or corporate actions, and its realizations. Recall from the discussion in Section 3 that the trading record of manager i in this case is given by

$$H_T^i \equiv (G, Q, X)_{[0,T]} \equiv \left\{(G_1^i, Q_1, X_0), \ldots, (G_T^i, Q_T, X_{T-1})\right\} \tag{18}$$

where G_t^i is the P&L of manager i in period t, and X_t is a state variable correlated with next-period stock return Q_{t+1}. Rejection of the null hypothesis (15) about the conditional distribution of P&L for manager i would imply that this manager has information about X.

We need to condition on the stock return first, and then compare the distribution of manager's P&L conditioned on the ex-post realizations of X. The implementation of this idea is straightforward if we have unlimited amount of data on the managers' trading records. In practice, such data is always limited, and we again face the challenge of finding a powerful test with limited amount of data.

We consider the following algorithm. Given a manager's trading record $(G, Q, X)_{[0,T]}$, we divide the sample into m equal-sized bins according to the value of Q. Each bin now provides a controlled sample in which the values of Q are clustered within a close range. Within each bin, manager's P&L is scaled by the average stock return for that bin. This makes P&L observations from different bins comparable to one another. Let $n \equiv T/m$ be the number of observations in each bin. One can then construct a test based on the sample $\{(X_{t_{k1}-1}, G_{t_{k1}}), \ldots, (X_{t_{kn}-1}, G_{t_{kn}})\}$, where $k = 1, \ldots, m$ denotes the k-th bin. We compute the average realization of the state variable $\bar{X}_k$ for the sub-sample k and then divide the sub-sample into two groups: $J_+(k) \equiv \{j : X_{t_{kj}} > \bar{X}_k\}$ and $J_-(k) \equiv \{j : X_{t_{kj}} \leq \bar{X}_k\}$. Conditional distributions of manager's P&L, $\widehat{F}_+^G(k)$ and $\widehat{F}_-^G(k)$, can be estimated based on the two groups in the sub-sample k as for the IID case. In particular, we can apply the Kolmogorov-Smirnov test to $\widehat{F}_+^G(k)$ and $\widehat{F}_-^G(k)$. However, this approach has the disadvantage of using a small sub-sample, giving the test of low power. In addition, it is hard to aggregate the test results over different bins. We instead adopt an revised procedure. We pool together the two groups from different sub-samples. Specifically, we aggregate the sample points in $J_+(k)$ from all the sub-samples into one sample $J_+ = \cup_{k=1}^m J_+(k)$. Similarly, we set $J_- = \cup_{k=1}^m J_-(k)$. From these two samples, we estimate the distribution functions $\widehat{F}_+^G$ and $\widehat{F}_-^G$. The Kolmogorov-Smirnov test is then conducted on these two conditional distributions. This approach has the advantage of utilizing all available data in a single test.

We apply this testing procedure in the context of the two settings considered in Sections 4.1 and 4.2, when a manager may have private information on the mean and volatility of asset returns, respectively. We generate returns for 400 weeks by using equations (1) and (16), respectively. The simulation is repeated 500 times. We set m, the number of bins used to control for stock return, to 10.

Panel A of Table 6 shows the results for the IID/Mean case. The four strategies considered here have been described in Section 4.1. As a reminder, stock returns in this setup are generated by $Q_{t+1} = \mu + \sigma_Z Z_t + \sigma_u u_{t+1}$, and the informed manager observes Z_t before deciding how much of her portfolio to invest in the stock. Because we do not have access to managers' positions, it is no longer possible to identify the manager who follows a passive buy-and-hold strategy without doing the actual test. However, the test clearly reveals that the uninformed manager has no information about Z, whether he follows a passive or an active strategy. The test also correctly identifies the informed manager; the rejection rate of the null hypothesis that this manager is uninformed is 100% both at the 1% and the 5% confidence level. Moreover, test results do not depend on the manager's trading intensity: test statistics when $\lambda = \lambda^*$ are almost identical to those when $\lambda = 5\lambda^*$.

Panel B of Table 6 shows the results for the IID/Volatility case. Again, the trading strategies of the informed and uninformed managers considered here are the same as the ones analyzed in Section 4.2. In this setting, stock returns are given by $Q_{t+1} = \mu + \sigma_t\, u_{t+1}$, and the informed manager gets a signal about σ_t. The test results show that the uninformed manager has no information about the volatility of the stock return, regardless of the strategy he follows. For the informed manager, the null hypothesis is strongly rejected even when her signal about volatility contains noise. The rejection rate for both partially- and fully-informed strategies is 100%. Not surprisingly, the average value of the Kolmogorov-Smirnov statistic is higher when the manager's signal is more precise.

Of course, the particular implementation developed here needs further examination, especially when applied in alternative information settings. We do so in the next two sections.

6 Extensions and Additional Applications

In this section, we show how our approach works in the context of three additional settings. First, we extend our analysis to allow for non-IID returns. Next, we apply our methodology to a situation where both managers might have private information about future stock return. Finally, we examine the case where a manager can trade in-between the times at which we observe his positions.

6.1 Non-IID Returns

In the previous two sections, we have assumed that stock returns are IID based on public information. This is a restrictive assumption. There is accumulating evidence that asset return distributions are time-varying. In such a situation, our task becomes to identify managers whose information about future returns is not in the public information set. For example, if it is known that the dividend yield predicts future returns, the question is whether a manager has predictors in addition to the dividend yield. Ferson and Khang (2002) have discussed in detail how to extend the methodology of Grinblatt and Titman (1989, 1993) to allow for non-IID returns. They rely on parametric specifications of the return process. The non-parametric nature of our methodology allows to extend it to the situation of non-IID returns without relying on parametric models of asset returns.

In the context of the setting considered in Section 4.1, we assume that stock price increments are generated by the following process:

$$Q_{t+1} = \mu + \sigma_Y Y_t + \sigma_Z Z_t + \sigma_u u_{t+1} \tag{19}$$

where μ, σ_Y, σ_Z and σ_u are positive constants, Y_t, Z_t and u_{t+1} are IID normal with zero mean and volatility 1. Moreover, we assume that Y_t is public information at time t and Z_t is private information, known only to the informed manager. The price process (19) is a straightforward extension of the simple case (1). However, since Y_t is public information at t, stock returns conditioned on this information are no longer IID over time.

As discussed at the end of Section 3.1, our methodology can be easily extended to analyze this case. The implementation of the test follows the algorithm described in Section 5. To control for public information, we first partition the sample into m equal-sized bins according to the value of Y. Each bin now provides a controlled sample in which the values of Y are clustered within a close range. We then estimate the conditional distributions $\widehat{F}_+$ and $\widehat{F}_-$ and apply the Kolmogorov-Smirnov test.

Table 7 reports the results of our test with $\mu = 0.12$, $\sigma_Q = 0.15$, as before, and $\sigma_Y^2/\sigma_Q^2 = \sigma_Z^2/\sigma_Q^2 = 0.04$. For the uninformed manager, we consider two strategies, a passive strategy with $N_t^{0'} = a^0 = 1$, and an active strategy that utilizes public information and sets $N_t^0 = a^0(1 + \lambda^* Y_t)$, where $a^0 = 1$ and $\lambda^* = 1.803$. For the informed manager, we consider her optimal strategy $N_t^1 = a^1\big[1 + \lambda^*(Y_t + Z_t)\big]$ with $a^1 = 1$. Since we have seen earlier that the intensity of the informed manager's trading on private information does not affect the results of our test (as long as $\lambda \neq 0$), we only consider the case when $\lambda = \lambda^*$. We set m, the number of bins used to control for public information, to 10. Sensitivity analysis presented in section 7.2 indicates that, in this setup, using at least five bins does an adequate job of controlling for the common signal. The trading records of the two managers are simulated for 800 weeks. We again run 500 simulations to obtain standard deviations of our estimates.

