Aeroelastic Tailoring using Additively Manufactured Lattice Structures

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Lattice structures are of interest to the aerospace industry due to their high specific stiffness and the large design freedom that they offer. For instance, this large design freedom can be used to aeroelastically tailor the structure to mitigate flutter. This paper describes a methodology for designing the internal structure of a wing as an additively manufactured lattice structure, while enforcing aeroelastic stability as a design constraint. A low-order model is developed for the dynamics of the lattice structure, which is then coupled to a physics-based transonic flutter model to yield a complete aeroelastic model of the wing. The approach is demonstrated on a test case of a wing in transonic flow, where it is shown that constraining the aeroelastic stability of the system adds only 3.5% weight while the flutter speed is increased by 3%.

Nomenclature

\begin{align*}
A, A_f, A_k & \quad \text{Coefficients of } \Gamma \text{ evolution equation} \\
E & \quad \text{Young's modulus} \\
F & \quad \text{Force vector} \\
\bar{I}_{ij} & \quad \text{Mass moment of inertia per span} \\
K & \quad \text{Stiffness matrix} \\
L & \quad \text{Lift per span} \\
M & \quad \text{Mass matrix} \\
\bar{M} & \quad \text{Mass per span} \\
M_\infty & \quad \text{Freestream Mach number} \\
\bar{M}_{c/2} & \quad \text{Aerodynamic moment per span around mid-chord} \\
\bar{s}_{ij} & \quad \text{Mass unbalance per span} \\
T_{(\cdot)\to\{\cdot\}} & \quad \text{Map between low-order and full-order lattice model} \\
V_L & \quad \text{Wing-perpendicular velocity} \\
V_{\text{cruise}} & \quad \text{Cruise speed} \\
V_f & \quad \text{Flutter speed} \\
\bar{a}_{ik} & \quad \text{Cross-sectional area of member connecting node } i \text{ and } k \\
c & \quad \text{Airfoil chord length} \\
h & \quad \text{Vertical deflection, positive downward} \\
l_{ik} & \quad \text{Length of member connecting node } i \text{ and } k \\
\bar{l} & \quad \text{Effective wing length} \\
m & \quad \text{Number of members in lattice} \\
n & \quad \text{Number of nodes in lattice} \\
t & \quad \text{Time} \\
\bar{u} & \quad \text{Displacement vector} \\
w & \quad \text{Vertical (downward) velocity} \\
\bar{x}, \bar{y}, \bar{z} & \quad \text{Coordinates aligned with swept wing} \\
\Gamma & \quad \text{Circulation strength} \\
\theta & \quad \text{Pitch angle} \\
\lambda & \quad \text{Eigenvalue} \\
\sigma & \quad \text{Stress} \\
\sigma_C & \quad \text{Maximum compressive stress} \\
\sigma_T & \quad \text{Maximum tensile stress} \\
\phi & \quad \text{Basis function} \\
\chi & \quad \text{Maximum real eigenvalue of aeroelastic system}
\end{align*}

Symbols

\begin{align*}
\vec{\mathbf{u}} & \quad \text{Displacement vector} \\
V_{\text{f}} & \quad \text{Flutter speed} \\
\bar{a}_{ik} & \quad \text{Cross-sectional area of member connecting node } i \text{ and } k \\
c & \quad \text{Airfoil chord length} \\
\Gamma & \quad \text{Circulation strength} \\
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I. Introduction

Novel manufacturing techniques open up additional design space for aerospace vehicles. High aspect ratio wings in novel aircraft concepts, for example, offer the benefits of higher aerodynamic efficiency, but present the challenge of being more susceptible to aeroelastic problems such as flutter. Novel manufacturing techniques present an opportunity to address this challenge via improved material properties (e.g., increased stiffness) and also by enabling non-conventional internal wing layouts. This paper examines the possibility of
designing the internal structure of aircraft wings as a lattice structure, while mitigating flutter constraints. The fabrication of these lattice structures is enabled by advances in additive manufacturing technology.

