Where Force Matters: Embedding Epistemic Modals and Attitudes*

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There has been much recent interest in the analysis and distribution of embedded epistemic modals (Yalcin 2007, Anand and Hacquard 2013, a.o.). We present novel data using the embedding verb *dopuščati* (‘to allow for the possibility that’) from Slovenian, analysed as an existential doxastic attitude, and argue for a new analysis of epistemic modals that captures their restricted distribution under doxastic attitudes.

1 Introduction

Suppose you wake up late one morning. It’s already bright outside but you are too lazy to open your eyes. You could entertain the following two thoughts about the light:

(1) a. Mislim, *da* utegne biti zunaj *sončno*.
    *I think that might be outside sunny*
    ‘I think it might be sunny outside.’

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b. Mislim, da mora biti zunaj sončno.
   I think that must be outside sunny
   ‘I think it must be sunny outside.’

As noted previously (Stephenson 2007, a.o.), doxastic attitudes like think can embed two kinds of modal verbs: possibility epistemic modals (e.g. might) and necessity epistemic modals (e.g. must). The same facts hold in Slovenian, as illustrated above. Put differently then, universal doxastic quantifiers can embed existential as well as universal epistemic modals.

In addition to verbs like misliti (‘think’) or verjeti (‘believe’), Slovenian also has a weaker doxastic verb: dopuščati (‘to allow for the possibility’). Imagine the same situation as before. As (2) illustrates, embedding a possibility epistemic modal under dopuščati is acceptable (albeit slightly redundant), while embedding a necessity one leads to an odd utterance.

(2) a. Dopuščam, da utegne biti sončno.
   I allow that might be sunny
   ‘I allow for the possibility that it might be sunny.’

b. #Dopuščam, da mora biti sončno.
   I allow that must be sunny
   ‘I allow for the possibility that it must be sunny.’

This data yields the generalization that (in a situation where the evidence under consideration is the attitude holder’s) it is odd to combine a strong embedded modal with a weak doxastic attitude, while the other three combinations are acceptable. This idea, that existential doxastic attitudes can only embed existential (and not universal) epistemic modals, is not new (see Anand & Hacquard 2013), but it has so far only been discussed in the context of attitude verbs involving preferences (hope and fear) or negative orientation (doubt). Since dopuščati involves neither, it delimits the space of possible analyses in an important way.

The paper has two goals. The first one is empirical: to discuss dopuščati and the formally relevant data in a non-technical way (§2). This section can hopefully be of use to a broader audience. The second goal (§3-4) is

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1 “Epistemic modal” is used for modals when under the epistemic construal.
to propose that (2b) is odd because it contextually expresses the same proposition as (1b), using a weak constituent (dopuščati) in comparison. The challenge lies in re-thinking epistemic modals and doxastic attitudes (§3) so that such an equivalence does not arise between (2a) and (1a).

2 Embedding under Doxastic Attitudes

I first discuss some properties of dopuščati and argue that it is a weak doxastic attitude (§2.1). Building on the data in (1) and (2), I show that dopuščati cannot embed epistemic necessity modals or their equivalents (negated possibility modals) (§2.2). Section §2.3 discusses matrix clause negation with doxastic attitudes – embedded necessity under don’t think and negated dopuščati is odd. Finally, dopuščati reveals an important difference in acceptability between embedding epistemic modals and embedding doxastic attitudes (§2.4).

2.1 What it Means to ‘Dopuščati’

Consider (3). The sentence conveys that Othello considers it possible that Desdemona loves Cassio, but he leaves it open as to whether or not she actually does. That is, the proposition that Desdemona loves Cassio is consistent with Othello’s beliefs, but he is understood to not have made up his mind as to whether he should believe it.

(3) Othello dopušča, da Desdemona ljubi Cassija.
   Othello allows that Desdemona loves Cassio
   ‘Othello allows for the possibility that Desdemona loves Cassio.’

Below are some natural occurring examples to the same point. Example (4c) illustrates that dopuščati can appear with the (always optional) noun možnost (‘possibility’).

