Problem 11.1

(a) \[ G(s) = \frac{1}{s(s-5)} \]

\( G(s) \) is not stable.

(b) \[ H(s) = \frac{1}{s^2 - 5s + K_p} \]

Using the Routh-Hurwitz criteria, notice that for \( s^2 - 5s + K_p \), \( a_1 = -5 \neq 0 \) which means that \( H(s) \) cannot be stabilized using a controller \( K(s) = K_p \). If \( K(s) = K_ds + K_p \) is used, then with a proper choice of \( K_p \) and \( K_d \), the system is stabilized as seen in the lecture.

Problem 11.2

(a) The steady state tracking error, \( e(\infty) \), is zero. Therefore, as long at \( K \) is positive \( e(\infty) \) does not change.

(b) The steady state tracking error is infinity when \( K \leq 0 \) and \( e(\infty) = \frac{10}{K} \) when \( K > 0 \).

Problem 11.3

(a) Open-loop pole: 1. Open-loop zero: one at infinity. Characteristic equation: \( 0 = 1 + KL_0(s) = 1 + \frac{K}{s-2} = s - (1 - K) \). Thus, the single closed-loop pole is at \( 1 - K \). We need \( K > 1 \) to make the closed-loop system stable. The root locus plot is shown below:

(b) Open-loop poles: two at 2. Open-loop zeros: one at -1, one at infinity. Characteristic equation: \( 0 = 1 + KL_0(s) = 1 + \frac{K(s+1)}{(s-2)} = s^2 + (K-4)s + (K+4) \). Thus, the closed-loop poles are: 

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\[ p_\pm = \frac{-(K - 4) \pm \sqrt{(K - 4)^2 - 4(K + 4)}}{2}. \]

When \(0 < K < 12\), \(p_\pm\) are both complex. When \(K = 12\), \(p_+ = p_- = -4\) (both poles meet at -4). When \(K > 12\), \(p_+ \to -1\) and \(p_- \to -\infty\) (both real). When \(K < 0\), \(p_+ \to +\infty\) and \(p_- \to -1\) (both real). According to Routh and Hurwitz, we need \(K > 4\) to make the closed-loop system stable. The root locus plot is shown below:

(c) Open-loop poles: -2, -3. Open-loop zeros: two at infinity. Characteristic equation: \(0 = 1 + KL_0(s) = 1 + \frac{K}{(s+2)(s+3)} = s^2 + 5s + (6 + K)\). Thus, the closed-loop poles are at:

\[ p_\pm = \frac{-5 \pm \sqrt{1 - 4K}}{2}. \]

When \(0 < K < \frac{1}{4}\), \(p_\pm\) are between -3 and -2. When \(K = \frac{1}{4}\), \(p_+ = p_- = -2.5\) (both poles meet at -2.5). When \(K > \frac{1}{4}\), both poles are complex and have -2.5 as their real part; the imaginary parts go to \(\pm\infty\). When \(K < 0\), both closed-loop poles are real and grow further apart. According to Routh and Hurwitz, we need \(K > -6\) to make the closed-loop system stable. The root locus plot is shown below:
(d) Open-loop poles: -2, -3. Open-loop zeros: -1, 2. Each open-loop pole is paired up with an explicit open-loop zero, so there are no open-loop zeros or poles at infinity. Characteristic equation: \(0 = 1 + KL_0(s) = 1 + \frac{K(s+1)(s-2)}{(s+2)(s+3)} = (K + 1)s^2 + (5 - K)s + (6 - 2K)\). Thus, the closed-loop poles are at:

\[
p_{\pm} = \frac{K - 5 \pm \sqrt{9K^2 - 26K + 1}}{2(K + 1)}.
\]

When \(K > 0\), the closed-loop poles move toward each other. When \(K \approx 0.3899\), the poles meet at around -2.387 (on the real axis). As \(K\) is increased further, the poles become complex and move in a curved manner back toward the real axis. When \(K \approx 2.8499\), they meet on the real axis again at around -0.279. As \(K\) is increased, they move apart toward the open-loop zeros. When \(K < 0\), both closed-loop poles are real. One moves from -2 (open-loop pole) to -1 (open-loop zero). The other one starts at -3 and heads towards negative infinity. When \(K = -1\), it hits infinity and starts coming back (“wrap around”) on the positive real axis toward 2 (open-loop zero). Overall, the closed-loop system is stable when \(-1 < K < 3\). The root locus plot is shown below:

The wrap around effect for \(p_-\) is interesting, and it can be verified that it actually happens. The following shows the location of \(p_-\) as a function of \(K\). We see that as \(K\) goes from 0 to -1, \(p_-\) goes from -3 down to negative infinity. As \(K\) goes from -1 down to negative infinity, \(p_-\) goes from positive infinity down to 2.