To date, the aeroelastic optimization of aircraft wings has mostly focused on conventional internal wing structures (i.e., an orthogonal array of ribs and spars).\textsuperscript{1–5} The structural efficiency of the wing can be further improved by using novel manufacturing techniques, which allow for moving away from the conventional orthogonal rib-spar layout. Several ways to parametrize the internal structure of the wing have been demonstrated. For example, Balabanov and Haftka,\textsuperscript{6} and Locatelli et al.\textsuperscript{7} retained the rib-spar layout but allowed for curvilinear components, which is therefore an inherently two-dimensional parametrization. Alternatively, three-dimensional parametrizations are used where the optimizer is allowed to place material anywhere in the wing. These problems are typically solved using solid isotropic material with penalization (SIMP) methods\textsuperscript{8,9} or level-set methods,\textsuperscript{10,11} as demonstrated by Kim et al.\textsuperscript{12} and James and Martins.\textsuperscript{13} A combination of both methods is investigated by Stanford and Dunning\textsuperscript{14} where an orthogonal rib-spar layout is used but where topology optimization is applied to the ribs and spars. Aage et al. used topology optimization to design the internal structure of an aircraft wing using giga-voxel resolution.\textsuperscript{15}

Other studies have shown the potential benefits of novel manufacturing techniques on wing material properties. For example, Haddadpour et al.\textsuperscript{16} and Stodieck et al.\textsuperscript{17} demonstrated increased wing stiffness for tow-steering with composite materials.\textsuperscript{18,19} Stanford et al.\textsuperscript{20} investigated the differences between several novel tailoring schemes on wings subject to transonic flutter constraints and found that a spatially varying thickness distribution—through additive manufacturing—yielded more benefit than tow-steering alone.

Aeroelastic optimization has also been conducted for helicopter and propeller blades. Ganguli et al.\textsuperscript{21} used high-fidelity flow and structural simulations to minimize rotor vibrations, while constraining aeroelastic stability. Glaz et al.\textsuperscript{22} developed a design methodology to minimize vibrations in forward flights, this time using surrogate models for the structural and aerodynamic loads. Pagano et al.\textsuperscript{23} coupled aeroelastic simulations to aeroacoustic evaluations for the design of propeller blades that minimize noise. In this work, however, we focus on aircraft wings, and simply note that the design methodology could also be applied to helicopter and propeller blades in the future, as they could also take advantage of lightweight aeroelastically tailored lattice structures.

Lattice structures are of particular interest in aircraft design because of their high stiffness to weight ratio.\textsuperscript{24} Walker et al.\textsuperscript{25} optimized the internal structure of a wing using lattice structures utilizing commercial software. Lattice structures have also been used in morphing wing applications.\textsuperscript{26,27} In this paper we focus on an additively manufactured lattice structure as a possible solution to mitigating flutter in high aspect ratio wings. These lattice structures are designed such that they are fine where stresses in the structure are high and coarse where the stresses are low, while still respecting flutter constraints. Furthermore, they are designed to be self-supporting by considering the manufacturing constraints directly in the optimization.

The paper is organized as follows. Section II discusses our design methodology for lattice structures in detail, together with the aerodynamic model, as well as a low-order beam model from the full dynamic lattice model. Section III discusses the validation of this low-order beam model. Section IV demonstrates the design methodology for a wing operating in transonic flow. The main findings are discussed in Section V.

## II. Methodology

This section details the aeroelastic tailoring approach, which is summarized in Section II.A. Section II.B explains the flutter model used throughout this work. The structural model for the flutter model is described in detail in Section II.C. Section II.D describes the aeroelastic tailoring of a lattice structure through optimizing the cross-sectional area of each of the lattice’s members.