Summary statistics of the managers' P&L are given in Panel A of Table 7. Clearly, the active strategy of the uninformed manager that uses public information can generate a Sharpe ratio much higher than that of the passive strategy. It also generates positive skewness and higher kurtosis. The informed manager also generates higher Sharpe ratio than the passive strategy, and positive skewness. For the value of λ that we choose for the informed manager, she outperforms the uninformed manager's active strategy as measured by the Sharpe ratio. But this is not a general result. If she trades less aggressively on private information (or on public information for that matter), her Sharpe ratio can end up being lower than that of an active but uninformed manager.

In order to separate the managers according to their private information, we can compare the estimated distributions $\widehat{F}_+$ and $\widehat{F}_-$. Panel B of Table 7 reports the moments of these distributions conditioned on the managers' stock position being above and below average, controlling for the corresponding level of Y. It becomes obvious that the uninformed manager's position, after controlling for Y, contains little information about future returns. The moments of the two conditional distributions are almost identical. The situation is very different for the informed manager. Controlling for Y, the stock return conditioned on her position being above average has significantly higher mean than the stock return conditioned on her position being below-average.

Panel C of Table 7 reports the results of Kolmogorov-Smirnov test on $\widehat{F}_+$ and $\widehat{F}_-$ for both managers. Confirming the results in panel B for the uninformed manager, we cannot reject the null that the two conditional return distributions are the same. For the informed manager, however, we can reject the null at the rate of 92.2% at the 1% level. Such a high rejection rate supports the validity of our algorithm in implementing our methodology.

6.2 Differential Information

Suppose that two managers both might have private information about future returns. A natural question we face is whether one of the managers has information in addition to that of the other manager. For example, knowing that manager 0 has information about future returns, can we tell if another manager, manager 1, has information that is different from or superior to that of manager 0? In this subsection, we consider an example that illustrates how our methodology can be used to answer this question.

As in Section 6.1, we assume that stock returns follow the process given in (19). Before deciding how much to invest in the risky asset at time t, both managers receive signals, S_t^1 and S_t^0, respectively, that contain information about Z_t and/or Y_t. The three cases we consider differ in the types of signals the two managers receive:

$$
\begin{array}{lll}
\text{Scenario 1:} & S_t^1 = Z_t, & S_t^0 = Y_t \\
\text{Scenario 2:} & S_t^1 = Y_t, & S_t^0 = Y_t + e_t^0 \\
\text{Scenario 3:} & S_t^1 = Y_t + e_t^1, & S_t^0 = Y_t + e_t^0
\end{array}
$$

where e_t^1 and e_t^0 are two independent white noise sequences with $\sigma_{e1}^2 = \sigma_{e0}^2 = 2\sigma_Y^2$. Thus, the first scenario describes a situation when the private information sets of the two managers are disjoint. The second scenario corresponds to a setting in which both managers receive a signal about the same "event", but the signal of one of the managers is more precise than the signal of the other manager. The last scenario is similar to the second one, except that now the two managers receive equally precise signals.

The implementation of our approach in this context is similar to that described in Section 6.1. Instead of controlling for public information Y, we need to control for the position of one of the managers, say manager 0, N^0. Under the null hypothesis that the second manager (manager 1 in this case) does not have any additional information, we must have

$$
F_A\big(\cdot \mid N^0\big) = F_{A'}\big(\cdot \mid N^0\big) \quad \forall \;\; A,\; A' \text{ and } N^0 \tag{20}
$$

where $F_S\big(x \mid N^0\big) \equiv F\big(Q_t \leq x \,|\, N_{t-1}^1 \in S,\, N_{t-1}^0 = N^0\big)$ is the cumulative distribution of stock dollar return conditioned on the stock position of manager 0 being $N_{t-1}^0 = N^0$ and the stock position of manager 1 being in the set $S = A, A'$. Given the trading records of the two managers $(Q, N^1, N^0)_{[0,T]}$, we partition the sample into m equal-sized bins according to the value of N^0. We then proceed to estimate the conditional distributions $\widehat{F}_+$ and $\widehat{F}_-$ as in the non-IID case, and apply the Kolmogorov-Smirnov test to these estimated distributions. By interchanging the positions of the two managers in (20) and repeating this procedure, we can test whether manager 0 has information beyond that of manager 1. Notice that if a manager's position is linear in her signal, controlling for her position gives exactly the same results as controlling for the value of the signal itself. If her position is non-linear in the signal, our (unreported) results indicate that this method of controlling for her information is still valid, provided that we use enough bins. The only requirement is that the manager's stock position is monotone in her signal.

For the tests discussed in this section, we assume that the managers' stock position is linear in their signal: $N_t^i = a^i\big(1 + \lambda S_t^i\big)$, $i = 0, 1$. We set $a^0 = a^1 = 1$, $\lambda = 1.803$, and m, the number of bins, to 10. We use (19) to simulate managers' trading records for 800 weeks, and repeat the simulation 500 times.

Test results for the three scenarios are presented in panels A, B, and C, respectively, of Table 8. In the first case, the results in panel A indicate that observing positions of both managers is informative. That is, no matter which manager's position we observe first, knowing the position of the other manager provides additional information about future stock returns. This is not surprising, given that the two managers receive independent, equally precise signals about the expected stock return. Since the signals are uncorrelated, both signals are valuable, and the order of conditioning does not matter.

In the second scenario, manager 1 receives a more precise signal. Therefore, observing his position is informative even if we already know the position of manager 0. The reverse, however, is not true. Results in panel B show that the test correctly distinguishes between the two managers. Controlling for manager 1's position, we cannot reject the null that manager 0 has no additional information. In contrast, the test strongly rejects the null that manager 1's position is uninformative once we know the position of manager 0. The rejection rate is 78% at the 1% level and 92.4% at the 5% level.

In the last scenario we consider, the signals of both managers contain noise. Because the noise components are uncorrelated, observing positions of both managers is more informative than only knowing the position of one of them. The incremental benefit, however, is smaller than in the first scenario. The results in panel C confirm this intuition. The rejection rates are similar for the two managers and are much lower than for the first scenario.

6.3 Incomplete Trading Records

In the discussion of Section 4, we have assumed that we observe the managers' complete trading records, including the times of their trades and the matching P&L. Often, however, we only have a manager's trading record at a given time interval, which is longer than the interval at which he can trade. Revealing records only infrequently gives the uninformed manager more room to find strategies that look profitable. In particular, following option-like strategies, he can generate very high Sharpe ratios in the absence of any private information on future returns. Of course, as discussed earlier, the Sharpe ratio is not an appropriate measure of performance for option-type strategies since they can produce a rich set of return distributions.