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2 The Russian cognate dopuskat’, which is also used as a weak doxastic attitude, does not seem to do this as naturally. The Russian National Corpus (http://www.ruscorpora.ru/en/index.html, last accessed in May 2017) contains 406 tokens of dopuskaju (‘I allow’) immediately followed by a čto (‘that’) clause, compared to 20 tokens of dopuskaju immediately followed by a noun, of which 1 is ‘possibility’ and 19 are ‘thought’ (Rafael Abramovitz, p.c.).
(4) a. Dopuščam da je vaša laž posledica neznanja in ne
Lallow that is your lie consequence ignorance and not
zlonamernosti.
malevolence.
‘I allow for the possibility that your lie follows from ignorance
and not malevolence.’ (web)
b. To je seveda le moje mnenje, nikakor ga ne
this is of course only my opinion in no way it not
vsiljujem, tudi dopuščam, da se motim.
Limpose also I allow that I err
‘This is of course only my opinion, I definitely do not impose it,
I even allow for the possibility that I’m wrong.’ (web)
c. Tusk dopušča možnost, da brexita ne bo.
Tusk allows possibility that Brexit not will be
‘Tusk allows for the possibility that there will be no Brexit.’
(web)

In English, some speakers use the verb allow (without for the possibility) in a related way, as in (5).³ The difference, however, is that allow is more “discursive” (used, for example, to admit something to be true for the sake of the argument). Dopusčati, on the other hand, carries no such implication – it is used merely to report on your mental state.

(5) a. Othello allows that Desdemona might love Cassio.
b. I’ll allow that I’m wrong.

Unlike vanilla doxastic attitude (think/believe), dopuščati is weak in the sense that one can dopuščati something as well as its opposite, as in (6a).

Lallow that is inside and I allow that is outside
‘I allow that he’s inside and I allow that he’s outside.’
b. #Mislim, da jenotri, in mislim, da je zunaj.
I think that is inside and I think that is outside
‘I think that he’s inside and I think that he’s outside.’

³ Thanks to Matt Mandelkern (p.c.) for first pointing this out to me.
The relationship between *dopuščati* and attitudes like *think/believe* resembles that of *some* compared to *all*. In (7), we see that a *dopuščati* claim can be strengthened into a belief claim (cf. *some students passed the exam, in fact all of them did*).

(7) In a debate with Flat-Earthers, a scientist is asked: *Ali dopuščate, da je Zemlja okrogla?* (Do you allow for the possibility that the Earth is round?) The scientist replies:

Seveda dopuščam, da je – trdno verjamem, da je!
of course I allow that is firmly believe that is
‘Of course I allow for the possibility that it is – I firmly believe it!’

The reason why one might think that *dopuščati* talks about something that is consistent with our beliefs, rather than knowledge, is that it is commonly assumed that there is no such thing as false knowledge. There are, however, false *dopuščati* states:

(8) *Dežuje, ampak Janez ne dopušča, da dežuje.*
rains but John not allows that rains
‘It’s raining but John doesn’t allow for the possibility that it’s raining.’

In §3 I will assume that *dopuščati* is an existential quantifier over the doxastic (belief) state.

2.2 *Embedding Epistemic Modals*

Epistemic *possibility modals* can be embedded under strong doxastics like *misliti* or *think*, (1a), but also under weak ones like *dopuščati*, (2a). The examples in the introduction involved the verb *utegniti* but the same point can be made with an adverb like *mogoče* (‘maybe’), as in (9).

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4 The analogy with *some* and *all* runs into trouble with the example below, which is not odd (contradictory). There is plausibly, however, a contextual shift involved – I think x but I allow for the possibility that I’m wrong in which case not x. Alternatively, *think/believe* are in fact weaker than usually assumed (Hawthorne et al. 2016), in which case the analogy might be closer to, for example, *some* and *most*.