### II.A. Aeroelastic Tailoring Approach

The complete aeroelastic tailoring approach can be summarized as follows. This work uses a physics-based low-order model for transonic flutter. The first step, therefore, is to calibrate the aerodynamic coefficients of that model using 2D unsteady Euler simulations for the airfoil family used in the wing design. This approach is described in detail in Ref. 28.

Second, the load distribution over the wing is computed to obtain the baseline lift coefficient $c_{L0}$ at each beam section—which is needed to select the correct aerodynamic coefficients for the flutter model—and to generate a pressure field over the wing that is subsequently used as input to the structural analysis of the wing.

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internal structure. There are several ways of computing the flow over the wing, e.g., using a vortex-lattice method or a higher-fidelity Euler simulation. In the current work, we use a vortex-lattice method.

![Diagram of approach for design of lattice structures for additive manufacturing](image)

**Figure 1. Approach for design of lattice structures for additive manufacturing.**

The pressure distribution over the wing is used as the input for the structural FEM analysis of the internals of the wing, which allows for computing the stress tensor everywhere in the domain. From the stress tensor, then, a Riemannian metric field is computed, in which manufacturing constraints—such as overhang angle constraints or minimum feature size constraints—are incorporated. That metric is used to generate a background lattice that is optimal and manufacturable by using a metric-based mesher. This design methodology is illustrated in Fig. 1; more details are provided in Ref. 29. An example of a bracket designed using this design methodology is shown in Fig. 2.

![Example of a bracket designed using the design of additive manufacturing algorithm](image)

**Figure 2. Example of a bracket designed using the design of additive manufacturing algorithm in Ref. 29.**

Finally, the size of each member of this lattice is determined by solving an optimization problem that takes into account compatibility conditions, buckling, and flutter.
II.B. Flutter Model

We tailor the internal lattice structure of the wing such that the wing has minimum weight while mitigating flutter, potentially at transonic flow conditions. At transonic Mach numbers, Theodorsen theory with compressibility corrections is no longer accurate. Instead, we employ a physics-based low-order model for transonic flutter. This section briefly summarizes the physics-based model’s formulation; for more details, see Refs. 28 and 30.

This low-order flutter model is calibrated using 2D unsteady high-fidelity Euler simulations and validated against standard flutter benchmark cases, where it is shown to accurately capture the characteristic transonic dip behavior. The model’s application to the flutter characteristics of wings is described in Ref. 30.

The aerodynamic model uses strip theory, which is applicable to high aspect ratio wings. For instance, at the $i$th section on the wing, the evolution equation for the circulation perturbation $\Delta \Gamma_i$ can be written as

$$
\Delta \dot{\Gamma}_i = -\frac{A_F}{A_T} \Delta \Gamma_i - \frac{A_k}{A_F} \Delta \kappa_{x,i} + \frac{1}{A_F} \left( \Delta w_i + V_{\perp} \frac{\partial \Delta h_i}{\partial y_i} \tan \Lambda \right) \\
+ \frac{V_{\perp}}{A_T} \Delta \theta_i + \frac{c_i/4}{A_T} \left( \Delta \omega_i + V_{\perp} \frac{\partial \Delta \theta_i}{\partial y_i} \tan \Lambda \right),
$$

where $h$ is the (downward) deflection, $w$ is the vertical (downward) velocity, $\theta$ is the pitch angle, $\omega$ is the angular velocity, $c$ is the local chord, and $V_{\perp}$ is the wing-perpendicular velocity. $\kappa_x$ is the doublet strength, which is the second aerodynamic state per section. The aerodynamic coefficients $A_F$, $A_T$, $A_k$ are all calibrated from unsteady 2D transonic Euler simulations, as described in Ref. 28.

In Ref. 30, the flutter model is coupled to a one-dimensional Euler-Bernoulli beam model. Here, we replace that beam model with a low-order lattice model, which is further described in Section II.C.

II.C. Low-Order Structural Model for Lattices

We can define a dynamic model for the displacement of each node in lattice, however, the resulting system is potentially quite large and does not directly consider the vertical and angular displacement, which is needed for the aerodynamic model. Therefore, we build a low-order structural model for the lattice, which is described in detail here.

