Camera Control For Learning Nonlinear Target Dynamics via Bayesian Nonparametric Dirichlet-Process Gaussian-Process (DP-GP) Models

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Abstract—This paper presents a camera control approach for learning unknown nonlinear target dynamics by approximating information value functions using particles that represent targets’ position distributions. The target dynamics are described by a non-parametric mixture model that can learn a potentially infinite number of motion patterns. Assuming that each motion pattern can be represented as a velocity field, the target behaviors can be described by a non-parametric Dirichlet process-Gaussian process (DP-GP) mixture model. The DP-GP model has been successfully applied for clustering time-invariant spatial phenomena due to its flexibility to adapt to data complexity without overfitting. A new DP-GP information value function is presented that can be used by the sensor to explore and improve the DP-GP mixture model. The optimal camera control is computed to maximize this information value function online via a computationally efficient particle-based search method. The proposed approach is demonstrated through numerical simulations and hardware experiments in the RAVEN testbed at MIT.

I. INTRODUCTION

The problem of using position-fixed sensors to actively monitor and learn the behavior of targets with little or no prior information is relevant to a variety of fields, including monitoring urban environments [1] and detecting anomalies in manufacturing plants [2]. Many methods have been proposed to describe targets’ behaviors in a workspace, such as Gauss-Markov chains [3], [4], linear stochastic models [5]–[7], and nonholonomic dynamics models [8]–[10]. Position-fixed sensors, such as cameras, are often deployed to cooperatively track and surveil moving targets based on limited information that only becomes available when the target enters a sensor’s field-of-view (FoV). In many cases, the target environment is too large for complete sensor coverage, and thus a controller that accounts for the FoV geometry is necessary to determine sensor configurations that minimize uncertainty [11]–[13].

However, little work has been done for the case when sensors have limited FoVs and minimal information about the target model structure is known a priori. In this paper, the target behavior is described as a mixture of unknown velocity fields, the number of which is also unknown. The Dirichlet process-Gaussian process (DP-GP) mixture model provides the necessary flexibility to capture such behavior without overfitting and has been successfully applied in clustering time invariant spatial phenomena [14]. Therefore, the DP-GP mixture model is adopted to model the target movement behavior given noisy measurements.

The active sensing problem considered in this paper is coupled with the problem of tracking moving targets with unknown dynamics, where it is necessary to estimate the classification, dynamics, and position of each target. The objective is to maximize the accuracy of the learned target behavior (i.e., accuracy of the model associated with each target behavior type). Thus, an information value function is introduced for DP-GP model updates. The information functions quantify the amount of information associated with random variables such as velocity fields, and by optimizing the function, the uncertainty of the velocity field can be minimized to control the sensor [15]–[20]. Computing information value functions for one or more random variables or stochastic processes requires knowledge of their joint probability mass (or density) functions. To this end, a general approach was recently presented by the authors for estimating the expected information value of future sensor measurements in target classification problems [21].

In this paper, a particle filter using the Gaussian mixture model constructed from the DP-GP model as the proposal distribution, is adopted to estimate targets’ positions by a set of weighted particles [22]. The weight associated with each particle is obtained through Bayes’ rule from the prior distribution of the target position, the prior distribution of the target behavior classification, the target position-measurement and the measurement model. A computational efficient particle-based optimal control searching approach is proposed to optimize the DP-GP information value function by approximating it as a function of weighted particles. The proposed approach is demonstrated through numerical simulations and hardware experiments.

The paper is organized as follows. The problem formulation is presented in Section II. Section III provides background knowledge of the DP-GP model. The proposed approach is presented in Section IV by introducing (a) the DP-GP information value function, (b) particle filter, and (c) approximation of the DP-GP information value function, as well as the search strategy for optimal camera control. Simulation and hardware results are presented in Sections V and VI, respectively. Finally, conclusions are drawn in Section VII.