(i) *Marija misli/verjame, da je notri, ampak dopušča, da je zunaj.*

Mary thinks/believes that is inside but allows that is outside
‘Mary thinks/believes he’s inside but allows for the possibility that he’s outside.’
Othello is asked whether he thinks that Desdemona is cheating on him. He replies:

a. Dopuščam, da me (mogoče) vara.
   I allow that me maybe cheats.on
   ‘I allow for the possibility that she might be cheating on me.’

b. Mislim, da me mogoče vara.
   I think that me maybe cheats.on
   ‘I think she might be cheating on me.’

Is there a difference between (9a) and (9b)? The two are very close, but speakers report Othello to have perhaps some reason for suspecting Desdemona of cheating in (9b). Speakers also report the modal in (9a) to be somewhat redundant, i.e. not needed for conveying that she might in principle be unfaithful, but they note that its presence adds tentativeness.

In contrast to possibility modals, epistemic necessity modals do not behave uniformly with respect to the strength of the embedding verb, as illustrated in (1b) and (2b). What leads to oddness in (2b), however, seems to be the embedding of a wide-scope necessity meaning. Consider:

(10) Situation as in (1) and (2).
    #Dopuščam, da ne more deževati.
    I allow that not can rain
    ‘I allow for the possibility that it can’t be raining.’

On a fairly standard assumption, the force of a negated possibility modal, as in (10), equals that of a necessity modal with a negated complement. Given that (10) is odd, the culprit in (2b) is plausibly not morati (‘must’) per se, but the overall force in the embedded clause of dopuščati.

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5 Must under think is sometimes also claimed to be redundant.
6 It is difficult to find negated necessity modals to check whether those are acceptable, by analogy to (2a). Here is a potential candidate:

(i) Dopuščam možnost, da ni nujno, da sem prinesel “tako” hude
    I allow possibility that is not necessary that aux brought such bad
    poškodbe in bil zato lahek plenMOMa [...] injuries and aux therefore easy prayBPD
    ‘I allow for the possibility that it is not necessarily the case that having such bad
    wounds made me easy prey to BPD [...]’ (web)
2.3 *Negated Doxastic Attitudes*
Consider the following example:7

(11) Situation: You, me, and John see Bob go home from work early. We sit down on some couches in front of Bob’s office. John has his back turned to Bob’s door. He puts on some headphones and starts cheating on the latest homework. After a while, Bob, who has a secret entry to his office, which he used to come back, creeps out of his office and comes up behind John’s back. John, still immersed in cheating, does not notice this. I nudge you and whisper, with both of us staring at Bob:

a. John does not think that Bob might be behind his back.

This data is in line with Anand and Hacquard’s (2013, fn. 27) finding for Romance that main clause negation makes the embedding of a necessity modal under a doxastic attitude degraded.8,9 The same pattern occurs in Slovenian with *misliti*, which is not neg-raising.

(12) Situation: as in (11).

a. Janez ne misli, da je Bob mogoče za njegovim
   → John not thinks that is Bob maybe behind his
   back
   ‘John does not think that Bob might be behind his back.’

b. ??Janez ne misli, da mora biti Bob za njegovim
   → John not thinks that must be Bob behind his
   back
   ‘John does not think that Bob must be behind his back.’

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7 Judgments in this section vary somewhat; I mark the most cheritable interpretation (e.g. “???” should be read as “?? or worse”).
8 See also Crnič (2014) and Ippolito (2017).
9 Homer (2015) makes this observation for American English with (i) below. He notes that the British English *must* is acceptable under *don’t think* and argues that this is because it can participate in double neg-raising (i.e. think*>must*<not in (ii) below).

(i) #I don’t think that John must equal be very intelligent. (AmE)
(ii) I don’t think that John must equal be very intelligent. (BrE)
The situation in (11) is constructed so that the only evidence that the embedded epistemic can be felicitously sensitive to is the attitude holder’s, i.e. John’s (since you, me, and Bob know/see that Bob is behind John’s back). The statements in (11b)/(12b) also express something that is intuitively true, cf. (13). While (13) may be a somewhat awkward way of putting it, it does not feel odd in the same way as (11b)/(12b) does.

(13) It’s not the case that John thinks that Bob must be behind his back.