Consider a lattice with $n$ nodes and $m$ members. The dynamics of that lattice structure are described by

$$
M \ddot{u}(t) + K u(t) = F(t),
$$

where $M \in \mathbb{R}^{3n \times 3n}$ is a mass matrix, $K \in \mathbb{R}^{3n \times 3n}$ is the stiffness matrix, $F(t) \in \mathbb{R}^{3n}$ is a dynamic force vector, and $u(t) \in \mathbb{R}^{3n}$ is the displacement vector.

The mass matrix $M$ is obtained by lumping half the weight of each member connecting to a node into the mass of that node, while having massless connectors (Fig. 3).

The stiffness matrix $K$ is obtained from the connectivity of the lattice and stress-strain relations,

$$
K = AB^{-1}A^T
$$

where $A \in \mathbb{R}^{3n \times m}$ and $B \in \mathbb{R}^{m \times m}$. Each column of $A$ is the projection of a member of the lattice on the degrees of freedom of the nodes on the lattice that are connected to that member (Fig. 4). For example the matrix entry corresponding to the $i$th node and $j$th member is expressed as,

$$
A_{3(i-1)+1,j} = n_j \cdot \hat{x} \\
A_{3(i-1)+2,j} = n_j \cdot \hat{y} \\
A_{3(i-1)+3,j} = n_j \cdot \hat{z}.
$$

$B$ is a diagonal matrix where the $i$th diagonal entry corresponds to the deformation per unit force of the $i$th member of the lattice,

$$
B_{ii} = \frac{l_i}{Ea_i},
$$
where $l_i$ is the length of the $i$th member, $a_i$ is the cross-sectional area of the $i$th member, and $E$ is the Young’s modulus.

$K$ and $M$ are potentially quite large (sparse) matrices, resulting in a large aeroelastic system. As part of the lattice optimization, the eigenvalues of this aeroelastic system needs to be computed, which can become quite expensive. Therefore we reduce the size of the structural system by using an equivalent beam model.

For this beam model, we only consider bending around the $\bar{x}$ axis and torsion around the $\bar{y}$ axis. Therefore, we only have to consider displacements in $z$-direction. To find the stiffness matrices for the beam model, we consider the system

$$
\begin{bmatrix}
K & 0 & 0 \\
T_{\mathbf{u} \rightarrow \mathbf{h}} & -I & 0 \\
T_{\mathbf{u} \rightarrow \mathbf{\theta}} & 0 & -I \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{u} \\
\mathbf{h} \\
\mathbf{\theta} \\
\end{bmatrix}
= 
\begin{bmatrix}
T_{\mathbf{L} \rightarrow \mathbf{F}} \\
0 \\
0 \\
\end{bmatrix}
L
+ 
\begin{bmatrix}
T_{\mathbf{M} \rightarrow \mathbf{F}} \\
0 \\
0 \\
\end{bmatrix}
M_{c/2}.
$$

(4)

Through Schur complements of Eq. (4), this system can be rewritten as

$$
\begin{bmatrix}
T_{\mathbf{u} \rightarrow \mathbf{h}}K^{-1}T_{\mathbf{L} \rightarrow \mathbf{F}} \\
T_{\mathbf{u} \rightarrow \mathbf{\theta}}K^{-1}T_{\mathbf{L} \rightarrow \mathbf{F}} \\
T_{\mathbf{u} \rightarrow \mathbf{\theta}}K^{-1}T_{\mathbf{M} \rightarrow \mathbf{F}} \\
\end{bmatrix}
\begin{bmatrix}
L \\
M_{c/2} \\
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{h} \\
\mathbf{\theta} \\
\end{bmatrix}.
$$

(5)