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II. PROBLEM FORMULATION

The active sensing problem consists of determining the control input, denoted by $u$, for a sensor with a limited field-of-view (FoV), denoted by $S$, to surveil a two-dimensional convex workspace, $W \subseteq \mathbb{R}^2$. $S$ is assumed to be a subset of the workspace, and is determined by the control vector, such that $S(u(t)) \subseteq W$, at time $t$. The sensor has two possible FoV zoom levels, $L = \{1, 2\}$, where the first zoom level enables the sensor to make measurements of a small area with high accuracy, and the second zoom level enables the sensor to observe a larger area with less accuracy. Let $F_w$ denote a fixed inertial frame of reference embedded in $W$, and $F_r$ represent a moving frame of reference embedded in $S$, with origin $O_s$ as illustrated in Fig. 1. If the position of $O_s$ with respect to $F_w$ is denoted by $q(t) \in W$, and the FoV is assumed to translate in $W$ without rotation as a free-flying object, the control vector that fully determines the configuration of the sensor FoV is $u(t) = [q^T(t) \ l(t)]^T$, where $l(t) \in L$ denotes the choice of zoom level. Then, at any time $t$, a noisy vector measurement of the $j$th target position, $x_j(t) \in W$, and velocity, $\dot{x}_j(t)$, for $j = 1, \ldots, N(t)$, is obtained if $x_j(t) \in S[u(t)]$, where $N(t)$ is the number of targets that have entered the workspace up to time $t$, and “distributed as” denotes “is distributed as”. When $x_j(t) \not\in S[u(t)]$, the measurement of target $j$ at $t$ is an empty set, i.e., $m_j(t) = \emptyset$. Velocity measurements are obtained through target position difference in two consecutive video frames. It is assumed that data-target association is perfect and the noise vectors $n_x$ and $n_v$ are normal distributed with zero mean and covariances $\Sigma_x, \Sigma_v \in \mathbb{R}^{2 \times 2}$ with zero off-diagonal entries, respectively. For the two zoom levels, $\Sigma_x(1) \prec \Sigma_x(2)$ and $\Sigma_v(1) \prec \Sigma_v(2)$, where $\prec$ denotes comparing matrices by elements.

An unknown number of targets are allowed to travel through $W$. Although the true target states are unknown, it can be assumed that all target behaviors can be modeled by a possibly nonlinear time-invariant system,

$$x_j(t) = f_j(x_j(t)), \quad j = 1, \ldots, N(t).$$

The vector function $f_j : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, referred to as a velocity field, is also unknown, and is drawn from a set $\mathcal{F} = \{f_1, \ldots, f_M\}$ of unknown velocity fields to be learned from data, where $M$ is unknown. For simplicity, it is assumed that $N(t)$ can be determined without error. Although the set of velocity fields is assumed to capture all possible target behaviors, there does not exist a one-to-one correspondence between $\mathcal{F}$ and the set of targets. This is because one or more targets in $W$ may be described by the same velocity field in $\mathcal{F}$, while some velocity fields in $\mathcal{F}$ may not describe any of the targets in $W$.

Then, the problem considered in this paper is to determine $\mathcal{F}$, as well as the association between the velocity fields in $\mathcal{F}$ and the targets in $W$, based on sensor measurements obtained up to the present time according to the model in (1). It is assumed that every velocity field in $\mathcal{F}$ is of class $\mathcal{C}^1$, or continuously differentiable [23]. Let a discrete random variable $g_j$, with range $\mathcal{I} = \{1, \ldots, M\}$, denote the index of the velocity field that describes the behavior of the $j$th target. Then, the event $\{g_j = i\}$ represents the association of target $j$ with the velocity field $f_i \in \mathcal{F}$, as shown in (2). It is assumed that $g_j$ obeys an unknown $M$-dimensional categorical distribution [24], denoted by $\pi = [\pi_1, \ldots, \pi_M]^T$ describes the prior probabilities of every possible outcome of $g_j$, for any $j = 1, \ldots, N(t)$. Moreover, targets do not collaborate with each other, so the random variables $g_1, \ldots, g_{N(t)}$ can be assumed to be independent and identically distributed (i.i.d.), such that,

$$\Pr\{g_j = i\} = \pi_i, \forall i, j$$

where $\Pr\{g_j = i\}$ is the probability of event $\{g_j = i\}$. Then, the elements of vector $\pi$ satisfy the properties,

$$\sum_{i=1}^{M} \pi_i = 1, \quad \text{and} \quad \pi_i \in [0, 1], \forall i = 1, \ldots, M.$$

and $\pi$ can be used to represent the probability mass function (PMF) of the $M$-dimensional categorical distribution. Based on the above assumptions and problem formulation, the pair $\{\mathcal{F}, \pi\}$ is the sufficient statistic of the target dynamics.