Thus, we examine how our methodology works when applied to option-type strategies. Specifically, we consider the following two strategies. The first strategy consists of holding a portfolio of three put options in the proportion of -1000:1000:-8. This position resembles long put spread; the ratio is chosen so that higher moments of the strategy's P&L are similar to those of other strategies. When the position is initiated, all three puts have three weeks to maturity and strikes that are \$0.30, \$0.28, and \$0.26 below the current stock price, respectively. The position is rolled over one week before the options mature. The second strategy involves rolling over a short position in a deep out-of-the-money call option on the stock. When the position is initiated, the call has 12 weeks to maturity and a strike price \$1.24 above the current stock price. Again, the position is rolled over one week before the option expires. We use results in Brennan (1979) to price options under normality. For both option strategies, the positions are scaled so that mean and standard deviation of P&L are comparable to those produced by cash-and-stock strategies. The option strategies can be interpreted as a proxy for an uninformed manager who dynamically adjusts his position between observation times.

Weekly stock returns are generated by simulating (1) for 400 weeks, using the same parameter values as in Section 4.1. Simulation is repeated 500 times to obtain standard deviations of our estimates. Stock price is initialized to \$1 at the beginning of each simulation.

Panel A of Table 9 reports the summary statistics for the two option strategies. For comparison, we have also included the summary of the P&L of an uninformed manager who follows a passive strategy, and of an informed manager who trades actively on her private information. Clearly, the option-type strategies can generate very large Sharpe ratios. The put strategy gives a Sharpe ratio of 0.844, significantly higher than that of the informed manager. The Sharpe ratio of the call strategy (0.284) also exceeds that of the informed manager. In addition, the call strategy P&L has large positive skewness, a characteristic shared by the informed manager's P&L. In this situation, comparing the unconditional distributions of managers' P&L fails to distinguish an informed manager from an uninformed one. As pointed out before, this is one of the main reasons our methodology relies on conditional distributions of stock returns.

Suppose that we do not have complete information on managers' trading records. If, for example, an informed manager is dynamically implementing an option-type strategy that effectively rolls over a short position in calls, we may not be able to identify his strategy based only on his weekly trading record. However, if he has to report the delta of his position on a weekly basis, then our approach can be applied to evaluate him.

Panel B of Table 9 reports the moments of stock return conditioned on the manager's delta being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. The results indicate that there is practically no difference between the two sets of conditional moments. This is not surprising given that the manager has no information about future returns. His current position is only related to past stock price movements, but not to future ones.

Panel C of Table 9 gives the results of the Kolmogorov-Smirnov test. Clearly, we cannot reject the null that conditioned on different delta's of the uninformed manager, the return distributions are the same. Thus, the results in Table 9 confirm that while option-type strategies can generate high Sharpe ratios, our methodology detects that these strategies contain no information about future stock returns.

7 Robustness

In this section, we discuss several additional issues regarding the robustness of our methodology. In Section 7.1, we show how the precision of manager's private signal and the length of her trading record affect the power of our approach to identify the informed manager when she invests in a single risky asset. Section 7.2 presents results on choosing the number of bins used to control for common information.

7.1 Sample Size and Power of the Test

Up to this point, we have focused on developing a methodology to detect an informed manager assuming that we have enough observations to do so. In particular, for the cases where returns are IID to the uninformed investors, we have used a 400-week sample, which roughly corresponds to eight years of data. We have used a sample twice as long when less informed investors have information that makes returns non-IID from their perspective. Whether or not this is a reasonable assumption depends on the application at hand. For example, mutual fund holdings data is only available quarterly. Thus, a portfolio manager would have to have an unusually long career before we can collect a sample of 400 observations. On the other hand, a supervisor at an investment bank trying to assess the performance of a trading desk has access to traders' positions at least at a daily frequency, making our data availability assumptions more realistic. Another factor that affects the power of our tests is the amount of information reflected in a manager's trading strategy. Obviously, the better informed a manager is, the easier it is to identify his strategy as being informative.

In the IID setting of Section 4.1, Table 10 provides sensitivity results for our testing methodology with respect to the precision of manager's information and the available sample size. For three values of R^2, 5%, 2%, and 1%, we report the results of the Kolmogorov-Smirnov test applied to the conditional distributions $\widehat{F}_+$ and $\widehat{F}_-$ estimated when the number of observations per simulation, T, ranges from 50 to 400. When $R^2 = 5\%$, the rejection rate remains quite high, 35% at the 5% level, even when the sample size is reduced to 100 observations. As R^2 decreases, we need more data points to reliably identify the informed manager. For example, when $R^2 = 1\%$, the rejection rate at the 5% level is 31.8% for 400 observations, but drops to 17.4% when we use 200 observations. Overall, the results indicate that the test remains useful even when the sample size is reduced and when the manager's information is less precise.

7.2 Choosing Number of Bins to Control for Common Information

In Section 6.1, we develop a procedure to detect incremental information provided by a manager relative to some common information set. The procedure involves allocating available observations to bins (sub-samples) formed according to the value of the common signal (call it Y). Ideally, we would like Y to be constant within each sub-sample, but this is not possible when Y is a continuous random variable and we have a limited amount of data. The idea behind our approach is to limit variation of Y within each sub-sample. Then, in each sub-sample, most of the variation in manager's position must be caused by other signal, whether it is informative or not. This suggests using as many bins as possible, up to a maximum of $T/2$.[19] However, it might be interesting to see how many bins are sufficient to control for the common signal.

In the non-IID setting of Section 6.1, we show what happens to our test results as the

[19]Since we compute $\widehat{F}_+(k)$ and $\widehat{F}_-(k)$ for each sub-sample k, we need to have at least two observations in each bin.

number of bins used to control for the public signal Y_t increases from two to 100. Since we might not know a priori who the more informed manager is, we do two sets of tests: controlling for manager 0's position, and controlling for manager 1's position, similar to the differential information tests discussed in Section 6.2. Table 11 contains the results of the Kolmogorov-Smirnov test applied to $\widehat{F}_+$ and $\widehat{F}_-$. For comparison, we show the results for $T = 800$ (panel A), and for $T = 400$ (panel B). Apparently, using five or more bins to condition on manager 1's position is sufficient to conclude that manager 0 has no information beyond that of manager 1. If we use two bins, the variation of Y in each sub-sample is large enough to cause significant differences between the two estimated conditional distributions at least some of the time. Controlling for manager 0's position (which in this case is equivalent to controlling for Y directly), the test correctly detects that manager 1's position is informative. The rejection rate declines somewhat as we increase the number of bins, especially when we only have 400 observations. There is a simple explanation for this effect. When we use few bins, both the variation in X and the residual variation in Y contribute to the difference between $\widehat{F}_+$ and $\widehat{F}_-$. As we use more bins, the residual variation in Y diminishes, leaving X as the only source of difference between $\widehat{F}_+$ and $\widehat{F}_-$ and reducing the observed rejection rates. Not surprisingly, this effect is more pronounced for smaller samples.

8 Conclusion

It has been recognized for a long time that traditional performance evaluation methods for money managers have numerous drawbacks. Risk-based framework relies on a specific risk model. Because the risk of an asset typically depends on an investor's information set, any risk model an evaluator uses based on an information set different from that of the manager's will be mis-specified in general. To the extent that the value of a manager comes from the additional information she may bring into the investment process, the first step in evaluating a manager is to determine if she possesses private information. In this paper, we develop a general methodology to identify managers with information on future asset returns based on their trading records. Our methodology does not rely on a specific risk model, such as the Sharpe ratio, CAPM, or APT. It is robust with regard to the nature of private information the managers may have and the trading strategies they follow.

