Influence of Transonic Flutter on the Conceptual Design of Next-Generation Transport Aircraft

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Transonic aeroelasticity is an important consideration in the conceptual design of novel aircraft configurations. The influence of aeroelasticity on conceptual design tradeoffs is investigated by extending a previously developed low-order physics-based airfoil flutter model to swept high aspect ratio wings. This physics-based flutter model uses the flowfield’s lowest moments of vorticity and volume-source density perturbations as its states. The model is calibrated using off-line 2D unsteady transonic CFD simulations. The resulting aeroelastic system combines this calibrated aerodynamic model with a beam model. The low computational cost of the model permits its incorporation in a conceptual design tool for next-generation transport aircraft. The model’s capabilities are demonstrated by finding transonic flutter boundaries for different clamped wing configurations, and investigating the influence of transonic flutter on the planform design of next-generation transport aircraft.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Aspect ratio</td>
</tr>
<tr>
<td>$A_a$, $B_a$</td>
<td>State-space matrices of aerodynamic system</td>
</tr>
<tr>
<td>$A_\Gamma$, $A_F$, $A_\kappa$</td>
<td>Coefficients of $\Gamma$ evolution equation</td>
</tr>
<tr>
<td>$B_\Gamma$, $B_\kappa$, $B_\kappa$</td>
<td>Coefficients of $\kappa_x$ evolution equation</td>
</tr>
<tr>
<td>$C_{th}(\bar{\omega})$</td>
<td>Theodorsen’s function</td>
</tr>
<tr>
<td>$EI$</td>
<td>Bending stiffness</td>
</tr>
<tr>
<td>$GJ$</td>
<td>Torsional stiffness</td>
</tr>
<tr>
<td>$I_{\bar{y}}$</td>
<td>Mass moment of inertia per span around elastic axis</td>
</tr>
<tr>
<td>$L$</td>
<td>Lift per span</td>
</tr>
<tr>
<td>$M$</td>
<td>Internal bending moment</td>
</tr>
<tr>
<td>$M_{\infty}$</td>
<td>Freestream Mach number</td>
</tr>
<tr>
<td>$M_{CR}$</td>
<td>Cruise Mach number</td>
</tr>
<tr>
<td>$M_{ca}$</td>
<td>Aerodynamic moment per span around elastic axis</td>
</tr>
<tr>
<td>$S$</td>
<td>Internal shear force</td>
</tr>
<tr>
<td>$S_{\bar{y}}$</td>
<td>Mass unbalance per span</td>
</tr>
<tr>
<td>$T$</td>
<td>Internal torsion moment</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity field</td>
</tr>
<tr>
<td>$V_\mu$</td>
<td>Flutter speed index</td>
</tr>
<tr>
<td>$V_\infty$</td>
<td>Freestream velocity</td>
</tr>
<tr>
<td>$V_{\perp}$</td>
<td>Wing-perpendicular velocity</td>
</tr>
<tr>
<td>$b$</td>
<td>Wing span</td>
</tr>
<tr>
<td>$c$</td>
<td>Airfoil chord length</td>
</tr>
<tr>
<td>$c_{l_0}$</td>
<td>Baseline sectional lift coefficient</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between quarter-chord and elastic axis</td>
</tr>
</tbody>
</table>

| $h$ | Vertical position, positive downward |
| $\tilde{l}$ | Effective wing length |
| $m$ | Mass per span |
| $t$ | Time |
| $\boldsymbol{u}_a$ | State-space control vector of aerodynamic system |
| $x, y, z$ | Aircraft coordinate system |
| $\tilde{x}, \tilde{y}, \tilde{z}$ | Coordinates aligned with swept wing |
| $\mathbf{x}_a$ | State-space vector of aerodynamic system |
| $x_{cg}$ | Position of center of gravity relative to wing leading edge |
| $x_{ca}$ | Position of elastic axis relative to wing leading edge |

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\Gamma$</td>
<td>Circulation strength</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Local dihedral angle</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>$\kappa_x$</td>
<td>$x$-doublet strength</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Sweep angle</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Taper ratio</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Volume-source strength</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Thickness ratio of airfoil</td>
</tr>
<tr>
<td>$\chi_{fl}$</td>
<td>Maximum real eigenvalue of aeroelastic system</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular velocity</td>
</tr>
<tr>
<td>$\bar{\omega}$</td>
<td>Reduced frequency</td>
</tr>
</tbody>
</table>

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I. Introduction

Meeting NASA’s ambitious N+3 goals of reducing aircraft fuel burn by 60% by 2035, requires a move towards novel aircraft configurations. Such novel aircraft configurations usually have characteristically high aspect ratios, examples being the Truss-Braced Wing concept\(^1\) or D8.x aircraft.\(^2\) However, large aspect ratios lead to larger flexibility in the wing, and thereby more associated aeroelasticity problems. Flutter therefore becomes an even more important design consideration for these aircraft concepts, and requires flutter constraints to be included in the design of these aircraft as early as possible, preferably in the conceptual design stage. This will prevent costly design changes in later design phases, or during flight testing.

A wealth of literature is available on static aeroelastic optimization of aircraft wings for both conventional tube-wing configurations,\(^3–5\) and next-generation configurations.\(^6\) Dynamic aeroelasticity, however, is often not included in these studies. Kennedy et al.\(^7\) included flutter constraints in their aircraft wing optimization, but the compressibility effects were accounted for using Prandtl-Glauert’s rule, resulting in not being able to accurately predict the flutter boundary in transonic flow. Moreover, all aforementioned methods use high-fidelity aerodynamic and structural analysis methods, and are therefore too expensive to use in conceptual aircraft design. Conceptual aircraft design tools, such as SUAVE,\(^8\) often do not include flutter constraints. Mallik et al.\(^9\) did include transonic flutter constraints in an MDO setting for investigating Truss-Braced Wing designs, but the flutter model consisted of Theodorsen theory with compressibility corrections, limiting its accuracy for transonic flows.