To find the low-order stiffness matrices, the block inverse of Eq. (5) is computed, yielding the system

$$
\begin{bmatrix}
K_{hh} & K_{h\theta} \\
K_{\theta h} & K_{\theta\theta} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{h} \\
\mathbf{\theta} \\
\end{bmatrix}
= 
\begin{bmatrix}
-L \\
M_{c/2} \\
\end{bmatrix}.
$$

(6)

In order to compute the stiffness matrices in Eq. (6), the mapping between the deflections of the full-order lattice model and the beam model need to be defined. The mapping between the lift and moment force on a beam section to the force at each node of the lattice also needs to be defined.
Figure 5. Mapping the deflections from the low-order beam model to the full lattice model

To map the full displacement vector $u$ to the deflection of the beam model $h$, we consider the average $z$-displacement near the beam node,

$$T_{u_{z,k} \rightarrow h_j} = \frac{A_k \phi_j(\bar{y}_k)}{\sum_i A_i \phi_j(\bar{y}_i)}$$  \hspace{1cm} (7a)

where $\phi_j$ is the basis function of the $j$th node, $A_k$ is the projected area in $z$-direction of the $k$th lattice node and $\bar{y}$ is the coordinate along the elastic axis of the model. Throughout this work, hat functions are used as basis functions. These quantities are defined in Fig. 5. The full displacement vector $u$ can also induce a pitch deflection of the beam model, the mapping between those is defined to be

$$T_{u_{z,k} \rightarrow \theta_j} = \frac{1}{2} \frac{\ell_k \phi_j(\bar{y}_k)}{\sum_i \ell_i \phi_j(\bar{y}_i)} \frac{1}{\bar{x}_k - \bar{x}_j},$$  \hspace{1cm} (7b)

where $\ell_k$ is the length along the leading or trailing edge for the lattice node $k$.

The lift per span $L$ only acts on the surface of the lattice, and is mapped to the full force vector $F$ by considering the average pressure around the $j$th beam node,

$$T_{L_{j} \rightarrow F_{k,z}} = \frac{1}{c_j} \phi_j(\bar{y}_k) A_k.$$

Finally, the moment around mid-chord per span $M_{c/2}$ is mapped to the full force vector $F$ as,

$$T_{M_{j} \rightarrow F_{k,z}} = \ell_k \phi_j(\bar{y}_k) \frac{1}{\bar{x}_k - \bar{x}_j}.$$  \hspace{1cm} (7d)
To complete the dynamic low-order model for the lattice structure, the mass matrices for the beam model also need to be defined. We define the mass matrix $\tilde{M}$ as a diagonal matrix, where the $i$th diagonal entry corresponds to the mass per span of the $i$th beam section. Similarly, $\tilde{S}_y$ is a diagonal matrix where the diagonal represents the mass unbalance per span at each beam section. Finally, $\tilde{I}_y$ is a diagonal matrix with the mass moment of inertia per span at each beam section.

The beam model mass matrix $\tilde{M}$ is found by yet another mapping,

$$\text{diag} \left( \tilde{M} \right) = T_{M \rightarrow \tilde{M}} \text{diag} \left( M \right)$$

where $T_{M \rightarrow \tilde{M}}$ is

$$T_{M_k \rightarrow \tilde{M}_j} = \frac{\phi_j(\bar{y}_k)}{\int_0^1 \phi_j(\bar{y})d\bar{y}} \left( \bar{x}_k - \bar{x}_j \right).$$

Similarly, the mass unbalance matrix $\tilde{S}$ is found from a mapping from the mass matrix $M$,

$$\text{diag} \left( \tilde{S}_y \right) = T_{M \rightarrow \tilde{S}} \text{diag} \left( M \right)$$

with

$$T_{M_k \rightarrow \tilde{S}_j} = \frac{\phi_j(\bar{y}_k)}{\int_0^1 \phi_j(\bar{y})d\bar{y}} \left( \bar{x}_k - \bar{x}_j \right).$$