A possible fault with the scenario in (11) is that it gives John no reason for entertaining the thought that Bob must be behind his back.\(^\text{10}\) Given the contrast between (11b)/(12b) and (13), it is unclear why this should play a role. Nevertheless, consider a different scenario:

(14) Situation: You and I have had the opportunity to work as assistants to Sherlock Holmes, who is investigating a recent murder. Sherlock has taken an interest in the gardener and the butler. You and I are discussing what Sherlock thinks about who the murderer is. I say:

\[ ?\text{Sherlock does not think that the gardener must be innocent (since he followed him around this morning).} \]

Speakers still find something a bit odd about this example.\(^\text{11}\) Here, however, Sherlock presumably thinks that the gardener might, or possibly must, be guilty. It should therefore be felicitous, given his behaviour, to deny that he thinks that the gardener must be innocent.

Interestingly, this contrast between embedded possibility and embedded necessity modals is maintained with dopuščati, as illustrated in (15). Put differently, negation over dopuščati does not seem to play a role in the embedding of epistemic modals under dopuščati.

\(^{10}\) Thanks to Kai von Fintel (p.c.) for raising this issue.

\(^{11}\) Some speakers feel that (14) is as bad as (11b). The difference for the others could be related to the ability of because/since to suspend implicatures, e.g. Some students passed the exam because all of them did, bringing doesn’t think closer to it’s not the case that he thinks.
In this section we showed that embedding a necessity modal under a negated doxastic, be it a strong one like *think/misliti* or a weak one like *dopuščati*, leads to a certain degree of oddness. This does not occur with embedded possibility modals.

2.4 Embedding Doxastic Attitudes

I want to briefly touch upon a difference between attitudes and modals, most influentially discussed in Yalcin (2007). I will not review his data here, but I will make a similar point with different data. Consider again example (2). Recall that embedding a weak epistemic modal under a weak attitude is acceptable, in (2a), while embedding a strong epistemic modal under a weak attitude leads to oddness, in (2b). By contrast, embedding either kind of attitude strength under a weak attitude is odd:

(17) a. #Dopuščam, da dopuščam, da sem se zmotila.
I allow that I allow that I made a mistake.

b. #Dopuščam, da verjamem, da sem se zmotila.
I allow that I believe that I made a mistake.

Intuitively, what goes wrong in (17) is that the speaker (more generally, the attitude holder) fails to be an authority on his own beliefs; we take
belief to be something that we all ‘have privileged and immediate access to’ (Klein et al. 2015).\footnote{See also Dorr and Hawthorne (2013: 897–898).}

The point about this is minor but important: (2a) and (17a) both involve embedding under a weak expression, but only the case of embedded attitudes leads to oddness. (On a related note, the oddness in (17b) feels distinctively different from that in (2b).) Informally, we can conclude from this that while people are assumed to be authorities on their beliefs, they are not assumed to be authorities of the same kind on their evidence. Nevertheless, a tight connection needs to be maintained between belief and epistemic modals embedded under belief, for reasons discussed in Yalcin (2007).\footnote{The standard analysis (Hintikka (1962), Kratzer (1977, 1981)) predicts (2a) and (2b) to both be good, while a simple extension of Yalcin’s (2007) revised semantics predicts both to be odd. Our goal is to arrive at something intermediate for embedded modals, while keeping the common assumptions about belief introspection that explain the oddnesses in (17).}

3 A New Semantics for Doxastic Attitudes and Epistemic Modals

The generalization that we want to model is that embedded universal epistemic force is odd under dopuščati and negated doxastic attitudes (be it milštit or dopuščati). There are three key notions to the interaction: (i) doxastic states are structured in terms of prominence and the choice of what is made salient depends on the attitude verb, (ii) epistemic modals are “local” (Mandelkern, forthcoming), and (iii) epistemic modals are “total”. The rest of the interaction is carried by the assumptions about presupposition projection that I make.