There are several ways to predict unsteady loads on an aircraft.\(^10\) The three most common ones are strip theory (2D unsteady airfoil theory with 3D corrections),\(^11\) the doublet-lattice method,\(^12\) and the unsteady vortex lattice method (UVLM).\(^13,14\) Considering that our goal is to investigate aircraft concepts with high aspect ratios, the use of strip theory here is appropriate.

The theory of flutter behaviour of wings has been investigated thoroughly for decades now, for instance in the seminal work of Theodorsen,\(^15\) who considered flutter of a uniform, straight wing in incompressible flow. Barmby et al.\(^16\) extended Theodorsen’s theory to swept wings in incompressible flow. Yates extended this theory to subsonic, supersonic and hypersonic flow—the method loses accuracy for transonic flow—with a modified strip theory and validated the theoretical results with experimental results.\(^17\)

Thus, there exists a need for a flutter model that is cheap enough to allow for potentially thousands of flutter evaluations of wing designs in the conceptual design phase, but which is also accurate enough to capture subtle design trade-offs for commercial transport aircraft operating in the transonic flow regime. We therefore extend a previously developed and validated airfoil flutter model to swept aircraft wings and implement it in a physics-based conceptual aircraft design tool. Section II describes the airfoil flutter model used, together with its extension to high-aspect ratio swept wings. Section III discusses the aeroelastic behavior of several wing configurations computed using the flutter model, while Section V concludes the paper.

II. Methodology

In this section, the 2D transonic flutter model developed in Ref. 18 is extended to swept aircraft wings. Section II.A briefly explains the 2D flutter model, while Section II.B discusses its extension to 3D. Section II.C presents the structural model for the aircraft wing. The flutter model is implemented in a conceptual aircraft design tool which is explained in Section II.D.

II.A. Airfoil Transonic Flutter Model

We use strip theory to predict flutter for aircraft wings, thereby relying on a locally 2D approximation of the aerodynamics and structures of the wing. The seminal work by Theodorsen\(^15\) is widely used for flutter prediction of airfoils, but only works for incompressible flows. Instead, we use a physics-based low-order model\(^18\) for transonic flutter that is calibrated using high-fidelity Euler/RANS simulations. The approach for constructing that low-order model is shown in Fig. 1, where the model is derived from first principles of transonic unsteady flow, and this model’s unknown coefficients are calibrated with high-fidelity unsteady CFD simulations using Dynamic Mode Decomposition.\(^19\)
The low-order model uses the circulation and $x$-doublet perturbation $\Delta \Gamma$, $\Delta \kappa_x$ from their steady transonic flow values as its aerodynamic state variables, such that the aerodynamic state-space model can be written as,\textsuperscript{18}

$$
\frac{d}{dt} \begin{bmatrix}
\Delta \Gamma(t) \\
\Delta \kappa_x(t) \\
\Delta \dot{\kappa}_x(t)
\end{bmatrix}
= \begin{bmatrix}
-\frac{A_{\Gamma}}{A_{\Gamma}} & -\frac{A_{\kappa}}{A_{\Gamma}} & 0 \\
0 & 0 & 1 \\
B_{\Gamma} & B_{\kappa} & B_{\dot{\kappa}}
\end{bmatrix}
\begin{bmatrix}
\Delta \Gamma(t) \\
\Delta \kappa_x(t) \\
\Delta \dot{\kappa}_x(t)
\end{bmatrix}
+ \begin{bmatrix}
\frac{V_{\infty}}{A_{\Gamma}} & \frac{c/4}{A_{\Gamma}} & \frac{1}{A_{\Gamma}} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \theta(t) \\
\Delta \omega(t) \\
\Delta w(t)
\end{bmatrix},
$$

\text{(1)}

where $\Delta \theta$ is the pitch angle perturbation, $\Delta \omega$ is the pitch rate perturbation, $\Delta w$ is the vertical velocity perturbation, $V_{\infty}$ is the free-stream velocity, and $c$ is the chord. The coefficients ($A_{\Gamma}$, $A_{\kappa}$, $B_{\Gamma}$, $B_{\kappa}$, and $B_{\dot{\kappa}}$) are calibrated using high-fidelity CFD simulations. For the 2D aerodynamic model, these coefficients are calibrated as a function of freestream Mach number $M_{\infty}$, baseline lift coefficient $c_{\ell_0}$, and thickness ratio $\tau$.

For the airfoil flutter model, the aerodynamic model (1) is combined with the typical-section structural model which has four states – heave perturbation $\Delta h$, vertical velocity perturbation $\Delta w$, pitch angle perturbation $\Delta \theta$, and pitch rate perturbation $\Delta \omega$.\textsuperscript{20,21} This gives one state-space system with seven states ($\Delta h$, $\Delta \theta$, $\Delta w$, $\Delta \omega$ for the structural model; $\Delta \Gamma$, $\Delta \kappa_x$, $\Delta \dot{\kappa}_x$ for the aerodynamic model). An example of a flutter boundary with transonic dip found using this airfoil flutter model is shown in Fig. 1. This model has been validated for a standard transonic flutter case in Ref. 18.

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**Figure 1.** Summary of transonic airfoil low-order flutter model construction, detailed in Ref. 18.
II.B. Wing Flutter Model

To assess the flutter characteristics of aircraft wings, we rely on strip theory\textsuperscript{11,22} using the low-order model described in Section II.A. Note that we focus on wings with high aspect ratios, therefore the use of strip theory is appropriate. For our model, this means that we use the perpendicular-plane Mach number $M_\perp$ and lift coefficient $c_{\ell_\perp}$ for selecting the appropriate coefficients for the transonic aerodynamics model.