This paper presents an approach to determining the optimal control, $u^*(t)$, that enables the sensors to collect the most valuable measurements for learning $\{\mathcal{F}, \pi\}$.

III. DIRICHLET PROCESS-GAUSSIAN PROCESS MIXTURE MODEL

This section presents a Dirichlet process-Gaussian process mixture model for describing target behaviors [14]. Based on the model of targets’ behaviors (2), every velocity field, $f_i$, projects the $j$th target position, $x_j(k)$, to the target velocity, $\dot{x}_j(k)$. Thus, they can be viewed as two-dimensional spatial phenomena that can be modeled by multi-output Gaussian processes (GPs) [25]. Let a Gaussian process $GP_i$ represent the distribution of velocities over the workspace specified by the $i$th velocity field, $f_i$, such that

$$f_i(x) \sim GP_i, \quad \forall x \in W,$$
for \( i = 1, \ldots, M \). Then, \( \text{GP}_i \) is completely specified by the
mean function and the kernel function \([26]\):

\[
\theta_i(x) = \mathbb{E}[f_i(x)], \quad \forall x \in \mathcal{W},
\]

\[
c(x_i, x_j) = \mathbb{E}\{[f_i(x_i) - \theta_i(x_i)][f_i(x_j) - \theta_i(x_j)]^T\}
\]

(7)

for all \( x_i, x_j \in \mathcal{W} \), where \( \mathbb{E}[\cdot] \) denotes the expectation
operator. In this paper, it is assumed that all of the \( M \)
GPs share the same kernel function, which is known to
be the sensor. Therefore, the \( i \)-th Gaussian process \( \text{GP}_i \) can be
parameterized by \( \text{GP}_i = \text{GP}(\theta_i, c) \). For a rigorous
definition and a comprehensive review of Gaussian processes the reader is
referred to \([26]\).

From Section II, the PMF \( \pi \), which describes the proba-
blity of an association between a target and a velocity field, is
unknown and must be learned from data using classification
approaches \([27]\). Furthermore, because different targets may
be described by the same velocity field in \( \mathcal{F} \), a prior distribu-
tion on \( \pi \) is required to augment the ability of learning the
clustering effect of target behaviors learned from the
sensor measurements. Dirichlet processes (DPs) have been
successfully applied in clustering data without specifying
the number of clusters \textit{a priori}, because they allow the
creation and deletion of clusters when necessary as new data
is obtained over time. Let \( \mu \) denote a random probability
measure over a support set, \( \mathcal{A} \). A DP is a distribution for \( \mu \)
such that marginals on any finite partitions of \( \mathcal{A} \) are
Dirichlet-distributed \([28]\). A DP can be described by two parameters,
the base distribution denoted by \( H(A) \), and the strength
parameter \( \alpha \) \([29]\). The base distribution is analogous to the
Dirichlet distribution on \( \mathcal{A} \). A DP can be described by two parameters,
\( \{A_1, \ldots, A_n\} \) of \( \mathcal{A} \) obeys,

\[
[\mu(A_1), \ldots, \mu(A_n)]^T \sim \text{Dir}[\alpha H(A_1), \ldots, \alpha H(A_n)],
\]

(9)

where “\( \text{Dir} \)” denotes the Dirichlet distribution. For a rigorous
definition and a comprehensive review of DPs, the reader is
referred to \([30]\). In this paper, the base distribution, \( H \), is
chosen to be a Gaussian process, \( \text{GP}_0 = \text{GP}(0, c) \), and the
support set, \( \mathcal{A} \), is chosen to be the set consisting of all the
admissible velocity fields. The resulting model is a Dirichlet-
process-Gaussian process (DP-GP) mixture model \([14]\),

\[
\{\theta_i, \pi\} \sim \text{DP}(\alpha, \text{GP}_0), \quad i = 1, \ldots, \infty
\]

\[
g_j \sim \text{Cat}(\pi), \quad j = 1, \ldots, N
\]

\[
f_{g_j}(x) \sim \text{GP}(\theta_{g_j}, c), \quad \forall x \in \mathcal{W}, \quad j = 1, \ldots, N,
\]

(10)

that can be utilized to learn the behaviors of the targets in
(2) from data.