Formulas will be evaluated with respect to an index that consists of three parameters: (i) the information state (a set of worlds), as in Yalcin (2007), the salient state (a subset of the information state), and (iii) the world of evaluation. I will use intensional semantics à la von Fin
tel and Heim (2016) over these evaluation triples.

\[
\begin{align*}
\text{(18) a. extension: } & \llbracket \gamma \rrbracket^\text{e}(s, s', w) \\
\text{b. intension: } & \lambda(s, s', w)[\llbracket \gamma \rrbracket^\text{e}(s', w)] \\
& \text{(abbreviated as } \llbracket \gamma \rrbracket^\text{e} \text{)}
\end{align*}
\]
In (18), \( c \) is the context set, \( g \) the assignment function, \( s \) the information state, \( s' \) the salient state, and \( w \) the world of evaluation. We can follow Lewis (1980) in assuming that assertions would simply be a set of worlds, letting the context close off the values of \( s \) and \( s' \). Here is then the proposed meaning for the relevant two doxastics in Slovenian:

(19) **Semantics of doxastic attitudes:**

a. \([\text{misliti}]^{\lambda (s, s', w) \in B_x^w \rightarrow \lambda p \lambda x \forall w' \in B_x^w \{ p((B_x^w, B_s^w, w')) = 1 \}]\)

b. \([\text{dopuščati}]^{\lambda (s, s', w) \in B_x^w \rightarrow \lambda p \lambda x \exists w' \in B_x^w \{ p((B_x^w, s', w')) = 1 \}]\)

Following Hintikka (1962), the verbs in (19) are analysed as quantifiers over the set of worlds compatible with what the attitude holder \( x \) believes in \( w \). Following Yalcin (2007), they both shift the information state parameter to the doxastic set. The difference lies in what is made salient: *misliti* is neutral in prominence in that it makes the whole doxastic state salient, while *dopuščati* brings to attention the witness(es) to its existential statement. This distinction is passed on to the embedded proposition (evaluated with respect to an updated point of evaluation). Notice then that the two verbs are duals for any expression that is not sensitive to the second coordinate (the salience parameter).

(20) **Semantics of epistemic modals:**

a. \([\text{morat}i]^{\lambda (s, s', w) \in B_x^w \rightarrow \lambda R \forall v \in s(R(v) \subseteq s) \& \exists v \in s' \{ R(v) \lambda p \forall w' \in R(w) \{ p((s, s', w')) = 1 \}]\)

b. \([\text{možno}i]^{\lambda (s, s', w) \in B_x^w \rightarrow \lambda R \forall v \in s(R(v) \subseteq s) \& \exists v \in s' \{ R(v) \lambda p \exists w' \in R(w) \{ p((s, s', w')) = 1 \}]\)

The entries in (20) follow the standard truth-conditional content of modals as quantifiers over a contextually-determined set of accessible worlds (Kratzer, 1977, 1981). The modal combines with a free (modal base) variable of type \( ss \) (von Fintel, 1994), which maps the world of

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14 Assertion: \( \lambda w, [\_ \in B_x^w \wedge s \in s'] \) where \( s \) is the contextually determined \( s \), etc.

15 Thanks to Irene Heim (p.c.) for suggesting a simplification. The earlier semantics had the following formulation (which is slightly weaker when we take into account the constraint on epistemic modals, but, as far as I can see, the choice between the two proposals is not important for these two verbs):

\([\text{misliti}]^{\lambda (s, s', w) \in B_x^w \rightarrow \lambda p \lambda x \forall W \subseteq B_s^w \exists w' \in W \{ p((B_x^w, B_s^w, w')) = 1 \}]\)

\([\text{dopuščati}]^{\lambda (s, s', w) \in B_x^w \rightarrow \lambda p \lambda x \exists W \subseteq B_s^w \forall w' \in W \{ p((B_x^w, W, w')) = 1 \}]\)
evaluation to a set of accessible worlds (sloppily switching between function-talk and set-talk). There are two presuppositions on the modal base function in (20a) and (20b), which I discuss in turn.