![Figure 2. Swept wing considered in model. Figure adapted from Ref. 16,18.](image)

Commercial transport aircraft commonly have swept wings to enable higher cruise Mach numbers. We consider the wing geometry in Fig. 2 with sweep angle $\Lambda$. The effective length of the wing $\bar{l} = l / \cos \Lambda$, and all section parameters such as chord length, moment of inertia, etc., are based on sections perpendicular to the elastic axis.

For the aerodynamics model, we must account for the changes to the velocity distribution due to the swept wing. The vertical velocity at any point on the wing is found to be\textsuperscript{16}

$$U_n(\bar{x},t) = w + V_\infty \frac{\partial h}{\partial \bar{y}} \sin \Lambda + V_\infty \theta \cos \Lambda + \bar{x} \omega + \bar{x} V_\infty \frac{\partial \theta}{\partial \bar{y}} \sin \Lambda,$$

(2)

where $\bar{x}, \bar{y}$ are the coordinates aligned with the swept wing. From Eq. (2), Barmby et al.\textsuperscript{16} found the circulatory lift per span for swept wings in incompressible flow to be

$$L_c = \pi \rho V_\perp c C_{th}(\bar{\omega}) \left[ w + V_\perp \frac{\partial h}{\partial \bar{y}} \tan \Lambda + V_\perp \theta + \frac{c}{4} \left( \omega + V_\perp \frac{\partial \theta}{\partial \bar{y}} \tan \Lambda \right) \right],$$

(3)

where $V_\perp$ is the wing-perpendicular velocity and $C_{th}(\bar{\omega})$ is Theodorsen’s function\textsuperscript{15} with $\bar{\omega}$ the reduced frequency. Following the same approach, here the evolution equation for the circulation strength is defined to be,

$$\Delta \Gamma = -\frac{A_r}{A_f} \Delta \Gamma - \frac{A_w}{A_f} \Delta \kappa_x + \frac{1}{A_f} \left( \Delta w + V_\perp \frac{\partial h}{\partial \bar{y}} \tan \Lambda \right) + \frac{V_\perp}{A_f} \Delta \theta + \frac{c/4}{A_f} \left( \Delta \omega + V_\perp \frac{\partial \theta}{\partial \bar{y}} \tan \Lambda \right).$$

(4)
Lastly, the non-circulatory forces and moments used in the model are\cite{16}

\[
\Delta L_{nc} = \frac{1}{4} \pi \rho_{\infty} c^2 \left[ \Delta \ddot{h} + V_1 \frac{\partial \Delta \dot{h}}{\partial y} \tan \Lambda + V_1 \Delta \dot{\theta} \right] - \frac{1}{4} \pi \rho_{\infty} c^2 \left[ \Delta \ddot{\theta} + V_1 \frac{\partial \Delta \dot{\theta}}{\partial y} \right]
\]

(5)

\[
\Delta M_{nc} = -\frac{1}{4} \pi \rho_{\infty} c^2 V_n \left[ \left( \frac{3}{4} c - x_{ea} \right) \Delta \ddot{\theta} + \frac{1}{2} V_1 \frac{\partial \Delta \theta}{\partial y} \tan \Lambda \right] - \rho_{\infty} \pi c^4 \left[ \Delta \ddot{\theta} + V_1 \frac{\partial \Delta \dot{\theta}}{\partial y} \tan \Lambda \right]
\]

+ \frac{1}{4} \rho_{\infty} \pi c^2 \left( x_{ea} - \frac{c}{2} \right) \left[ \Delta \ddot{h} + V_1 \frac{\partial \Delta \dot{h}}{\partial y} \tan \Lambda - \left( x_{ea} - \frac{c}{2} \right) \left\{ \Delta \ddot{\theta} + V_1 \frac{\partial \Delta \dot{\theta}}{\partial y} \tan \Lambda \right\} \right] .
\]

(6)

Note that this formulation ignores the wing camber effect in the noncirculatory forces and moments, which is known to have a negligible effect on flutter.\cite{16}

Finally, note that—using the same assumptions as Barmby et al.\cite{16}—this unsteady aerodynamics model ignores (1) variations in the trailing vorticity, (2) any variation of the airloads in the spanwise direction, and (3) any three-dimensional wing tip effects. These assumptions are quite reasonable for such high reduced frequencies, because then the wake is not fully developed. Flutter is typically observed for high reduced frequencies, making these assumptions appropriate for this work.

II.C. Wing Structural Model

For the structural part of the model, we use Bernoulli-Euler beam theory. Following Bisplinghoff et al.,\cite{20} the beam equations are,

\[
m(\bar{y}) \Delta \ddot{h}(\bar{y}, t) + S_{\bar{y}}(\bar{y}) \Delta \ddot{\theta}(\bar{y}, t) + \frac{\partial^2}{\partial y^2} \left[ EI(\bar{y}) \frac{\partial^2 \Delta h(\bar{y}, t)}{\partial y^2} \right] = -\Delta L'(\bar{y}, t)
\]

(7a)

\[
I_{\bar{y}}(\bar{y}) \Delta \ddot{\theta}(\bar{y}, t) + S_{\bar{y}}(\bar{y}) \Delta \ddot{h}(\bar{y}, t) - \frac{\partial}{\partial y} \left[ GJ(\bar{y}) \frac{\partial \Delta \theta(\bar{y}, t)}{\partial y} \right] = \Delta M'_{\bar{y}}(\bar{y}, t)
\]

(7b)

with boundary conditions,

\[
\Delta h(0, t) = 0, \quad \frac{\partial \Delta h}{\partial y}(0, t) = 0, \quad \frac{\partial^2 \Delta h}{\partial y^2}(\bar{I}, t) = 0, \quad \frac{\partial^3 \Delta h}{\partial y^3}(\bar{I}, t) = 0, \quad \Delta \theta(0, t) = 0, \quad \frac{\partial \Delta \theta}{\partial y}(\bar{I}, t) = 0.
\]

(7c)