In many applications \([31]-[33]\), it can be assumed that the
sensor obtains measurements at a constant known interval,
\( \delta t \). Thus, the target model (2) can be discretized accordingly.
Letting \( k \) denote the discrete time index, the DP-GP model
in (10) can be updated when enough observations of a new
target trajectory are available, using algorithms such as the
Markov chain sampling methods \([34]\), or the variational
inference approach \([35]\). In this work, the Gibbs sampling
technique is applied to obtain the posterior parameters of
the DP-GP model following the work in \([14]\), \([36]\), \([37]\).
First, the indicator of the target-velocity association, \( g_j \), for
every target is sampled from the Chinese restaurant process
\([38]\). Then the GP parameters, \( \theta_i \), for every velocity field are
sampled given the target-velocity field association from the
previous step. The two steps are repeated a large amount of
times before the posterior distribution is acquired.

As mentioned in Section II, the measurement histories of all the
targets already used in updating the DP-GP model, and \( \mathcal{E}_i(k) \) denote the measurement histories assigned to the\( i \)-th velocity field. In other words, if \( k' \) denotes the last time
when the DP-GP model is updated, and vector \( m(k) = \{m_{1}^{T}(k) \ldots \cdot \cdot \cdot m_{N}^{T}(k)\}^T \) denotes measurements of all targets
at the \( k' \)-th time step, the following measure histories,

\[
\mathcal{E}(k) = \{m(\ell) \mid m(\ell) \neq \emptyset, 0 \leq \ell \leq k'\},
\]

(11)

and

\[
\mathcal{E}_i(k) = \{m_{j}(\ell) \mid m_{j}(\ell) \neq \emptyset, 0 \leq \ell \leq k', g_j = i\}.
\]

(12)

are employed in the expressions the mean and variance the
\( j \)-th target velocity at position \( x_j(k) \), such that

\[
\left. \begin{array}{l}
\mu_j(k) = \theta_j(x_j(k)) + C[x_j(k), P_j(k)] \\
\Sigma_j(k) = C[x_j(k), x_j(k)] - C[x_j(k), P_j(k)]
\end{array} \right\}
\]

(13)

(14)

where

\[
C(A, B) = \begin{bmatrix}
c(a_1, b_1) & \cdots & c(a_1, b_n) \\n\vdots & \ddots & \vdots \\c(a_m, b_1) & \cdots & c(a_m, b_n)
\end{bmatrix}
\]

(16)

is the cross-covariance matrix of matrices \( A = [a_1 \ldots a_m] \)
and \( B = [b_1 \ldots b_n] \).

IV. METHODOLOGY

This section first introduces the DP-GP information value
function based on Kullback-Leibler (KL)-divergence be-
tween the prior and posterior of the DP-GP given an ad-
tional future measurement. Then, it describes a particle
filter that includes a set of particles sampled from the prior
(predicted) target position distribution at \( k + 1 \); this filter is
used to obtain posterior distribution of the targets’ positions.
In the remainder of this section, these sampled particles are
further used to approximate information values, and thus to
facilitate searching for optimal camera control.
A. DP-GP Information Value

Information theoretic functions, particularly the KL-divergence have been shown to be effective at representing information value for probabilistic models [39]. Expected information value functions have been proposed in [15] to represent the benefit of future sensor measurements. This paper develops one expected information value function that is applicable to DP-GP mixture models, which is referred to as the DP-GP information value function.

Since the DP-GP mixture model can be viewed as a distribution over probability distributions [14], [36], [40], [41], a KL-divergence function is employed here to represent the distance between the prior (current) DP-GP mixture and the posterior DP-GP mixture model updated with an additional sensor measurement. Let \( \xi_i, \ i = 1, \ldots, G \), denote the \( G \) points of interest selected to represent the velocity field over the workspace. For example, they can be \( G \) uniformly distributed grid points in workspace. Let \( \mathbf{X} = [\xi_1 \ \ldots \ \xi_G] \) be a shorthand notation of the points of interest, such that

\[
f_i(\mathbf{X}) = [f_i(\xi_1)^T \ \ldots \ f_i(\xi_G)^T]^T
\]

Let \( \mathbf{F} \) denote the vector function by stacking \( f_i, \ i = 1, \ldots, M \), column-wise, such that

\[
\mathbf{F}(\mathbf{X}) = [f_1(\mathbf{X})^T \ \ldots \ f_M(\mathbf{X})^T]^T
\]

where \( \mathbf{F}(\mathbf{X}) \) denotes the function values evaluated at \( \mathbf{X} \). Notice that \( \mathbf{F}(\mathbf{X}) \) is the vector of random variables investigated in this paper.