Finally, the mass matrix $M$ maps to the mass moment of inertia matrix $\tilde{I}_y$ as

$$\text{diag} \left( \tilde{I}_y \right) = T_{M \rightarrow \tilde{I}} \text{diag} \left( M \right)$$

with

$$T_{M_k \rightarrow \tilde{I}_j} = \frac{\phi_j(\bar{y}_k)}{\int_0^1 \phi_j(\bar{y})d\bar{y}} \left[ (\bar{x}_k - \bar{x}_j)^2 + (\bar{z}_k - \bar{z}_j)^2 \right].$$

We can therefore write the full dynamic beam model as

$$\begin{bmatrix} \tilde{M} & \tilde{S}_y \\ \tilde{S}_y & \tilde{I}_y \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} K_{hh} & K_{h\theta} \\ K_{\theta h} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} -L \\ M_c/2 \end{bmatrix}.$$  \hspace{1cm} (11)

This structural model therefore has four states per beam section: the vertical (downward) deflection $h_i$, the vertical (downward) velocity $w_i$, the pitch angle $\theta_i$ and the pitch rate $\omega_i$. Combined with the aerodynamic states ($\Delta \Gamma$, $\Delta \kappa_x$, $\Delta \dot{\kappa}_x$) the aeroelastic system has seven states per beam section.

**II.D. Lattice Optimization**

To find the optimal area of each member in the lattice, we solve a nonlinear optimization problem, which includes limits on the stresses in each member, buckling constraints, and constraints on flutter behavior. This optimization problem for a lattice with $m$ members and $n$ nodes in the physical dimension $d$ is written as

$$\min_{u,a,f,\sigma} V = \mathbf{a}^T \mathbf{f}$$

subject to

$$A\mathbf{f} = \mathbf{F} \quad \text{(force balance)}$$

$$B\mathbf{\sigma} + A^T\mathbf{u} = \mathbf{0} \quad \text{(stress-strain compatibility)}$$

$$f_i = \sigma_i a_i \quad \text{(stress definition)}$$

$$-\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{(stress limits)}$$

$$f_i \geq -\frac{\pi E a_i^2}{4(K_{li})^2} \quad \text{(buckling)}$$

$$\chi \theta < \chi_{\theta,\text{max}} \quad \text{(flutter)}$$

$$a_{\text{min},i} \leq a_i \leq a_{\text{max},i} \quad \text{(buckling)}$$

$$\mathbf{u} \in \mathbb{R}^n, \mathbf{f} \in \mathbb{R}^m, \mathbf{\sigma} \in \mathbb{R}^m,$$
where $a_i$ is the cross-sectional area of the $i$th lattice member, $f_i$ is the force in the $i$th lattice member, $\sigma_i$ is the stress in the $i$th lattice member, and $l_i$ is the length of the $i$th member. $\sigma_T$ is the maximum allowable tensile stress and $\sigma_C$ is the maximum allowable compressive stress. $\chi_{fl}$ is the maximum real part of the eigenvalues of the aeroelastic system, which is constrained to be lower than $\chi_{fl,max}$ (typically $-0.005/s$). Finally, $K$ is the column effective length factor, which is taken to be 1.2 here.

Note that the optimization problem in Eq. (12) is nonconvex, if only because the constraint $f_i = \sigma_i a_i$ is nonconvex. The problem is therefore solved using a generic NLP solver, in this case Ipopt.\textsuperscript{31} Such lattice optimization problems are known to suffer from vanishing constraints,\textsuperscript{32} i.e., when the area $a_i \to 0$, the relationship between $f_i$ and $\sigma_i$ is no longer satisfied. However, this is not a problem in this work, because we set a minimum required area for each member of the lattice. This has been done to ensure the manufacturability of the lattice, as removal of members from the lattice could result in the lattice no longer being self-supporting.