This system can be rewritten as

\[
m(\bar{y}) \Delta \ddot{h}(\bar{y}, t) - S_{\bar{y}}(\bar{y}) \Delta \ddot{\theta}(\bar{y}, t) + \frac{\partial \Delta S(\bar{y}, t)}{\partial \bar{y}} = -\Delta L'(\bar{y}, t)
\]

(8a)

\[
I_{\bar{y}}(\bar{y}) \Delta \ddot{\theta}(\bar{y}, t) + S_{\bar{y}}(\bar{y}) \Delta \ddot{h}(\bar{y}, t) - \frac{\partial \Delta T(\bar{y}, t)}{\partial \bar{y}} = \Delta M'_{\bar{y}}(\bar{y}, t)
\]

(8b)

where

\[
\frac{\partial \Delta M}{\partial \bar{y}} = \Delta S, \quad \frac{\partial \Delta \gamma}{\partial \bar{y}} = \frac{\Delta M}{ET}, \quad \frac{\partial \Delta h}{\partial \bar{y}} = \Delta y, \quad \frac{\partial \Delta \theta}{\partial \bar{y}} = \frac{\Delta T}{GJ},
\]

(8c)

with boundary conditions

\[
\Delta h(0, t) = 0, \quad \Delta \gamma(0, t) = 0, \quad \Delta M(\bar{I}, t) = 0, \quad \Delta S(\bar{I}, t) = 0, \quad \Delta \theta(0, t) = 0, \quad \Delta T(\bar{I}, t) = 0.
\]

(8d)

To solve the system in Eq. (8), the wing is discretized in several sections, as shown in Fig. 3. The sectional properties are all computed using the assumed wingbox shape as shown in Fig. 4. The thickness ratio \( \tau \) is used to determine the coefficients of the aerodynamic model, as explained in Section II.A. It is assumed that the elastic axis is at the center of the wing box. Wing box design variables such as spar and skin thicknesses are obtained from the conceptual design tool in which this flutter model is included.

The system in Eq. (8) is solved using finite differences, specifically the trapezoidal rule. However, Eq. (8) has \( 6 \times n_{beam} \) structural parameters in the aeroelastic system, of which only two have inertial terms (\( \Delta h \)
Figure 3. Discretized beam model used in structural part of the flutter model.

and $\Delta \theta$). Such a large system slows down the eigenvalue computation. Instead, we can reduce the size of the system through Schur complements. Consider the discretized system of Eq. (8),

$$
\begin{bmatrix}
\dot{\mathbf{E}}_{hh} & 0 & 0 & \dot{\mathbf{E}}_{h\theta} & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\mathbf{E}_{\theta h} & 0 & 0 & \dot{\mathbf{E}}_{\theta \theta} & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\Delta \dot{h} \\
\Delta \dot{\gamma} \\
\Delta \dot{M} \\
\Delta \dot{S} \\
\Delta \dot{\theta} \\
\Delta \dot{T} \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{\gamma h} & A_{\gamma \theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{\gamma h} & A_{\gamma \theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{\gamma h} & A_{\gamma \theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{\gamma h} & A_{\gamma \theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\Delta h \\
\Delta \gamma \\
\Delta M \\
\Delta S \\
\Delta \theta \\
\Delta T \\
\end{bmatrix}
= 
\begin{bmatrix}
-\Delta L' \\
-\Delta M_{e_{a}} \\
\end{bmatrix}
$$

(9)

The matrices $\tilde{A}_{\gamma h}$, $\tilde{A}_{\gamma \theta}$, $\tilde{A}_{\gamma M}$, and $\tilde{A}_{\gamma T}$ are all tightly banded and therefore cheap to invert. This allows for taking the Schur complement of the system in Eq. (9) to obtain a system with only $h, \theta$,

$$
\begin{bmatrix}
\dot{\mathbf{E}}_{hh} & \dot{\mathbf{E}}_{h\theta} \\
\dot{\mathbf{E}}_{\theta h} & \dot{\mathbf{E}}_{\theta \theta} \\
\end{bmatrix}
\begin{bmatrix}
\Delta \dot{h} \\
\Delta \dot{\theta} \\
\end{bmatrix}
+ 
\begin{bmatrix}
A_{hh} & 0 \\
0 & A_{\theta \theta} \\
\end{bmatrix}
\begin{bmatrix}
\Delta h \\
\Delta \theta \\
\end{bmatrix}
= 
\begin{bmatrix}
-\Delta L' \\
-\Delta M_{e_{a}} \\
\end{bmatrix}.
$$

(10)

This structural model is then coupled to aerodynamic model described in Section II.B through the aerodynamic lift and moment, yielding a $7 \times n_{beam}$ descriptor state-space system.

The engine adds a large amount of inertia and mass to the wing, and it is therefore important to represent in a flutter model. The engine is modeled as a point mass $m_{eng}$ with inertia $\bar{I}_{eng}$ and angular momentum $H_{eng}$, which are rotated into the swept coordinate system. As with the rest of the model, we only consider
perturbations from the steady state. The thrust and moment generated by the engine are thus not taken into account.

The influence of the engine on the structural model comes through a discontinuity in $\Delta S$, $\Delta M$, and $\Delta T$ at the section to which the engine is attached, as shown in Fig. 5. These discontinuities are captured with a zero-length grid interval at the engine attachment point.