Similar to \( \mathcal{E}(k) \), \( \mathcal{M}(k) \) denotes the measurement histories of all the targets that have not been used in updating the DP-GP model as follows,

\[
\mathcal{M}(k) = \{ \mathbf{m}(\ell) \mid \mathbf{m}(\ell) \neq \emptyset, k' < \ell \leq k \}. \tag{20}
\]

\( \mathcal{M}_j(k) \) denotes the set of unlabeled measurement history of the \( j \)th target. Since the sensor obtains measurements from multiple targets simultaneously, the total information value is the sum of the information value of each target in the workspace, as follows

\[
\varphi[\mathbf{F}(\mathbf{X}): \mathbf{m}(k + 1) \mid \mathcal{M}(k), \mathcal{E}(k), \mathbf{u}(k)] = \sum_{\{j \mid \mathbf{m}_j(k) \in \mathcal{S}\}} \varphi_j[\mathbf{F}(\mathbf{X}): \mathbf{m}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}(k), \mathbf{u}(k)], \tag{21}
\]

In addition, the velocity field-target association is unknown. Therefore, the information value for the \( j \)th target is obtained through a weighted summation of information value conditioned on all possible associations,

\[
\varphi_j[\mathbf{F}(\mathbf{X}): \mathbf{m}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}(k), \mathbf{u}(k)] = \sum_{i=1}^{M} \varphi_j[f_i(\mathbf{X}): \mathbf{m}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}(k), \mathbf{u}(k), g_j = i]
\times p[g_j = i \mid \mathcal{M}_j(k), \mathcal{E}(k), \mathbf{u}(k)], \tag{22}
\]

where \( p[\cdot] \) denotes probability. Notice that the condition \( (g_j = i) \) is implied by \( \mathcal{E}_i \), therefore it will be dropped when there is no confusion. Because at \( k \) the value of \( \mathbf{m}_j(k + 1) \) is unknown, the information value is estimated by marginalizing over \( \mathbf{m}_j(k + 1) \) to obtain the expected conditional sub-information value as follows, [11]

\[
\varphi_j[f_i(\mathbf{X}): \mathbf{m}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]
\simeq \int_{x_j \in S} \left\{ \int_{\mathbf{m}_j} D[p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k + 1), \mathcal{E}_i(k), \mathbf{u}(k)] \mid p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)] \right\}
\times p[\mathbf{m}_j(k + 1) \mid \mathbf{x}_j(k + 1), \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]d\mathbf{m}_j
\times p[\mathbf{x}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]d\mathbf{x}_j, \tag{23}
\]

where \( D(\cdot|\cdot) \) denotes the KL-divergence between the posterior and prior distributions of \( f_i(\mathbf{X}) \) given the measurement \( \mathbf{m}_j(k + 1) \) [42], and can be computed as follows:

\[
D(p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k + 1), \mathcal{E}_i(k), \mathbf{u}(k)] \mid p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)])
= \int_{-\infty}^{\infty} \left( \ln[p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k + 1), \mathcal{E}_i(k), \mathbf{u}(k)]) - \ln[p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]) \right) \times p[f_i(\mathbf{X}) \mid \mathcal{M}_j(k + 1), \mathcal{E}_i(k), \mathbf{u}(k)]dW_i. \tag{24}
\]

If it can be assumed that the position measurement noise is negligible compared to the velocity measurement noise, the inner integral of (23), denoted by \( h_i \), can be evaluated analytically [16], as

\[
h[\mathbf{x}_j(k + 1)] = \text{tr}([\mathbf{Q}^{-1}]^T \mathbf{R}^T \mathbf{R} \mathbf{Q}^{-1} \mathbf{\Sigma}_v) \tag{25}
\]

\[
\mathbf{R} = -\mathbf{C}[\mathbf{X}, \mathbf{P}(k)](\mathbf{C}[\mathbf{P}(k), \mathbf{P}(k)] + \mathbf{\Sigma}_v)^{-1}
\times \mathbf{C}[\mathbf{P}(k), \mathbf{x}(k + 1)] + \mathbf{C}[\mathbf{X}, \mathbf{x}(k + 1)] \tag{26}
\]

\[
\mathbf{Q} = \mathbf{\Sigma}_j(k) + \mathbf{\Sigma}_v \tag{27}
\]

\( \text{tr}[\cdot] \) denotes the trace of a matrix. \( \mathbf{Q} \) is the covariance matrix of the target velocity regulated by the measurement noise, and \( \mathbf{P} \) can be seen as the covariance between the points of interest and the target position at the next time step.