Following Mandelkern (2017, forthcoming), who builds on Yalcin (2007), epistemic modals carry a definedness condition called **Locality** (the first presupposition). Under belief, this constraint requires the modal base function to map belief worlds onto subsets of the doxastic state. Locality, which constrains admissible modal bases, ensures that epistemic modals only access the information that is locally provided to them. A way to intuitively understand Mandelkern’s and Yalcin’s idea is to think of our beliefs as pieces of evidence that we use to navigate the world. Epistemic modals under belief predicates are constrained by this kind of evidence.

There is a second constraint on epistemic modals, called **Totality**. Under belief, it requires that the modal base function finds at least one world (v) in the salient part of the belief state (s') and map the modal base from that world (R(v)) to a superset of the belief state (s). (Together with Locality, this means that the modal base from that world and the doxastic state coincide.) Intuitively, epistemic modals under belief predicates are not allowed to be completely constrained by the outside evidence – there are salient worlds from which only belief-evidence is accessed.

I will write the two presuppositions as intermediately accommodated into the restrictor of the attitude predicate (as is customary, restrictors will be assumed to be non-empty). This is only crucial when Totality ends up containing a variable bound by the attitude (as with dopuščati), in which case we need intermediate accommodation to derive the right truth-

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16 This constraint is weaker than Yalcin’s but can account for Yalcin’s data (and more).
17 Like Yalcin (2007) and Mandelkern (2017, forthcoming), I would need to assume a covert shifting operator for the cases in which the embedded epistemic modal is not sensitive to the attitude holder’s evidence.
18 Informally, this is the main piece that enables us to construct an intermediate system, see footnote 12.
19 Thanks to Daniel Rothschild (p.c.) for suggesting I adopt this weaker version of Totality.
conditions. The formulas are, however, more transparent if Locality and Totality are kept together (it is easier to see how the negation is “pushed in”, for example).

Consider first examples like (1b), repeated in (21). Here is how we derive the truth-conditions using the entries in (19a) and (20a).

(21) Mislim, da mora biti sončno.

I think that must be sunny

‘I think it must be sunny.’

Let us now turn to (2b) and show that we get something equivalent.

(22) #Dopuščam, da mora biti sončno.

I allow that must be sunny

‘I allow for the possibility that it must be sunny.’

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20 Locating them in the antecedent of the conditional statement is logically equivalent to writing them as conditions on the set.

21 On this semantics must is strong: Bp and B p are equivalent.
In Figure 2, Locality is as before whereas Totality contains the variable \( w' \), bound by the existential quantification contributed by \textit{dopuščati}. This means that it is the world that \textit{dopuščati} talks about ('there is a world in the belief state such that...') that is such that the modal base maps it onto the doxastic state. Since the truth-conditions require it to be sunny in every world from that modal base \( (g(i)(w')) \), it again follows that the attitude holder believes that it is sunny. It is then easy to see how the two entail each other.\footnote{Similarly for the truth-conditions of (10), which mirror Figure 2:}

\[(2b)|^{i,s',w} = 1 \text{ iff } \exists w' \in B^w_{loc} \left[ \forall v \in B^w_{loc} [g(i)(v) \subseteq B^w_{loc}] \land \exists v \in B^w_{loc} [B^w_{loc} \subseteq g(i)(v)] \land \forall w'' \in g(i)(w') [\text{it's sunny at } w''] \right] \]

\[(2b)|^{i,s',w} = 1 \text{ iff } \exists w' \in B^w_{loc} \left[ \forall v \in B^w_{loc} [g(i)(v) \subseteq B^w_{loc}] \land \exists v \in B^w_{loc} [B^w_{loc} \subseteq g(i)(v)] \land \forall w'' \in g(i)(w') [\text{it's sunny at } w''] \right] \]

\[\text{Figure 2: Truth-conditions of (22)/(2b)}\]

Let us now turn to embedded existential modals, as in (1a) and (2a), repeated in (23).