III. Validation of Low-Order Structural Beam Model

The low-order beam model has to be validated, especially against its full-order lattice model. We compare the frequencies and mode shapes for a long and slender wing, as computed by the full-order lattice model and the low-order beam model. A symmetric lattice that is used in the first part of this section is shown in Fig. 6. This lattice is symmetric with respect to the $yz$ plane with the elastic axis and center of gravity coinciding with the mid-chord line.

![Figure 6. Lattice with 136 nodes and 535 members used for validation.](image)

The frequencies of the model are listed in Table 1 for the first four bending modes and first two torsion modes using a beam model with 10 beam sections. The error between low-order and full-order model is below 4\% in all cases, which is deemed sufficiently accurate in this study, as the error is substantially lower than the spacing of frequencies between different modes. Note that the error increases for higher mode numbers, which is the result of the discretization in the beam model. If the number of beam sections in the beam model is increased, the error in the higher modes goes down substantially.

The mode shapes are also compared between the low-order beam model and full-order lattice model. The second bending mode shape is shown in Fig. 8. These shapes also match well between the low-order model and full-order model.

A comparison between the low-order model and full-order model for the first torsion mode are shown in Fig. 8, and again these shapes match quite well between the low-order and full-order model.

Lastly, we also compare the structural frequencies between the low-order and full-order model for a lattice where neither the elastic axis, nor the center of gravity position coincides with the mid-chord line. This lattice has an asymmetric orientation for its members as well as different thicknesses for several members (Fig. 9).

The structural frequencies for the lattice in Fig. 9 obtained from both the low-order model (10 beam sections) and full-order model are compared in Table 2. Again, the maximum error is less than 4\%, with the error increasing for higher frequencies.
Table 1. Comparison between frequencies obtained from the full-order lattice model and the low-order beam model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Lattice model, Hz</th>
<th>Beam model, Hz</th>
<th>Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending (I)</td>
<td>0.9075</td>
<td>0.9055</td>
<td>0.22</td>
</tr>
<tr>
<td>Bending (II)</td>
<td>5.263</td>
<td>5.306</td>
<td>0.81</td>
</tr>
<tr>
<td>Torsion (I)</td>
<td>9.152</td>
<td>9.111</td>
<td>0.45</td>
</tr>
<tr>
<td>Bending (III)</td>
<td>14.24</td>
<td>14.52</td>
<td>1.9</td>
</tr>
<tr>
<td>Bending (IV)</td>
<td>26.73</td>
<td>27.82</td>
<td>3.9</td>
</tr>
<tr>
<td>Torsion (II)</td>
<td>28.38</td>
<td>28.42</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 7. Comparison between bending modes of full-order lattice model and low-order beam model for the second bending mode.

Table 2. Comparison between frequencies obtained from the full-order lattice model and the low-order beam model for a case where the center of gravity and elastic axis do not coincide with the midchord.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Lattice model, Hz</th>
<th>Beam model, Hz</th>
<th>Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode I</td>
<td>1.103</td>
<td>1.112</td>
<td>0.81</td>
</tr>
<tr>
<td>Mode II</td>
<td>6.620</td>
<td>6.398</td>
<td>3.5</td>
</tr>
<tr>
<td>Mode III</td>
<td>9.716</td>
<td>9.433</td>
<td>3.0</td>
</tr>
<tr>
<td>Mode IV</td>
<td>17.59</td>
<td>16.94</td>
<td>3.7</td>
</tr>
</tbody>
</table>
Figure 8. Comparison between bending modes of full-order lattice model and low-order beam model for first torsion mode.

Figure 9. Asymmetric lattice with 126 nodes and 489 members used for validation of structures. The cross-sectional area of the red members is twice as large as that of the white members, which in turn is twice the size of the cross-sectional area of the blue members.
IV. Flutter Mitigation through Aeroelastic Tailoring

This section demonstrates the aeroelastic tailoring of lattice structures to mitigate flutter. First, we optimize the internal lattice of the wing without considering flutter and show the flutter boundary and structural modes of that design. Second, we optimize the internal lattice again, but now including a flutter constraint in the optimization. We then compare the flutter characteristics of the two wings as well as their relative mass. It is found that optimizing the wing to obtain a 3% higher flutter speed only makes the structure 3.5% heavier.