In (23), \( p[\mathbf{x}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)] \) is the probability density function of the target position at the next time step, and can be obtained as follows,

\[
p[\mathbf{x}_j(k + 1) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]
= \int_{\mathcal{V}} p[\mathbf{v}_j(k) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]
\times p[\mathbf{x}_j(k) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)]d\mathbf{v}_j(k), \tag{28}
\]

where

\[
\mathbf{x}_j(k) = \mathbf{x}_j(k + 1) - \mathbf{v}_j(k)\delta t, \tag{29}
\]

\( p[\mathbf{v}_j(k) \mid \mathcal{M}_j(k), \mathcal{E}_i(k), \mathbf{u}(k)] = N[\mathbf{\mu}_j(k), \mathbf{\Sigma}_j(k)], \tag{30}\)

\( \delta t \) is the time step size, and \( \mathcal{V} \) is the set of possible target velocities.

Note that given \( \mathcal{M}_j(k) \) the probability of the \( j \)th target dynamics following the \( i \)th velocity field in \( F \) is determined
by
\[ w_{ji} \triangleq p[g_j = i \mid M_j(k), E(k), u(k)] \]
\[ = \frac{\pi_i \prod_{k=k'}^k N[z_j(\ell); \hat{\mu}_j(\ell), \hat{\Sigma}_j(\ell)]}{\sum_i \pi_i \prod_{k=k'}^k N[z_j(\ell); \hat{\mu}_j(\ell), \hat{\Sigma}_j(\ell)]}, \quad (31) \]
where the estimated mean, \( \hat{\mu}_j(\ell) \), and variance, \( \hat{\Sigma}_j(\ell) \), of the target velocity are calculated by replacing \( x_j(\ell) \) with \( y_j(\ell) \) in (15-16). The approximation of DP-GP information value via particles is introduced in Section IV-C, where these particles are used to present the target position distribution. This approximation further reduces computational complexity of determining the optimal camera control policy.

B. Particle Filter

The position of the FoV centroid and zoom levels of all cameras need to be planned ahead of time in order to obtain measurement of moving targets. Thus, the estimation of every target’s position propagation at one time step ahead (at \( k+1 \)), denoted as \( x_j(k+1^-) \), must be obtained for the purpose of planning, where \( k+1^- \) denotes the moment right before \( k+1 \) when \( m_j(k+1) \) is not available. Measurements can be empty sets when the camera loses sight of targets, resulting in a nonlinear observation model. Therefore, a classical particle filter algorithm, sequential importance resampling (SIR) [43], is adopted to estimate prior and posterior target position distribution using a Gaussian mixture model as a proposal distribution. This Gaussian mixture model is the transition prior probability distribution of targets’ positions at \( k+1^- \), which is built upon the learned DP-GP model and particles representing posterior distributions of targets’ positions at \( k \).

The position propagation of target \( j \) under the estimated behavior \( \{F, \pi\} \) from \( k \) to \( k+1 \) is
\[ x_j(k+1) = x_j(k) + f_j[x_j(k)]dt \quad (32) \]
where \( w_{ji} \) is given by (31). In the first step, the samples from the posterior distribution of the \( j \)th target position at time step \( k \) given \( M_j(k) \) and \( E_i(k) \) are represented by a set of particle and weight pairs,
\[ P_{ji}(k) \triangleq \{(\omega_{jis}(k), x_{jis}(k)) : 1 \leq s \leq S\} \quad (33) \]
where \( S \) is the number of particles for each velocity field, \( x_{jis}(k) \) represents \( s \)th particle for velocity filed \( j \) and target \( i \), and \( \omega_{jis}(k) \) represents the associated weight, such that
\[ \sum_{s=1}^{S} \omega_{jis}(k) = w_{ji}. \quad (34) \]