\[(23)\]

a. Mislim, da utegne biti sončno.
   I think that might be sunny.
   \('I think it might be sunny.'\)

b. Dopuščam, da utegne biti sončno.
   I allow that might be sunny.
   \('I allow for the possibility that it might be sunny.'\)

\[[(1a)]^{i,s',w} = 1 \text{ iff } \forall w' \in B^w_{loc} \left[ \forall v \in B^w_{loc} [g(i)(v) \subseteq B^w_{loc}] \land \exists v \in B^w_{loc} [B^w_{loc} \subseteq g(i)(v)] \land \exists w'' \in g(i)(w') [p(w'') = 1] \right] \]

\[[(2a)]^{i,s',w} = 1 \text{ iff } \exists w' \in B^w_{loc} \left[ \forall v \in B^w_{loc} [g(i)(v) \subseteq B^w_{loc}] \land \exists v \in B^w_{loc} [B^w_{loc} \subseteq g(i)(v)] \land \exists w'' \in g(i)(w') [p(w'') = 1] \right] \]

\[\text{Figure 3: Truth-conditions of (1a) and (2a)}\]

To see why the two are not equivalent, consider the model in Figure 4. First, notice that Locality is satisfied (the modal base from either world

\[\text{\footnotemark[22]}\]
maps onto a subset of the doxastic state). Then, looking at Figure 3, notice that Totality is satisfied for (2a) and (2a) is true: there is a world (the one on the left), whose modal base is mapped onto the doxastic state (a non-proper superset) and there is a p-world in that modal base, namely itself. For (1a), Totality is also satisfied since this same world can be used to satisfy it. However, (1a) is false. It is not the case that for every world there is a p-world in its modal base, cf. the world on the right hand side (whose modal base contains only itself, which is not a p-world).

On the other hand, it is easy to see that the converse relationship holds, i.e. (1a) entails (2a). Thus, when dopuščati embeds an existential modal, this statement is strictly weaker than the one obtained with misliti.

In sum, I proposed a semantics on which embedded universal modals collapse the distinction in the attitude force, while embedded existential modals preserve it. I will not go through the proofs, but this property is maintained under negation, as illustrated schematically in Figure 6.23 Figure 5 provides the remaining truth-conditions.

\[
\begin{align*}
[-\Diamond \Box p]^{D,J,i,=1} & \text{ iff } \forall w' \in B_j[i] \left( \forall v \in B_j[i] \left[ g(i)(v) \subseteq B_j[i] \land g(i)(w') \subseteq B_j[i] \right] \rightarrow \exists w'' \in g(i)(w') \left[ p(w'') = 0 \right] \right) \\
[-\Box \Diamond p]^{D,J,i,=1} & \text{ iff } \exists w' \in B_j[i] \left( \forall v \in B_j[i] \left[ g(i)(v) \subseteq B_j[i] \land g(i)(w') \subseteq B_j[i] \right] \land \exists w'' \in g(i)(w') \left[ p(w'') = 0 \right] \right) \\
[-\Diamond \Box p]^{D,J,i,=1} & \text{ iff } \exists w' \in B_j[i] \left( \forall v \in B_j[i] \left[ g(i)(v) \subseteq B_j[i] \land g(i)(w') \subseteq B_j[i] \right] \land \exists w'' \in g(i)(w') \left[ p(w'') = 0 \right] \right) \\
[-\Box \Diamond p]^{D,J,i,=1} & \text{ iff } \forall w' \in B_j[i] \left( \forall v \in B_j[i] \left[ g(i)(v) \subseteq B_j[i] \land g(i)(w') \subseteq B_j[i] \right] \rightarrow \forall w'' \in g(i)(w') \left[ p(w'') = 0 \right] \right)
\end{align*}
\]

Figure 5: Truth-conditions for modals under negated attitudes
(D=dopuščati, B=misliti, J=attitude holder, i=modal base)

23 Figure 4 can be re-used (¬Dj\Diamond p is true while ¬Dj\Diamond p is false).
4 How to Derive Oddness?

As illustrated in Figure 6, the expressions with embedded strong modals are equivalent, regardless of what the embedding doxastic attitude is. In this section I want to give an idea as to why this leads to oddness (the details are left for future work).