As mentioned in the introduction, our methodology has its limitations. First, it identifies informed managers, but does not assess the value of a manager to a particular investor. Second, the non-parametric nature of the methodology allows it to detect any information a manager may have about future asset returns. The drawback is that it does not tell us what this information is about. Further analysis is needed to answer this question. Third, the implementation of the methodology requires data on the managers' trading records in addition to their P&L. In the two cases we considered, one requires information on the managers' positions and the other requires information regarding the events on which they trade. Moreover, the amount of data needed can be quite substantial in order for the test to have power. For some applications, such as internal evaluation of a trading desk or the evaluation of a hedge fund by its investors, the required data is available. But for other applications, such as the evaluation of mutual funds, the data can be hard to come by. For these reasons, our methodology should be viewed as a complement to the conventional methods.

References

Admati, Anat R., and Stephen A. Ross, 1985, Measuring investment performance in a rational equilibrium model, *Journal of Business* 58, 1-26.

Bollerslev Tim, Ray Y. Chou and Kenneth F. Kroner, 1992, ARCH modeling in finance: A review of the theory and empirical evidence, *Journal of Econometrics* 52, 5-59.

Brennan, Michael J., 1979, The pricing of contingent claims in discrete time models, *Journal of Finance* 34, 53-68.

Campbell, John Y., and Robert J. Shiller, 1988, Stock prices, earnings, and expected dividends, *Journal of Finance* 43, 661-676.

Chance, Don M., and Michael L. Hemler, 2001, The performance of professional market timers: Daily evidence from executed strategies, *Journal of Financial Economics* 62, 377-411.

Chen, Zhiwu, and Peter J. Knez, 1996, Portfolio performance measurement: Theory and applications, *Review of Financial Studies* 9, 511-555.

Copeland, Thomas E., and David Mayers, 1982, The Value Line enigma (1965-1978): A case study of performance evaluation issues, *Journal of Financial Economics* 10, 289-321.

Cornell, Bradford, 1979, Asymmetric information and portfolio performance measurement, *Journal of Financial Economics* 7, 381-390.

Cumby, Robert E., and David M. Modest,1987, Testing for market timing ability: A framework for forecast evaluation, *Journal of Financial Economics* 19, 169-189.

Dybvig, Philip H., and Stephen A. Ross, 1985, Differential information and performance measurement using a security market line, *Journal of Finance* 40, 383-399.

Dybvig, Philip H., and Gregory A. Willard, Empty promises and arbitrage, *Review of Financial Studies* 12, 807-834.

Fama, Eugene F., and Kenneth R. French, 1988a, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3-25.

Fama, Eugene F., and Kenneth R. French, 1988b, Permanent and temporary components of stock prices, *Journal of Political Economy* 96, 246-273.

Farnsworth, Heber K., Wayne E. Ferson, David Jackson, and Steven Todd, 2002, Performance evaluation with stochastic discount factors, *Journal of Business* 75, 473-503.

Ferson, Wayne E., 2002, Tests of multifactor pricing models, volatility bounds and portfolio performance, Forthcoming in *Handbook of the Economics of*

Finance, George M. Constantinides, Milton Harris and Rene M. Stulz, editors, Elsevier Science Publishers, North Holland.

Ferson, Wayne E., and Kenneth Khang, 2002, Conditional performance measurement using portfolio weights: Evidence for pension funds, *Journal of Financial Economics* 65, 249-282.

Ferson, Wayne E., and Rudi W. Schadt, 1996, Measuring fund strategy and performance in changing economic conditions, *Journal of Finance* 51, 425-461.

Glosten, Lawrence R., and Ravi Jagannathan, 1994, A contingent claim approach to performance evaluation, *Journal of Empirical Finance* 1, 133-160.

Goetzmann, William N., Jonathan Ingersoll Jr., and Zoran Ivkoviĉ, 2000, Monthly measurement of daily timers, *Journal of Financial and Quantitative Analysis* 35, 257-290.

Graham, John R., and Campbell R. Harvey, 1996, Market timing ability and volatility implied in investment newsletters' asset allocation recommendations, *Journal of Financial Economics* 42, 397-421.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393-408.

Grinblatt, Mark, and Sheridan Titman, 1989, Portfolio performance evaluation: Old issues and new insights, *Review of Financial Studies* 2, 393-421.

Grinblatt, Mark, and Sheridan Titman, 1993, Performance measurement without benchmarks: An examination of mutual fund returns, *Journal of Business* 66, 47-68.

Henriksson, Roy D., and Robert C. Merton, 1981, On market timing and investment performance. II. Statistical procedures for evaluating forecasting skills, *Journal of Business* 54, 513-533.

Hollander, Myles, and Douglas A. Wolfe, 1999, *Nonparametric Statistical Methods*, New York, NY: John Wiley.

Jagannathan, Ravi, and Robert A. Korajczyk, 1986, Assessing the market timing performance of managed portfolios, *Journal of Business* 59, 217-235.

Jensen, Michael, 1969, Risk, the pricing of capital assets, and the evaluation of investment portfolios, *Journal of Business* 42, 167-247.

Jensen, Michael, 1972, Optimal utilization of market forecasts and the evaluation of investment portfolio performance, In Giorgio P. Szegö and Karl Shell, *Mathematical Methods in Investment and Finance*, Amsterdam: North Holland.

Kim, P.J., and Robert I. Jennrich, 1973, Tables of the exact sampling distribution of the two-sample Kolmogorov-Smirnov criterion, D_{mn}, $m \leq n$. In H.

Leon Harter and Donald B. Owen, *Selected Tables in Mathematical Statistics*, Volume 1, Providence, RI: American Mathematical Society and Institute of Mathematical Statistics, 79-170.

Kothari, S.P., and Jay Shanken, 1997, Book-to-market, dividend yield, and expected market returns: A time-series analysis, *Journal of Financial Economics* 44, 169-203.

Kyle, Albert S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1335.

Lo, Andrew, Harry Mamaysky, and Jiang Wang, 2001, Foundations of technical analysis: Computational algorithms, statistical analysis and empirical results, *Journal of Finance* 55, 1705-1770.

Lo, Andrew and A. Craig MacKinlay, 1988, Stock prices do not follow random walks: Evidence from a simple specification test, *Review of Financial Studies* 1, 41-66.

Merton, Robert C., 1981, On market timing and investment performance. I. An equilibrium theory of value for market forecasts, *Journal of Business* 54, 363-406.

Milgrom, Paul R., and Nancy L. Stokey, 1982, Information, trade, and common knowledge, *Journal of Economic Theory* 26, 17-27.

Poterba, James, and Lawrence Summers, 1988, Mean reversion in stock prices: Evidence and implications, *Journal of Financial Economics* 22, 27-59.

Roll, Richard, 1978, Ambiguity when performance is measured by the securities market line, *Journal of Finance* 33, 1051-1069.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 8, 343-362.

Sharpe, William F., 1966, Mutual fund performance, *Journal of Business* 39, 119-138.

Treynor, Jack L., and Fischer Black, 1973, How to use security analysis to improve portfolio selection, *Journal of Business* 46, 66-86.

Wang, Jiang, 1993, A model of intertemporal asset prices under asymmetric information, *Review of Economic Studies* 60, 249-282.