In the second step, according to the target position propagation (32) and the DP-GP model, \( x_j(k+1^-) \) can be obtained and represented by a Gaussian mixture,
\[ x_j(k+1^-) \sim \sum_{i=1}^{M} \sum_{s=1}^{S} \omega_{jis}(k)N[\eta_{jis}(k+1^-), \Lambda_{jis}(k+1^-)] \quad (35) \]
where
\[ \eta_{jis}(k+1^-) = \chi_{jis}(k) + \mu_{jis}(k) \delta t \quad (36) \]
\[ \Lambda_{jis}(k+1^-) = \Sigma_{jis}(k) \delta t^2 \quad (37) \]
and where \( \mu_{jis} \) and \( \Sigma_{jis} \) are the mean and variance of \( s \)th Gaussian component of target \( j \) for velocity field \( i \) at \( \chi_{jis}(k) \), which can be calculated from (15-16) by replacing \( x_j(\ell) \) with \( y_j(\ell) \). This Gaussian mixture is used as the optimal proposal distribution to sample transient particles representing the probability distribution of \( x_j(k+1^-) \), as follows,
\[ \chi_{jis}(k+1^-) \sim \sum_{s=1}^{S} \frac{\omega_{jis}(k)}{w_{ji}} N[\eta_{jis}(k+1^-), \Lambda_{jis}(k+1^-)] \quad (38) \]
\[ \omega_{jis}(k+1^-) = w_{ji}/S \quad (39) \]

Finally, when a non-empty measurement \( m_j(k+1) \) is obtained, the weights associated with particles are updated as follows,
\[ \omega_{jis}(k+1) = \frac{w_{ji} \omega_{jis}(k+1^-)N[z_j(k+1); \chi_{jis}(k+1^-), \Sigma_j(k+1)]}{\sum_{s=1}^{S} \omega_{jis}(k+1^-)N[z_j(k+1); \chi_{jis}(k+1^-), \Sigma_j(k+1)]} \quad (40) \]
via measurement model (1). When an empty measurement of target \( j \) is obtained, i.e., \( m_j(k+1) = \emptyset \), \( \omega_{jis}(k+1) \) is set to zero if \( \chi_{jis}(k+1^-) \in S[u(k+1)] \). Then, weights of all particles for target \( j \) are normalized. The particles stay the same as the transient particles, such that \( x_{jis}(k+1) = \chi_{jis}(k+1^-) \). Therefore, similar to (33), the samples from posterior probability distribution of target \( j \) at time \( k+1 \) can also be represented by the weighted particles
\[ P_{ji}(k+1) = \{(\omega_{jis}(k+1), x_{jis}(k+1)) : 1 \leq s \leq S\} \quad (41) \]

In the following sections, all transient particle sets obtained by (38-39), denoted by \( P_{ji}(k+1^-) \), are utilized to facilitate the calculation of DP-GP information values for the search of optimal camera control.

C. Approximation of DP-GP Information Value

The resultant weighted particles from the particle filter, \( P_{ji}(k+1^-) \), representing samples from the prior position distribution of target \( j \) at \( k+1 \) are utilized to reduce the computational complexity for determining the DP-GP information value. Let \( h[x_j(k+1^-)] \) denote the integrand in (23) in the curly bracket, the evaluation of DP-GP information value becomes,
\[ \varphi_j[F(X); m(k+1) \mid M_j(k), E(k), u(k)] \]
\[ \approx \sum_{i=1}^{M} \int h[x_j(k+1^-)] p[x_j(k+1^-) \mid M_j(k), E_i(k), u(k)] dx_j(k+1^-) \times w_{ji} \quad (42) \]
By using the weighted particles, \( P_{ji}(k+1^-) \), (42) can be approximated by a finite sum, such that

\[
\hat{\varphi}_j[F(X); m(k+1)] = \sum_{i=1}^{M} h[\chi_{jis}(k+1^-)] \omega_{jis}(k+1^-)
\]

where \( S[u(k+1)] \) is abbreviated as \( S(k+1) \) hereinafter.