The inertial-reaction forces and moments on the $i$th section on the wing due to the engine are,

$$\Delta F_{\text{eng}} = -m_{\text{eng}} \Delta a_{\text{eng}} \quad (11a)$$

$$\Delta M_{\text{eng}} = \Delta r_{\text{eng}} \times \Delta F_{\text{eng}} - \Delta \omega_i \times H_{\text{eng}} - \bar{I}_{\text{eng}} \Delta \omega_i - \Delta \omega_i \times \left( \bar{I}_{\text{eng}} \Delta \omega_i \right) \approx 0 \quad (11b)$$

where $\Delta r_{\text{eng}} = [\Delta x_{\text{eng}}, \Delta y_{\text{eng}}, \Delta z_{\text{eng}}]^T$ is the distance vector between the $i$th beam section and the engine, and $\Delta F_{\text{eng}}$ and $\Delta M_{\text{eng}}$ are the force and moment, respectively, on the $i$th beam section. $\Delta \omega_i$ is the rotation rate
perturbation of the $i$th beam section with respect to the aircraft body axes, defined as $\Delta \mathbf{\omega}_i = [-\mathbf{\Delta} \gamma_i, \mathbf{\Delta} \dot{\vartheta}, 0]^T$. We observe that $\Delta \mathbf{\omega}_i \times \mathbf{H}_{\text{eng}}$ yields only an in-plane bending moment for unswept wings, while in-plane motion is not modeled here. Note that the last term in Eq. (11b) is a quadratic term in the perturbation rates, and can therefore be neglected. Finally, $\Delta a_{\text{eng}}$ is the inertial acceleration perturbation of the engine, defined as

$$\Delta a_{\text{eng}} = \Delta a_i + \Delta \mathbf{\omega}_i \times \Delta r_{\text{eng}} + \Delta \mathbf{\omega}_i \times (\Delta \mathbf{\omega}_i \times \Delta r_{\text{eng}}) \approx 0$$  \hspace{1cm} (11c)

where $\Delta a_i$ is the acceleration perturbation of the $i$th beam section. Again, the last term in Eq. (11c) is a higher-order term in the perturbation rates and hence is omitted. Note that in this formulation, the aircraft fuselage is assumed to not participate significantly in the flutter modes.

The discontinuity in the shear force and bending moments results in additional terms in $\mathbf{E}_{hh}, \mathbf{E}_{h\theta}, \mathbf{E}_{\theta h},$ and $\mathbf{E}_{\theta\theta}$ in Eq. (10).

### II.D. Conceptual Aircraft Design Tool

We use the Transport Aircraft System OPTimization (TASOPT)$^{24}$ tool for the conceptual design of next-generation aircraft concepts. TASOPT was developed at MIT as part of a NASA project to design aircraft to meet aggressive fuel burn, noise, and emission reduction goals for the 2035 timeframe.$^{25,26}$ To assess the performance of next-generation aircraft concepts, this conceptual design tool is primarily developed from first principles rather than using historical correlations. It uses low-order physical models implementing fundamental structural, aerodynamic, and thermodynamic theory, and relies on historical correlations only for the weight of secondary structure and aircraft equipment.

TASOPT can both size the aircraft for a particular mission (range and payload) and optimize the aircraft by varying, for instance, cruise altitude, cruise lift coefficient, aspect ratio, wing sweep, etc., to minimize mission fuel burn. TASOPT can therefore be used to model existing aircraft, assess their off-design performance, and perform a sensitivity analysis for an aircraft. TASOPT is also used to assess the influence of new technologies, such as advanced materials, on an airframe design. Finally, this tool is used to design entirely new aircraft for a set of missions.

As with most aircraft conceptual design tools, dynamic aeroelasticity is currently not a design consideration in TASOPT. To assess the influence of transonic flutter on the conceptual design of next-generation aircraft, the developed transonic flutter model is implemented in TASOPT here.

Flutter is indicated using the eigenvalues of the aeroelastic system—the overall system that consists of the wing aerodynamics model and beam model. If the largest real part of the eigenvalues of the aeroelastic system—denoted here as $\chi_{\text{fl}}$—becomes positive, flutter occurs. Flutter constraints are included in a conceptual design tool by constraining the value of $\chi_{\text{fl}}$ for several different points in the flight envelope.

The design routine in this conceptual design tool wraps an optimizer around a weight-convergence routine. During the optimization several design parameters, such as cruise altitude, wing sweep, aspect ratio, etc., are varied to minimize the fleet-wide fuel energy consumption per payload-range. Constraints can be incorporated in this optimization loop, such as minimum balanced field length and maximum wing span. Here, the requirement for no flutter occurring for $K$ operating points is included in the design process as a sum of constraints in this optimization loop via a penalty term in the objective function,

$$f(\mathbf{x}) = \bar{f}(\mathbf{x}) + \sum_{k=1}^{K} \nu \{ \max [0, \chi_{\text{fl,k}} - \chi_{\text{fl,max}}] \}^2 ,$$  \hspace{1cm} (12)

where $\mathbf{x}$ are the design variables, $f(\mathbf{x})$ is the overall objective function, $\bar{f}(\mathbf{x})$ is a function that includes mission fuel burn with penalty terms for, e.g., balanced field length and maximum wing span, and $\nu$ is a suitable weighting factor. $\chi_{\text{fl,k}}$ is largest real part of the eigenvalues of the aeroelastic system for the $k$th operating point, and $\chi_{\text{fl,max}}$ is the largest value tolerated, typically $-0.005$/$s$.

The flutter model is included in TASOPT using a database of flutter coefficients with $c_{\ell_0}, M_L, \tau$ as the database independent variables. We calibrate 2D low-order models, using the methodology described in Section II.A, for several thickness ratios $\tau$ of an airfoil family (Fig. 6b) at different baseline lift coefficients $c_{\ell_0}$ and Mach numbers $M_L$ and store these in a database. An example of how such a flutter coefficient varies with $c_{\ell_0}$ and $M_L$ is shown in Fig. 6a. Whenever TASOPT queries a flutter evaluation, the appropriate flutter coefficients are found through spline interpolation using the flutter coefficient database.
TASOPT uses a parameterized spanwise loading $p(y)$, whose shape is either specified, or optimized for the best tradeoff between structural weight and induced drag (computed via a Trefftz-Plane analysis). Here, this same loading is used to obtain the local lift coefficient,

$$c_{\ell\perp}(\bar{y}) = \frac{2p(\bar{y}) \cos \Lambda}{\rho_\infty V_\perp^2 c_{\perp}(\bar{y})},$$

where $c_{\perp}(\bar{y})$ is the local chord length perpendicular to the elastic axis. This $c_{\ell\perp}$ is then used to get the appropriate flutter model for each section of the wing using the flutter coefficient database.