![Von Mises stress throughout design space for wing in transonic flow](image)

**Figure 10. Von Mises stress throughout design space for wing in transonic flow.**

For the load case considered, the Von Mises stress through the solid internal domain of the wing is shown in Fig. 10. The stress tensor for this load case is then used to generate a metric field, which in turn is used to generate a background lattice. The members of that lattice are subsequently optimized for minimum weight of the wing, subject to stress constraints—note that we will consider flutter constraints later in this section. The original optimized lattice—without considering flutter—is shown in Fig. 11. In this lattice the largest cross-sectional area of a member is 2.5x larger than the minimum cross-sectional area in the lattice.

![Original lattice that is optimized without considering aeroelastic instabilities](image)

**Figure 11. Original lattice that is optimized without considering aeroelastic instabilities.**
The flutter boundary for the original lattice is shown in Fig. 12. The model clearly exhibits typical transonic dip behavior, where the flutter speed is lowest for transonic Mach numbers and higher for subsonic and high transonic Mach numbers.\textsuperscript{33}

More importantly, the flutter speed $V_f$ in Fig. 12 is lower than the cruise speed $V_{\text{cruise}}$, which needs to be remedied. We therefore optimize the lattice again, but now add a flutter constraint that ensures the wing’s flutter speed is at least 2% higher than the cruise speed. The resulting lattice therefore has a flutter speed that is about 3% higher than the lattice which was not optimized for flutter. The aeroelastically tailored lattice is 3.5% heavier than the original lattice. The flutter boundaries for both the original and the aeroelastically tailored structure are shown in Fig. 12, which shows that the entire flutter boundary moves up for the aeroelastically tailored lattice. Fig. 13 shows the final optimized lattice, where its members are colored according to whether their cross-sectional area increased or decreased compared to the original lattice. We see that at the root section the cross-sectional areas increase, except that cross-sectional areas are decreased for the top and bottom of the wing at positions slightly outboard of the root section. This part of the structure has a huge influence on the overall stiffness of that wing, which is why the cross-sectional areas of the members near this location on the wing changed considerably.

Another way the flutter speed can be increased is by increasing the mass of a structure uniformly. It is known that the flutter speed scales with $\sqrt{m_{\text{wing}}}$. Therefore, to increase the flutter speed for this problem by adding mass, we would need to make the entire lattice 7.5% heavier. However, because during the optimization the mass of each member is optimized—which changes the mass of the structure, its stiffness and its center of gravity position—the mass increase for the aeroelastically tailored lattice is substantially lower.
V. Conclusion

This paper presented a design methodology for using lattice structures for the internal structure of a wing and tailoring the area of each member of the lattice for minimum weight and mitigating aeroelastic instabilities. The lattice structures are designed to align with the stress direction in the structure and are designed with manufacturability in mind. For flutter prediction, a physics-based low-order aerodynamic model is coupled to a low-order structural model of the lattice. This low-order structural model is used to minimize computational cost and to couple the structural model directly to the aerodynamic model. The low-order model is computed using mappings between the deflections of the full-order lattice model and the equivalent beam model, and mappings between the forces on each node of the lattice to the moments and forces on the beam model. The structural frequencies of the low-order beam model have been compared against the structural frequencies of the full-order lattice model, where a maximum error of only 4% was observed for higher modes. This flutter model is then used to predict the flutter of the lattice structure and is included in the optimization problem to find the optimal cross-sectional area of each of the members of the lattice, while avoiding aeroelastic instabilities.

The design methodology is applied to the design of the internal structure of a wing in transonic flow. Including a flutter constraint in the optimization, increases the weight of the structure by only 3.5%, compared to the design where flutter was not constrained.

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References