D. Searching for Optimal Camera Control

Adopting all the weighted particles, the DP-GP information value function to be maximized at each step can be written as

\[
J = \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{s_i} h[\chi_{jis}(k+1^-)] w_{jis}(k+1^-)
\]

where \( h[\chi_{jis}(k+1^-)] \) is precalculated for every particle. Given a zoom level, the optimal control \( u^*(k+1) \) is obtained by the reduction to the following geometric covering problem: Given a set of points

\[
\cup_{i=1}^{N} \cup_{s_i}^{M} \{\chi_{jis}(k+1^-)\}
\]

in a two dimensional plane, each associated with a weight \( \omega_{jis}(k+1^-) \times h[\chi_{jis}(k+1^-)] \); find the position of an axis-parallel rectangle of given size \( L_x \) (horizontal size) and \( L_y \) (vertical size), such that \( J \) in (44) is maximized [44].

Algorithm performance is evaluated by relative error comparing the estimated target model to the real underlying velocity fields. The relative error, \( \xi \), is the root mean square error (RMS) of the DP-GP model in imitating the motion patterns, normalized by the velocity \( \dot{x}_j(k) \) at each point. To obtain \( \xi \), \( N_A = 1000 \) new test trajectories (distinct from those observed by the camera), \( \{T_j : 1 \leq j \leq N_A\} \), are generated according to the motion patterns, where \( T_j = \{x_j(k), \dot{x}_j(k)\}, k = 1, \ldots, N_T_j \), represents the \( j \)th new trajectory and \( N_T_j \) is the length of the \( j \)th trajectory. These trajectories are compared to the evolving DP-GP model. If \( \mu_{ji}[x_j(k)] \) is utilized to denote the mean speed at \( x_j(k) \) by the \( i \)th Gaussian process component in the DP-GP model, \( \xi \) can be expressed as follows,

\[
\xi = \frac{1}{N_T} \sum_{j=1}^{N_A} \sum_{i=1}^{M} \frac{w_{ji}}{N_T_j} \sum_{k=1}^{N_T_j} \|1 - \frac{\mu_{ji}[x_j(k)]}{\dot{x}_j(k)}\|_2
\]

Figure 2 shows the decreasing trend of the relative error of the evolving DP-GP model over time using the four approaches. It demonstrates the advantages of an information-theoretic approach for control actions to update a DP-GP model. Additionally, model error decreases faster when the DP-GP model uncertainty is considered via the proposed DP-GP information value function.

Figure 3-(a) shows the trajectories utilized in training the prior DP-GP model at \( t = 0 \), and Figure 3-(b) shows the set of new trajectories \( T_i \). Figure 3-(c) shows the observed
trajectories by maximizing the DP-GP information value function. The red arrows indicate measurements made by the camera at the zoomed-in level, while the blue arrows represent observations at the zoomed-out level. Figure 3-(d) shows the observed trajectories by maximizing mutual information only. It is included for comparison. Comparing with Figure 3-(c), it can be seen that the camera tends to observe the trajectories of which the prior DP-GP model has little knowledge.

VI. HARDWARE EXPERIMENTS

The proposed approach was also implemented in hardware using the Real-time indoor Autonomous Vehicle test Environment (RAVEN) at MIT. The domain was constrained to a 16m² square region, with two AXIS P5512 PTZ cameras performing target-tracking. Camera intrinsics were utilized to obtain desired square FoVs with correct zoom levels (0.16m² and 0.36m², respectively) across the domain.

Three iRobot Create ground robots were used as targets, each assigned to one of three underlying velocity fields. A given velocity field may be assigned to multiple targets, and re-assignment was performed upon completion of each vehicle’s trajectory (marked by the vehicle departing the domain). Figure 4 shows a superimposed view of camera FoVs and position estimates for the above hardware setup.

Figure 5 illustrates relative RMS error, ξ, of the DP-GP models in predicting vehicle trajectories using each of the four algorithms described in Section V. As in results from the simulated experiments, this plot illustrates the increasing predictive accuracy of the DP-GP using the information-theoretic control strategy developed in this paper. Specifically, optimizing the DP-GP information value function results in fast and significant reduction in model error, owing to more sufficient surveillance of targets exhibiting behaviors with little prior information.

VII. CONCLUSION

An optimal camera control policy is presented for an active sensing problem where a number of moving targets follow an unknown number of underlying velocity fields. The target behaviors are described by a DP-GP mixture model, and a particle filter is utilized to estimate the target positions. The policy derived maximizes the DP-GP information value function. Numerical simulations demonstrated a decreasing trend in DP-GP model error using the derived policy, and an advantage over heuristic policies. Hardware experiments using three targets demonstrated effective use of the algorithm in real-time active sensing and planning.

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REFERENCES