Consider the very first case we started with: (1b) and (2b). I proposed a semantics for doxastic attitudes and epistemic modals that resulted in (1b) and (2b) being equivalent. How does it follow from this that (1b) is odd? Magri (2009, 2011) considers sentences like the following:

(24)  a. #Some Italians come from a warm country.
      b. #Some students got an A. (situation: the professor is known to assign the same grade to all of his students)

These sentences have scalar alternatives (e.g. *All students got an A*) that are semantically stronger but pragmatically equivalent (e.g. in a context where the students get the same grade, if some get an A, then they all do, and vice versa). Magri argues that the sentences in (24) are odd because they denote contextual contradictions, which arise due to an obligatory scalar implicature (due to a covert exhaustification operator). For
instance, the covert exhaustification operator creates an enriched meaning of (24b) that can be paraphrased as *Some but not all students got an A*, which is contextually contradictory.

To adopt Magri’s proposal then, we would need a way of making the exhaustification operator blind to the newly proposed semantic ingredients (Totality, in particular), just as it is blind to contextual information. I leave this for future work and instead conclude the paper with a very simplistic generalization over the relevant oddnesses.

Let’s suppose that (24a) is **deviant** in a context where Italians come from the same country because there is a Horn competitor *All Italians come from a warm country* (obtained by substituting *some* with *all*) such that (i) the two propositions are contextually equivalent and, importantly, (ii) (24a) is “structurally weaker” than its all-version. More precisely, there is a shared constituent *come from a warm country* that could be replaced by another structure, such as *smiled*, where *All Italians smiled* entails that *Some Italians smiled* and not vice versa. In other words, we have access to the make-up of (24a), which uses a simpler expression than needed (to convey the same information).

Such an explanation would work well for the basic case in (2b). We would say that (2b) is deviant in that context because there is a Horn competitor (1b) (*I think it must be sunny*), which is equivalent to it and, furthermore, (2b) is structurally weaker than (1b). To see this, consider replacing the embedded clause *it must be sunny* (a shared constituent) with a non-modal proposition, e.g. *it’s sunny*. Intuitively as well as theoretically (such simple propositions are not sensitive to prominence) *Dopuščam, da je sončno* (*I allow for the possibility that it’s sunny*) is entailed by but does not itself entail *Mislim, da je sončno* (*I think that it’s sunny*). Thus, (2b) uses a weaker expression than needed.

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24 To account for the two examples in (17), for example, it is reasonable to assume that the introspection principles (e.g. KD45), which give rise to the idea that speakers are authorities on their beliefs, are part of the common ground (i.e. part of how we assume speakers to reason about other speakers’ beliefs) and that doxastic attitudes instantiate these principles.

25 We would probably need a more fine-grained notion of what kind of structure is a good replacement test.
For the negated cases, this explanation works in a parallel way for (12b) but it cannot explain the acceptability of its close paraphrase in (13). For the latter, we would need to appeal to the somewhat mysterious notion of meta-linguistic negation.

The main issue with this simplistic view of oddness is (15b) because it requires this deviance principle to be able to apply locally (and percolate upward). The best we can do at this point is to stipulate that a constituent or sentence is deviant if it contains a deviant subconstituent.

5 Conclusion

Using evidence from the Slovenian existential belief verb dopuščati (‘to allow for the possibility that’), the paper argued for a new analysis of doxastic attitudes and epistemic modals. I proposed a semantics on which embedded universal modals collapse the distinction in the attitude force, while embedded existential modals preserve it (while maintaining duality between the two modals). I concluded with some thoughts on why this leads to oddness of classically weaker expressions, leaving the details of that for future work.

Sources

(4c) http://www.delo.si/svet/evropa/tusk-dopusca-moznost-da-breixta-ne-bo.html (last accessed December 2017)
fn. 6 https://med.over.net/forum5/viewtopic.php?t=10981767 (last accessed May 2018)

Magri (2009, 2011) achieves a similar effect by postulating that the exhaust operator is obligatory at every scope site.
References


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