Wang, Jiang, 1994, A Model of Competitive Stock Trading Volume, *Journal of Political Economy* 102, 127-168.

Table 1: Moments of managers' P&L and stock return. Panel A shows weekly P&L moments for the uninformed and informed managers' strategies. The uninformed manager's stock position is $N_t = a^0$ (i.e. he follows a passive strategy). The informed manager's stock position is $N_t = a^1(1+\lambda Z_t)$, where λ, her trading intensity, takes on one of two values: $\lambda = \lambda^*$ or $\lambda = 5\lambda^*$. Panel B shows unconditional and conditional moments of weekly stock return, demeaned by the unconditional mean. We condition on the informed manager's stock position being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. Stock return is given by $Q_{t+1} = \mu + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Z^2/\sigma_Q^2 = 0.01$, $a^0 = a^1 = 1$, $\lambda^* = 0.901$.

A. Moments of Managers' P&L

	Uninformed Manager	Informed Manager	
		$\lambda = \lambda^*$	$\lambda = 5\lambda^*$
Mean (%)	0.231	0.418	1.168
S.D. (%)	2.080	2.830	9.727
Skewness	0.000	0.609	0.706
Kurtosis	3.000	7.496	9.421
Sharpe ratio	0.111	0.148	0.120

B. Moments of Stock Dollar Return

	Unconditional	Conditional			
		$\lambda = \lambda^*$		$\lambda = 5\lambda^*$	
		$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$
Mean (%)	0.000	0.166	-0.166	0.166	-0.166
S.D. (%)	2.080	2.073	2.073	2.073	2.073
Skewness (x100)	0.000	0.022	-0.022	0.022	-0.022
Kurtosis	3.000	3.000	3.000	3.000	3.000

Table 2: Summary statistics for weekly managers' P&L and stock return (IID/Mean case). Panel A shows moments of the uninformed and informed managers' P&L. We consider two strategies for the uninformed manager: passive ($N_t = a^0$), and active with noise ($N_t = a^0 + \tilde{n}_t$). The informed manager's stock position is $N_t = a^1(1 + \lambda Z_t)$, where λ, her trading intensity, takes on one of two values: $\lambda = \lambda^*$, or $\lambda = 5\lambda^*$. Panel B shows moments of stock return conditioned on active managers' stock position being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. Stock return is given by $Q_{t+1} = \mu + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Z^2/\sigma_Q^2 = 0.05$, $a^0 = a^1 = 1$, $\lambda^* = 2.016$, $\sigma_{\tilde{n}} = 0.5$. Average moments and the corresponding standard deviations (shown in parentheses) are based on 500 simulations.

A. Moments of Managers' P&L

	Uninformed		Informed	
	Passive	Active	$\lambda = \lambda^*$	$\lambda = 5\lambda^*$
Mean (%)	0.231	0.234	1.176	4.964
	(0.105)	(0.115)	(0.247)	(1.074)
S.D. (%)	2.084	2.331	4.842	21.635
	(0.076)	(0.119)	(0.354)	(1.725)
Skewness	0.003	0.114	1.280	1.250
	(0.123)	(0.313)	(0.550)	(0.570)
Kurtosis	2.997	5.038	9.262	9.236
	(0.247)	(1.138)	(3.470)	(3.473)
Sharpe Ratio	0.111	0.101	0.242	0.229
	(0.051)	(0.049)	(0.046)	(0.045)

B. Moments of Stock Dollar Return

	Unconditional	Conditional					
		Uninformed		$\lambda = \lambda^*$		$\lambda = 5\lambda^*$	
		$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$
Mean (%)	0.231	0.237	0.225	0.604	-0.141	0.604	-0.132
	(0.105)	(0.148)	(0.149)	(0.151)	(0.150)	(0.142)	(0.152)
S.D. (%)	2.084	2.082	2.084	2.049	2.050	2.035	2.051
	(0.076)	(0.104)	(0.107)	(0.105)	(0.099)	(0.110)	(0.101)
Skewness	0.003	-0.008	0.015	0.005	0.000	0.005	0.007
	(0.123)	(0.172)	(0.173)	(0.172)	(0.168)	(0.180)	(0.168)
Kurtosis	2.997	2.987	2.980	2.994	2.979	2.997	2.976
	(0.247)	(0.346)	(0.337)	(0.370)	(0.335)	(0.352)	(0.353)

Table 3: Kolmogorov-Smirnov test results for the conditional distribution of stock return (IID/Mean case). We estimate cumulative distribution of weekly stock return conditioned on manager's stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$). We also estimate the unconditional cumulative distribution of weekly stock return ($\widehat{F}$). We then apply the Kolmogorov-Smirnov test to three pairs of distributions: $\widehat{F}_+$ versus $\widehat{F}$, $\widehat{F}_-$ versus $\widehat{F}$, and $\widehat{F}_+$ versus $\widehat{F}_-$. Panel A reports the average Kolmogorov-Smirnov test statistic, δ, its standard deviation (shown in parentheses), and the rejection rates at the 1% and 5% level based on 500 simulations. The active uninformed manager's stock position is $N_t = a^0 + \tilde{n}_t$. The informed manager's stock position is $N_t = a^1(1 + \lambda Z_t)$, where λ, her trading intensity, takes on one of two values: $\lambda = \lambda^*$, or $\lambda = 5\lambda^*$. Panel B reports the covariance between the manager's stock holding and the stock return, which is the performance measure used in Grinblatt and Titman (1993). It also reports a revised measure, the correlation between stock holding and the stock return. Stock return is given by $Q_{t+1} = \mu + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Z^2/\sigma_Q^2 = 0.05$, $a^0 = a^1 = 1$, $\lambda^* = 2.016$, $\sigma_{\tilde{n}} = 0.5$.

Panel A. Kolmogorov-Smirnov Test On Conditional Stock Dollar Return Distributions

	Uninformed	Informed	
	Active	$\lambda = \lambda^*$	$\lambda = 5\lambda^*$
		$\widehat{F}_+$ vs. $\widehat{F}$	
Kolmogorov-Smirnov δ	0.51	1.08	1.08
	(0.15)	(0.25)	(0.25)
Rejection at 1% (%)	0.00	2.00	2.60
Rejection at 5% (%)	0.00	15.60	13.40
		$\widehat{F}_-$ vs. $\widehat{F}$	
Kolmogorov-Smirnov δ	0.51	1.08	1.08
	(0.15)	(0.26)	(0.25)
Rejection at 1% (%)	0.00	2.20	1.60
Rejection at 5% (%)	0.00	15.80	14.40
		$\widehat{F}_+$ vs. $\widehat{F}_-$	
Kolmogorov-Smirnov δ	0.87	1.87	1.86
	(0.25)	(0.44)	(0.43)
Rejection at 1% (%)	0.80	70.20	68.60
Rejection at 5% (%)	4.00	87.40	88.40

Panel B. Grinblatt-Titman Performance Measures

	Uninformed		Informed	
	Passive	Active	$\lambda = \lambda^*$	$\lambda = 5\lambda^*$
Covariance between holding and return	0.000	0.000	0.009	0.047
	(0.000)	(0.001)	(0.002)	(0.011)
Correlation between holding and return	0.000	0.003	0.224	0.224
	(0.000)	(0.048)	(0.049)	(0.049)