### III. Influence of Wing Geometry on Transonic Flutter Boundaries

The flutter model explained in Section II is used here to quantify the influence of several parameters of an aircraft wing on the flutter boundary. First, we compute the flutter boundary of a clamped wing in a wind tunnel-like set-up in Section III.A. Second, we use the flutter model coupled with TASOPT to evaluate the influence of the wing parameters on the flutter damping values in Section III.B where each aircraft on the flutter boundary is a weight-converged aircraft design.

#### III.A. Clamped Wing

We consider a wind tunnel-like set-up for a straight, swept, tapered wing as shown in Fig. 2. This problem has twelve nondimensional parameters, which are listed in Table 1.

**Table 1. Nondimensional parameters for straight, swept, tapered wing.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Aspect ratio</td>
</tr>
<tr>
<td>$M_\infty$</td>
<td>Freestream Mach number</td>
</tr>
<tr>
<td>$c_{\ell\alpha}$</td>
<td>Baseline lift coefficient</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Thickness ratio</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Taper ratio</td>
</tr>
<tr>
<td>$V_\mu$</td>
<td>Flutter speed index</td>
</tr>
</tbody>
</table>

$\frac{x_{\ell\alpha}}{c}$ Relative elastic axis position  
$\frac{x_{cg}}{c}$ Relative center of gravity position  
$\frac{\omega_\theta}{\omega_h}$ Ratio of uncoupled pitch and heave frequencies  
$\mu$ Apparent-mass ratio  
$r_\theta^2$ Radius of gyration

Note that we are not making any claims on the general applicability of these trends; the onset of flutter is sensitive to small changes in the wing parameters and the results described herein only apply to the particular nondimensional parameters of the models described.
The apparent-mass ratio $\mu$, flutter speed index $V_\mu$, and radius of gyration $r_\theta^2$ are defined as,

$$\mu = \frac{4m_{mac}}{\pi \rho_{\infty} c_{mac}^2}$$  \hspace{1cm} (14a)

$$V_\mu = \frac{2 V_{\infty} |_{\chi_\phi=0}}{\sqrt{\mu \omega_\theta c_{mac}}}$$  \hspace{1cm} (14b)

$$r_\theta^2 = \frac{4I_{\bar{y},mac}}{m_{mac} c_{mac}^2}$$  \hspace{1cm} (14c)

where,

$$\omega_\theta = \frac{\sqrt{4 (GJ)_{mac}}}{m_{mac} b^2 c_{mac}^2}$$  \hspace{1cm} (15a)

$$\omega_h = \frac{\sqrt{(EI)_{mac}}}{m_{mac} b^4}$$  \hspace{1cm} (15b)

These non-dimensional parameters determine the planform shape and sectional properties of the mean aerodynamic chord (MAC). To obtain the sectional properties over the entire wing, we scale the sectional properties with the local chord length: $m \sim c^2$, $I_{\bar{y}} \sim c^4$, $EI \sim c^4$, and $GJ \sim c^4$.

![Figure 7. Geometry for clamped straight swept tapered wing, including the nondimensional parameters of the problem.](image)

Figure 7. Geometry for clamped straight swept tapered wing, including the nondimensional parameters of the problem.

First, we show the influence of wing taper on the flutter boundary. Those results are shown in Fig. 8. It is observed that the flutter boundary as a function of Mach number is almost independent of $M_\infty$ and the taper ratio only determines the offset, with higher taper ratios being more susceptible to flutter. This is expected because for lower taper ratios the mass is concentrated closer to the fuselage.

The aspect ratio of the wing has a large influence on the flutter speed of the wing, which is a concern for next-generation aircraft designs. In order to look at a realistic variation of aspect ratio for an aluminum transport aircraft wing, we scale the properties of the wing as $b \sim \sqrt{R}$, $c \sim 1/\sqrt{R}$, $h \sim 1/\sqrt{R}$, $m \sim R$, $GJ \sim 1$, $EI \sim 1$, and $I_{\bar{y}} \sim 1$. This ensures that the wing has a fixed wing area, fixed lift, and fixed wingbox stress. Note that the nondimensional parameters are not kept constant with such a sweep over aspect ratio. Therefore we use the nondimensional parameters of $R = 15$ to scale the results.

The flutter boundary as a function of aspect ratio is shown in Fig. 9. We see that a higher aspect ratio reduces the flutter speed substantially, as expected. This influence does, however, plateau for high aspect ratios. Furthermore, the characteristic transonic dip is again observed.
Figure 8. Influence of taper ratio on the flutter boundary for $\mathcal{R} = 16$, $\Lambda = 0^\circ$, $\mu = 30$, $\omega_\theta/\omega_h = 35$, $r_0^2 = 0.15$, $(x_{ea}/c) = 0.400$, $(x_{cg}/c) = 0.401$, $\tau = 0.13$, and $c_{\ell_0} = 0.6$ (C-airfoil family).

Figure 9. Influence of aspect ratio and freestream Mach number on the flutter boundary. The nondimensional parameters for $\mathcal{R} = 15$ are $\Lambda = 0^\circ$, $\lambda = 0.75$, $\mu = 33$, $\omega_\theta/\omega_h = 40$, $r_0^2 = 0.125$, $(x_{ea}/c) = 0.300$, $(x_{cg}/c) = 0.301$, $\tau = 0.13$, and $c_{\ell_0} = 0.6$ (C-airfoil family).

The sweep of the wing has a profound effect on the flutter boundary, particularly for transonic flows because it lowers the Mach number seen by the section perpendicular to the elastic axis. Flutter boundaries
for various sweep angles and Mach numbers are shown in Fig. 10. We observe that for low subsonic Mach numbers the flutter boundary follows a shifted parabola for incompressible flows, similar to what was found by Barmby et al.\textsuperscript{16} That behavior changes completely with transonic flow, though. Whereas for low subsonic flows the sweep angle generally increases the flutter speed, for transonic flows the sweep angle decreases the flutter speed. Fig. 10a clearly shows that wing sweep delays the transonic dip to higher freestream Mach numbers, which is partly the reason for a decrease in flutter speed for transonic Mach numbers.

![Figure 10](image)

**Figure 10.** Influence of wing sweep angle on the flutter boundary for $\mathcal{A} = 15$, $\lambda = 0.3$, $\mu = 40$, $\omega_0/\omega_h = 35$, $r_a^2 = 0.125$, $(x_{ca}/c) = 0.400$, $(x_{cg}/c) = 0.401$, $\tau = 0.14$, and $c_{t0} = 0.6$ (C-airfoil family).

When the engine is included in the model, additional nondimensional parameters need to be considered. Specifically, we consider the mass ratio of the engine $\mu_{\text{eng}}$, the radius of gyration of the engine $\bar{r}_{\text{eng}}$, the relative spanwise location of the engine $\eta_{\text{eng}}$, the relative chord-wise location of the engine $\xi_{\text{eng},x}$, and the relative normal location of the engine $\xi_{\text{eng},z}$. These parameters are defined as,

\begin{align}
\mu_{\text{eng}} &= \frac{m_{\text{eng}}}{m_{\text{mac}}} \quad \text{(16a)}
\end{align}

\begin{align}
\bar{r}_{\text{eng}} &= \frac{4I_{\text{eng}}}{m_{\text{eng}}c_{\text{mac}}^2} \quad \text{(16b)}
\end{align}

\begin{align}
\eta_{\text{eng}} &= \frac{y_{\text{eng}}}{b} \quad \text{(16c)}
\end{align}

\begin{align}
\xi_{\text{eng},x} &= \frac{\Delta x_{\text{eng}}}{c_{\text{mac}}} \quad \text{(16d)}
\end{align}

\begin{align}
\xi_{\text{eng},z} &= \frac{\Delta z_{\text{eng}}}{c_{\text{mac}}} \quad \text{(16e)}
\end{align}

The influence of the mass of the engine on the flutter boundary is shown in Fig. 11. We see a similar reversal of trends between low subsonic and transonic flows; for low subsonic flows the additional mass of the engine increases the flutter speed, but for transonic conditions the additional mass suddenly decreases the flutter speed.
(a) Flutter speed index versus Mach number

(b) Flutter speed index versus engine weight

(c) Planform geometry

Figure 11. Influence of engine weight and freestream Mach number on the flutter boundary for $AR = 15, \lambda = 0.3, \Lambda = 25^\circ, \mu = 25, \omega_0/\omega_h = 35, r_0^2 = 0.125, (x_{ca}/c) = 0.350, (x_{cg}/c) = 0.351, r = 0.13, c_\ell_0 = 0.6$ (C-airfoil family), $\eta_{\text{eng}} = 1/4$, $\xi_{\text{eng},x} = -1/4$, $\xi_{\text{eng},z} = -1/8$, $r_{\text{eng},xx}^2 = 0.5$, $r_{\text{eng},yy}^2 = 0.3$, and $r_{\text{eng},zz}^2 = 0.3$ (off-diagonal terms zero).

III.B. Weight-converged aircraft

The results in Section III.A showed the influence of wing parameters on the flutter boundary for a wing clamped to the wall. However, for an aircraft design, any change in parameter of the wing cascades through the entire aircraft design, because the aircraft has to be weight converged. In other words, the lift the wing generates has to be equal to the weight of the aircraft. When this is taken into account, the flutter characteristics as a function of wing parameters also change. This section shows the influence of wing parameters on the flutter damping values while ensuring that the aircraft design is weight-converged at each flutter evaluation. These aircraft designs are generated using TASOPT.

Figure 12. Comparison of D8.0 and D8.2 configurations.²
We consider two different D8.x aircraft configurations, as described in Ref. 2. The D8.0 and D8.2 configurations are compared in Fig. 12. The D8.0 is a “fuselage-only” modification to the Boeing 737, keeping the wing the same as for the 737 – the engines therefore hang under the wing. The D8.2 has two changes compared to the D8.0: the engines are moved to the back of the fuselage to enable boundary layer ingestion and the cruise Mach number is reduced to allow for a lower-sweep wing.

Flutter is evaluated for several worst-case points on the \((M_\infty, h_a)\) envelope, as shown in Fig. 13. All of the flutter evaluation points are for the never-exceed dynamic pressure, because an increase in dynamic pressure always results in the aircraft being closer to flutter, even in transonic flight. Due to the transonic dip behaviour, it is not clear which freestream Mach number is the worst-case for flutter. Therefore several Mach numbers between the cruise Mach number and 10% above the cruise Mach number are sampled. For each flutter evaluation point we also consider empty wing tanks, half-full wing tanks, and full wing tanks, because these change the mass and center of gravity of the wing substantially.

![Figure 13. \((M_\infty, h_a)\) envelope for the D8.2 aircraft.](image)

Fig. 14 shows the aeroelastic eigenvalues as function of wing aspect ratio \(A\) and cruise Mach number \(M_\infty\). Note that \(\chi_B > 0\) indicates flutter. These results consider a freestream Mach number 10% higher than \(M_{CR}\) and full wing tanks as the flutter evaluation point. Note that the trends differ between the different flutter evaluation points shown in Fig. 13, but the one shown in Fig. 14 is one of worst cases. We see that the D8.0 and D8.2 have wildly different values for \(\chi_B\), while the trends are also dissimilar. For the D8.2 the increase in aspect ratio leads to an increase in \(\chi_B\) (i.e., closer to instability), whereas for the D8.0 the aspect ratio at first decreases \(\chi_B\) but for higher aspect ratios this trend reverses. The trend for the D8.2 is therefore similar to the results for the clamped wing configuration in Fig. 9, where a higher aspect ratio decreases the flutter speed because the wing is more flexible for higher aspect ratios. For the D8.0, the wing also gets more flexible for larger aspect ratios, but the engine also moves further outboard. For lower aspect ratios, putting the engine further outward moves the wing towards stability. However, for higher aspect ratios the effect of the larger flexibility in the wing again moves the wing away from stability. An increase in design cruise Mach number puts the D8.2 closer to instability. However, for the D8.0 for lower aspect ratios the cruise Mach number decreases \(\chi_B\), whereas for aspect ratios around \(A = 11\) higher cruise Mach numbers increase \(\chi_B\).