Table 4: Summary statistics and Kolmogorov-Smirnov test results for the IID/Volatility case. Panel A shows summary statistics for weekly P&L of the uninformed and informed managers. The two strategies of the uninformed manager are passive ($N_t = a^0$), and active with noise ($N_t = \mu/[\gamma(\sigma_t + \tilde{e}_{0t})^2]$). The informed manager is either fully informed, with $N_t = \mu/(\gamma\sigma_t^2)$, or partially informed, with $N_t = \mu/[\gamma(\sigma_t + \tilde{e}_{1t})^2]$. Panel B shows moments of weekly stock return conditioned on active managers' stock position being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. Panel C shows results of the Kolmogorov-Smirnov test on the distributions of weekly stock return conditioned on active managers' stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$). It also reports the correlation between the managers' holdings of the stock and its returns, a modified Grinblatt-Titman performance measure. Stock return is given by $Q_{t+1} = \mu + \sigma_t u_{t+1}$. Here, σ_t is either 0.20 or 0.07, with equal probability, and $\tilde{e}_t$ can be $\sigma_l - \sigma_h$, 0, or $\sigma_h - \sigma_l$ with probabilities $0.5(1-p)$, p, and $0.5(1-p)$, respectively, where $p_0 = 0.5$, $p_1 = 0.75$. Other parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $a^0 = 1$. Average statistics, standard deviations (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations.

A. Summary Statistics of Managers' P&L

	Uninformed		Informed	
	Passive	Active	with Noise	without Noise
Mean (%)	0.222	0.562	0.570	0.574
	(0.103)	(0.323)	(0.264)	(0.172)
S.D. (%)	2.076	6.715	5.261	3.354
	(0.098)	(0.479)	(0.442)	(0.159)
Skewness	-0.023	0.129	0.177	0.298
	(0.223)	(0.487)	(0.767)	(0.232)
Kurtosis	4.721	8.972	11.538	4.698
	(0.526)	(1.464)	(2.826)	(0.565)
Sharpe Ratio	0.107	0.084	0.109	0.171
	(0.050)	(0.048)	(0.050)	(0.051)

B. Moments of Conditional Stock Dollar Return

	Unconditional	Conditional					
		Uninformed (Active)		Informed w/Noise		Informed w/o Noise	
		$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$
Mean (%)	0.222	0.223	0.221	0.226	0.217	0.228	0.214
	(0.103)	(0.141)	(0.148)	(0.114)	(0.168)	(0.069)	(0.189)
S.D. (%)	2.076	2.090	2.057	1.619	2.445	0.986	2.764
	(0.098)	(0.143)	(0.137)	(0.135)	(0.137)	(0.049)	(0.133)
Skewness	-0.023	-0.044	0.000	-0.042	-0.010	-0.008	-0.012
	(0.223)	(0.307)	(0.320)	(0.526)	(0.216)	(0.170)	(0.161)
Kurtosis	4.721	4.653	4.666	6.244	3.624	2.961	2.959
	(0.526)	(0.732)	(0.779)	(1.512)	(0.456)	(0.342)	(0.320)

C. Kolmogorov-Smirnov Test On Conditional Stock Dollar Return Distributions ($\widehat{F}_+$ vs. $\widehat{F}_-$)

	Uninformed		Informed	
	Passive	Active	with Noise	without Noise
K.-S. δ		0.89	1.64	2.73
		(0.25)	(0.28)	(0.26)
Rej. rate at 1% (%)		1.60	49.40	100.00
Rej. rate at 5% (%)		5.00	84.00	100.00
Correlation of holding and return	0.000	0.001	-0.003	-0.003
	(0.000)	(0.050)	(0.048)	(0.051)

Table 5: Summary statistics and Kolmogorov-Smirnov test results for the Multiple Assets case. Panel A shows summary statistics for monthly P&L of the uninformed and informed managers. Managers invest in 50 stocks. Monthly returns for the first 25 stocks are generated by $Q_{s,t+1} = \mu_s + \sigma_{s,z} Z_{s,t} + \sigma_s u_{s,t+1}$. Monthly returns for the remaining 25 stocks are generated by $Q_{s,t+1} = \mu_s + \sigma_{s,t} u_{s,t+1}$. Expected annual stock returns (μ_s) range from 6 to 20 percent. Unconditional volatility of annual returns ranges from 10 to 19 percent. Passive uninformed manager holds one share of each stock. The position of active uninformed manager in stock s is given by $N^0_{s,t} = a^0 + \tilde{n}_{s,t}$, where $a^0 = 1$ and $\tilde{n}_{s,t}$ is normal with a mean of zero and a volatility of 50%. The position of the informed manager in any of the first 25 stocks is given by $N^1_{s,t} = a^1(1 + \lambda Z_{s,t})$. Here, $a^1 = 1$, $\lambda = 2$, and R^2, the ratio of variance of the manager's signal Z_s to total variance, ranges between 0 and 5 percent. Her position in the remaining 25 stocks is given by $N^1_{s,t} = \mu_s/(\gamma\, v^2_{s,t})$. Here, $\gamma = 5\frac{1}{3}$ and $v_{s,t} = \sigma_{s,t} + \tilde{e}_{s,t}$. See Section 4.2 and Table 4 for the description of $v_{s,t}$. p, the probability that the manager's signal about the volatility state is correct, ranges from 50 to 75 percent. Panel B shows the results of the Kolmogorov-Smirnov test. The average Kolmogorov-Smirnov test statistic for the comparison of $\widehat{F}_+$ vs. $\widehat{F}_-$, δ, its standard deviation (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations.

A. Summary Statistics of Managers' P&L

	Uninformed		Informed
	Passive	Active	
Mean (%)	0.970	0.970	2.355
	(0.102)	(0.113)	(0.267)
S.D. (%)	4.284	4.808	11.335
	(0.092)	(0.140)	(0.425)
Skewness	0.105	0.382	0.818
	(0.114)	(0.279)	(0.388)
Kurtosis	4.415	7.575	11.664
	(0.336)	(1.488)	(2.179)
Sharpe Ratio	0.227	0.202	0.208
	(0.024)	(0.024)	(0.023)

B. Kolmogorov-Smirnov Test on Conditional Stock Dollar Return Distributions ($\widehat{F}_+$ vs. $\widehat{F}_-$)

	Uninformed	Informed
Kolmogorov-Smirnov δ	0.88	1.75
	(0.26)	(0.40)
Rejection rate at 1% (%)	1.20	60.80
Rejection rate at 5% (%)	6.40	84.60

Table 6: Kolmogorov-Smirnov test results for the conditional distribution of manager's P&L when managers' positions are unobservable. After controlling for the observed stock return, we estimate the cumulative distribution of manager's weekly P&L conditioned on the realized value of a state variable being above average ($\widehat{F}_{+}^{G}$) and below average ($\widehat{F}_{-}^{G}$). We use 10 bins to control for the variation in stock return. Panel A shows the results for the IID/Mean case. Stock returns are generated by $Q_{t+1} = \mu + \sigma_Z Z_t + \sigma_u u_{t+1}$. The state variable is Z_t. See Tables 2 and 3 for the description of managers' strategies and parameter values. Panel B shows the results for the IID/Volatility case. Stock returns are generated by $Q_{t+1} = \mu + \sigma_t u_{t+1}$. The state variable is σ_t. See Table 4 for the description of managers' strategies and parameter values. The average Kolmogorov-Smirnov test statistic for the comparison of $\widehat{F}_{+}^{G}$ vs. $\widehat{F}_{-}^{G}$, δ, its standard deviation (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations.