Fig. 15 shows the aeroelastic eigenvalues of weight-converged aircraft designs for different sweep angles of the wing \(\Lambda\) and different design cruise Mach numbers \(M_{CR}\). These results consider a freestream Mach number 10% higher than \(M_{CR}\) and empty wing tanks as the flutter evaluation point. The results for the D8.2 show that an increase in sweep angle moves the design away from instability, as does a decrease in cruise Mach number. For the D8.0, however, these trends are mostly reversed, showing the large influence of the inertia and mass of the engine on wing flutter. As explained in Section II.C, with an increase in sweep angle, \(\Delta x_{eng}\) decreases and \(\Delta y_{eng}\) increases. The additional stability from changing the engine position therefore outweighs the push towards instability by the increase in sweep angle.
Figure 14. Influence of aspect ratio $\mathcal{R}$ on maximum aeroelastic eigenvalue $\chi_{fl}$ for two different aircraft; the D8.0 has a wing with a large sweep angle and wing-mounted engines, the D8.2 has a lower-sweep wing and fuselage-mounted engines. These results are for a flutter evaluation at $M_\infty = 1.10M_{CR}$ with full wing tanks.

Figure 15. Influence of sweep angle $\Lambda$ on maximum aeroelastic eigenvalue $\chi_{fl}$ for two different aircraft; the D8.0 has a wing with a large sweep angle and wing-mounted engines, the D8.2 has a lower-sweep wing and fuselage-mounted engines. These results are for a flutter evaluation at $M_\infty = 1.10M_{CR}$ with empty wing tanks.
**IV. Influence of Transonic Flutter on Aircraft Designs**

This section discusses the influence of transonic flutter on novel aircraft designs. We will focus here on several variants of the D8.0. All results in this section are generated using TASOPT and with the flutter evaluation points as described in Fig. 13.

First, we show the influence of transonic flutter on the planform design of an advanced-technology version of the D8.0. For this design, better engine technology, laminar-bottom wings, and carbon-fiber structures are used, allowing for higher aspect ratios leading to a higher susceptibility to flutter. The optimized planforms with and without flutter constraints are shown in Fig. 16. The flutter constraints essentially serve as a span constraint, as we have already seen that higher aspect ratios lead to lower flutter speeds in Fig. 9. Due to the lower aspect ratio, the flutter-constrained design has worse aerodynamic performance, and therefore a 3.3% higher fuel burn. The lower span does result in a lower maximum take-off weight, but that does not offset the higher fuel burn due to poorer aerodynamic performance.

![Figure 16. Advanced-technology D8.0 designed with and without flutter constraints.](image)

Finally, we investigate specifically the influence of newer material technology on the performance of the D8.0 and describe the influence of transonic flutter constraints on this design. The specific allowable stress is used as a surrogate for new material technology, which is quantified here by a factor multiplying the baseline value, corresponding to aluminum in this case. The varying planform designs and influence on fuel burn and maximum take-off weight are shown in Fig. 17 for specific allowable stress up to 50% higher values than aluminum.

As expected, an increase in specific allowable stress results in a lower fuel burn and maximum take-off weight, which was already observed by Drela\textsuperscript{27} for Boeing 737-class aircraft. The inclusion of the flutter constraints limits the efficiency gains seen by an increase in specific allowable stress, because the aspect ratio is limited by the flutter constraints. As already observed in Fig. 16, the lower aspect ratio does result in a lower maximum take-off weight, but the fuel burn is still higher than for the design without flutter constraints. The trends in Fig. 17 are similar to those described by Drela\textsuperscript{27} for the Boeing 737-class aircraft with and without a span constraint.

**V. Conclusion**

A transonic flutter model applicable to high-aspect ratio swept wings was presented in this paper. This work extends a previously developed 2D transonic flutter model to 3D and implements it in a conceptual design tool. This model discretizes the wing in several sections and builds an aeroelastic system for the whole wing using calibrated flutter coefficients for transonic flow. Such a system is relatively low-dimensional,
allowing for fast computation of the eigenvalues, making it applicable for use in a conceptual design tool. The approach is demonstrated to find transonic flutter boundaries for several wing configurations, showing the influences of Mach number, taper ratio, and sweep.

The model is also used to investigate the aeroelastic characteristics of different aircraft designs. The flutter behaviour for two different aircraft configurations—one with a high-sweep wing with an engine attached below, one low-sweep wing with no engines attached to the wing—is quite different, and several trends of flutter damping values with respect to wing design parameters are reversed between the two configurations. For example, a larger sweep angle pushes the aircraft with wing-mounted engines towards instability, whereas for the aircraft with fuselage-mounted engines a larger sweep angle moves the design away from instability. Furthermore, it was found that the inclusion of flutter constraints in the aircraft design optimization limits the aspect ratio of the wings, resulting in higher fuel burn. Transonic flutter therefore limits the performance gains seen by using more advanced materials in the wing. Note that these trends are not generally applicable to any aircraft design; the onset of flutter is quite subtle and is sensitive to small changes in wing geometry, engine parameters, or aerodynamic forces. It is therefore crucial to include an accurate, fast aeroelastic analysis early in the conceptual design phase.

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References