A. IID/Mean case

	Uninformed		Informed	
	Passive	Active	$\lambda = \lambda^*$	$\lambda = 5\lambda^*$
Kolmogorov-Smirnov δ	0.87	0.89	7.82	7.95
	(0.27)	(0.26)	(0.41)	(0.44)
Rejection rate at 1% (%)	0.80	0.80	100.00	100.00
Rejection rate at 5% (%)	6.00	5.80	100.00	100.00

B. IID/Volatility case

	Uninformed		Informed	
	Passive	Active	with Noise	w/o Noise
Kolmogorov-Smirnov δ	0.97	0.95	4.72	9.39
	(0.25)	(0.27)	(0.43)	(0.24)
Rejection rate at 1% (%)	1.80	2.60	100.00	100.00
Rejection rate at 5% (%)	8.00	7.80	100.00	100.00

Table 7: Summary statistics and Kolmogorov-Smirnov test results for the non-IID case. Panel A shows weekly P&L moments for the uninformed and informed managers' strategies. We consider two strategies for the uninformed manager: passive ($N_t = a^0$), and active ($N_t = a^0(1 + \lambda^* Y_t/\sigma_Y)$). The informed manager's stock position is $N_t = a^1\left[1 + \lambda^*(Y_t + Z_t)\right]$. Panel B shows moments of weekly stock return conditioned on active managers' stock position being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. For comparison, we also provide moments of the unconditional weekly stock returns. Panel C shows results of the Kolmogorov-Smirnov test on the distributions of weekly stock return conditioned on active managers' stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$). Stock return is given by $Q_{t+1} = \mu + \sigma_Y Y_t + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Y^2/\sigma_Q^2 = \sigma_Z^2/\sigma_Q^2 = 0.04$, $a^0 = a^1 = 1$, $\lambda^* = 1.803$. Average statistics, standard deviations (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations.

A. Summary Statistics of Managers' P&L

	Uninformed		Informed
	Passive	Active	($\lambda = \lambda^*$)
Mean (%)	0.230	0.982	1.294
	(0.074)	(0.155)	(0.164)
S.D. (%)	2.084	4.421	4.490
	(0.052)	(0.219)	(0.243)
Skewness	-0.003	1.187	1.499
	(0.087)	(0.385)	(0.375)
Kurtosis	2.994	9.225	9.691
	(0.166)	(2.481)	(2.937)
Sharpe Ratio	0.110	0.222	0.288
	(0.036)	(0.033)	(0.031)

B. Moments of Conditional Stock Dollar Return

	Unconditional	Conditional			
		Uninformed		Informed	
		$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$
Mean (%)	0.230	0.247	0.213	0.564	-0.104
	(0.074)	(0.100)	(0.109)	(0.103)	(0.106)
S.D. (%)	2.084	2.081	2.086	2.052	2.062
	(0.052)	(0.073)	(0.073)	(0.075)	(0.070)
Skewness	-0.003	0.001	-0.007	0.004	-0.005
	(0.087)	(0.123)	(0.115)	(0.115)	(0.125)
Kurtosis	2.994	2.988	2.984	2.982	2.983
	(0.166)	(0.240)	(0.240)	(0.250)	(0.231)

C. Kolmogorov-Smirnov Test on Conditional Stock Dollar Return Distributions ($\widehat{F}_+$ vs. $\widehat{F}_-$)

	Uninformed	Informed
Kolmogorov-Smirnov δ	0.88	2.21
	(0.26)	(0.41)
Rejection Rate at 1% (%)	0.60	92.20
Rejection Rate at 5% (%)	5.80	98.80

Table 8: Kolmogorov-Smirnov test results for the conditional distribution of stock return when both managers have private information. After controlling for stock position of one of the two managers, we estimate the cumulative distribution of weekly stock return conditioned on the second manager's stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$). Panels A, B, and C show the results of applying the Kolmogorov-Smirnov test to $\widehat{F}_+$ and $\widehat{F}_-$ in three different information settings. In panel A, stock positions of managers 0 and 1 are given by $N_t^0 = a^0(1+\lambda Y_t)$, $N_t^1 = a^1(1+\lambda Z_t)$. In panel B, stock positions of managers 0 and 1 are given by $N_t^0 = a^0\big[1+\lambda(Y_t + e_t^0)\big]$, $N_t^1 = a^1(1+\lambda Y_t)$. In panel C, stock positions of managers 0 and 1 are given by $N_t^0 = a^0\big[1+\lambda(Y_t + e_{0t})\big]$, $N_t^1 = a^1\big[1+\lambda(Y_t + e_t^1)\big]$. Stock return is given by $Q_{t+1} = \mu + \sigma_Y Y_t + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Y^2/\sigma_Q^2 = \sigma_Z^2/\sigma_Q^2 = 0.04$, $a^0 = a^1 = 1$, $\lambda = 1.803$. Here, e_t^1 and e_t^0 are two independent white noise sequences with $\sigma_{e1}^2 = \sigma_{e0}^2 = 2\sigma_Y^2$. Average test statistics, standard deviations (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations.

A. Manager 0 observes Y, Manager 1 observes Z

	First Controlling for	
	Manager 1's Position	Manager 0's position
Kolmogorov-Smirnov δ	2.22	2.22
	(0.43)	(0.43)
Rejection rate at 1% (%)	91.60	91.60
Rejection rate at 5% (%)	98.20	98.60

B. Manager 0 observes Y with noise, Manager 1 observes Y

	First Controlling for	
	Manager 1's Position	Manager 0's position
Kolmogorov-Smirnov δ	0.87	1.94
	(0.26)	(0.43)
Rejection rate at 1% (%)	1.20	78.00
Rejection rate at 5% (%)	4.40	92.40

C. Manager 0 observes Y with noise, Manager 1 observes Y with noise

	First Controlling for	
	Manager 1's Position	Manager 0's position
Kolmogorov-Smirnov δ	1.22	1.25
	(0.37)	(0.35)
Rejection rate at 1% (%)	14.00	15.20
Rejection rate at 5% (%)	32.60	36.40

Table 9: Summary statistics and Kolmogorov-Smirnov test results for the Incomplete Trading Records case. Panel A shows weekly P&L moments for the option and non-option strategies. The two non-option strategies are the uninformed manager's passive strategy ($N_t = a^0$), and the informed manager's active strategy ($N_t = a^1(1+\lambda^* Z_t)$). The first option strategy consists of holding a portfolio of three 3-week puts in proportion of -1000:1000:-8. The respective strikes are set \$0.30, \$0.28, and \$0.26 below the current stock price. The position is rolled over one week before the options mature. The second option strategy involves shorting a 12-week call with a strike \$1.24 above the current stock price, and rolling over the short position one week before the option expires. The implied stock position for both strategies is obtained by computing the positions' delta. Panel B shows moments of weekly stock return conditioned on active managers' stock position being above ($N > \bar{N}$) and below ($N \leq \bar{N}$) its average. Panel C shows results of the Kolmogorov-Smirnov test on the distributions of weekly stock return conditioned on active managers' stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$). Stock return is given by $Q_{t+1} = \mu + \sigma_Z Z_t + u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Z^2/\sigma_Q^2 = 0.05$, $a^0 = a^1 = 1$, $\lambda^* = 2.016$. Average statistics, standard deviations (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations.

A. Summary Statistics of Managers' P&L

	Uninformed	Informed	Put Strategy	Call Strategy
Mean (%)	0.231	1.176	0.531	0.221
	(0.105)	(0.247)	(0.005)	(0.000)
S.D. (%)	2.084	4.842	1.734	4.763
	(0.076)	(0.354)	(11.582)	(32.021)
Skewness	0.003	1.280	-0.055	2.553
	(0.123)	(0.550)	(0.838)	(0.693)
Kurtosis	2.997	9.262	25.199	30.986
	(0.247)	(3.470)	(45.271)	(54.031)
Sharpe Ratio	0.111	0.242	0.844	0.284
	(0.051)	(0.046)	(0.219)	(0.085)

B. Moments of Conditional Stock Dollar Return

	Unconditional	Conditional					
		Informed		Put Strategy		Call Strategy	
		$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$	$N > \bar{N}$	$N \leq \bar{N}$
Mean (%)	0.231	0.604	-0.141	0.228	0.233	0.229	0.251
	(0.105)	(0.151)	(0.150)	(0.151)	(0.156)	(0.113)	(0.339)
S.D. (%)	2.084	2.049	2.050	2.082	2.085	2.085	2.072
	(0.076)	(0.105)	(0.099)	(0.103)	(0.112)	(0.082)	(0.272)
Skewness	0.003	0.005	0.000	0.003	-0.004	0.001	0.026
	(0.123)	(0.172)	(0.168)	(0.182)	(0.187)	(0.126)	(0.396)
Kurtosis	2.997	2.994	2.979	2.980	2.994	2.992	2.876
	(0.247)	(0.370)	(0.335)	(0.336)	(0.375)	(0.257)	(0.715)

C. Kolmogorov-Smirnov Test on Conditional Stock Dollar Return Distributions ($\widehat{F}_+$ vs. $\widehat{F}_-$)

	Informed	Put Strategy	Call Strategy
K.-S. δ	1.87	0.89	0.88
	(0.44)	(0.27)	(0.25)
Rej. Rate at 1% (%)	70.20	1.80	0.60
Rej. Rate at 5% (%)	87.40	7.60	5.00

Table 10: Sensitivity of the Kolmogorov-Smirnov test results (IID/Mean case). We estimate the sensitivity of rejection rates of the Kolmogorov-Smirnov test with respect to the precision of informed manager's information (as measured by R^2) and to the length of her trading record (T) available. The test is applied to the distributions of weekly stock return conditioned on the manager's stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$). The average Kolmogorov-Smirnov test statistic, δ, its standard deviation (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations. The informed manager's stock position is $N_t = a^1(1 + \lambda Z_t)$. Stock return is given by $Q_{t+1} = \mu + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $a^1 = 1$, $\lambda = 2.016$. $R^2 \equiv \sigma_Z^2/\sigma_Q^2$ is 5% in panel A, 2% in panel B, and 1% in panel C.

	A. $R^2 = 5\%$			
	$T = 400$	$T = 200$	$T = 100$	$T = 50$
Kolmogorov-Smirnov δ	1.87	1.48	1.23	1.08
	(0.44)	(0.39)	(0.38)	(0.33)
Rejection rate at 1% (%)	70.20	35.20	15.60	6.20
Rejection rate at 5% (%)	87.40	62.40	35.00	19.20
	B. $R^2 = 2\%$			
	$T = 400$	$T = 200$	$T = 100$	$T = 50$
Kolmogorov-Smirnov δ	1.39	1.17	1.04	0.98
	(0.41)	(0.36)	(0.35)	(0.28)
Rejection rate at 1% (%)	29.60	10.40	7.00	2.20
Rejection rate at 5% (%)	51.40	28.40	18.20	9.40
	C. $R^2 = 1\%$			
	$T = 400$	$T = 200$	$T = 100$	$T = 50$
Kolmogorov-Smirnov δ	1.17	1.04	0.96	0.94
	(0.38)	(0.33)	(0.32)	(0.27)
Rejection rate at 1% (%)	12.60	5.60	5.20	1.80
Rejection rate at 5% (%)	31.80	17.40	12.40	6.60

Table 11: Sensitivity of the Kolmogorov-Smirnov test results (non-IID case). We estimate the sensitivity of the Kolmogorov-Smirnov test with respect to the number of bins used to control for the information of one of the two managers. The test is applied to the distributions of weekly stock return conditioned on the second manager's stock position being above average ($\widehat{F}_+$) and below average ($\widehat{F}_-$), *after* controlling for the position of the first manager. The average Kolmogorov-Smirnov test statistic, δ, its standard deviation (shown in parentheses), and the rejection rates at the 1% and 5% level are based on 500 simulations. We assume that the trading record available is either 800 observations (panel A), or 400 observations (panel B). Stock positions of managers 0 and 1 are given by $N_t^0 = a^0(1 + \lambda^* Y_t)$, $N_t^1 = a^1\big[1 + \lambda^*(Y_t + Z_t)\big]$. Stock return is given by $Q_{t+1} = \mu + \sigma_Y Y_t + \sigma_Z Z_t + \sigma_u u_{t+1}$. Parameter values are: $\mu = 0.12$ (annual), $\sigma_Q = 0.15$ (annual), $\sigma_Y^2/\sigma_Q^2 = \sigma_Z^2/\sigma_Q^2 = 0.04$, $a^0 = a^1 = 1$, $\lambda^* = 1.803$.

A. $T = 800$

	# Bins = 100		# Bins = 50		# Bins = 10		# Bins = 5		# Bins = 2	
First Conditioning on:	Mgr 1	Mgr 0	Mgr 1	Mgr 0	Mgr 1	Mgr 0	Mgr 1	Mgr 0	Mgr 1	Mgr 0
Kolmogorov-Smirnov δ	0.87	2.13	0.88	2.17	0.86	2.25	0.90	2.28	1.24	2.48
	(0.27)	(0.43)	(0.26)	(0.43)	(0.26)	(0.44)	(0.27)	(0.44)	(0.37)	(0.44)
Rejection rate at 1% (%)	1.40	87.60	1.40	90.00	0.80	92.20	2.00	94.00	15.00	98.00
Rejection rate at 5% (%)	6.00	96.80	6.40	97.20	5.20	98.80	6.00	99.60	34.60	99.60

B. $T = 400$

	# Bins = 100		# Bins = 50		# Bins = 10		# Bins = 5		# Bins = 2	
First Conditioning on:	Mgr 1	Mgr 0	Mgr 1	Mgr 0	Mgr 1	Mgr 0	Mgr 1	Mgr 0	Mgr 1	Mgr 0
Kolmogorov-Smirnov δ	0.87	1.55	0.87	1.64	0.89	1.70	0.89	1.74	1.07	1.88
	(0.25)	(0.38)	(0.25)	(0.40)	(0.26)	(0.41)	(0.25)	(0.41)	(0.35)	(0.41)
Rejection rate at 1% (%)	1.20	41.60	1.20	51.40	0.80	56.00	1.40	62.00	7.20	71.80
Rejection rate at 5% (%)	4.20	69.40	4.00	77.40	6.20	80.80	5.00	82.80	21.60	90.00