Higher-Order Confidence with Epistemic Modals

Maša Močnik
Talk at Expressing Evidence

June 6, 2019
Konstanz

There has been much recent interest[1] in the contribution of embedded epistemic modals. Consider:

(1) You wake up and you're too lazy to open your eyes. But you can nevertheless tell that it's bright. What should you make of that? You might say to yourself:

a. ***Mislim**, da **utegne** biti sončno.* $B\Diamond p$
I.think that might be sunny
'I **think** it **might** be sunny.'

b. ***Mislim**, da **mora** biti sončno.* $B\Box p$
I.think that must be sunny
'I **think** it **must** be sunny.'

On the traditional picture, there is a double layer of modality ($w^* \rightarrow$ doxastic worlds $\rightarrow$ epistemic worlds). Stephenson (2007), Yalcin (2007), Hacquard (2006) explore the idea that the epistemic worlds are constrained[2] by the attitude worlds: they propose a collapse between the attitude worlds and the epistemic worlds ($w^* \rightarrow$ doxastic worlds $=$ epistemic worlds). That is, the two nested modalities **collapse** into a single one.

Some views (see also Mandelkern (2019b)):

1. the unconstrained view (von Fintel and Heim 2016)
$w^* \rightarrow$ doxastic worlds $\rightarrow$ epistemic worlds

2. the constrained+no-collapse view (Mandelkern 2019a)[3]
$w^* \rightarrow$ doxastic worlds $\circlearrowleft$ epistemic worlds $\mathcal{B} \supseteq MB$

3. the constrained+collapse view (Stephenson 2007, Yalcin 2007, Hacquard 2006)
$w^* \rightarrow$ doxastic worlds $=$ epistemic worlds $\mathcal{B} = MB$

[1] Yalcin (2007, 2012), Mandelkern (2019a), Ippolito (2017), Anand and Hacquard (2013), Willer (2013), a.o.
[2] This takes care of epistemic contradictions: **Suppose that it's raining and suppose that it might not be* (Yalcin 2007).
[3] See also Hacquard (2010) for a brief mention of the possibility of adding an ordering source to Hacquard (2006).

My data will require a view that is in between 2 and 3: a **constrained + partial-collapse** view. The data involves the Slovenian verb *dopuščati* ('let'; 'to allow for the possibility that'), which I analyse as an **existential doxastic attitude verb**.

(2) a. *Janez* ***dopušča****, da se je zmotil.*
John allows that REFL AUX erred
'John allows for the possibility that he made a mistake.'

b. *seveda* ***dopuščam*** *da obstajajo določene izjeme. ampak jih še nisem srečala*
of.course I.allow that exist certain exceptions but them yet not.AUX met
'of course I consider it possible that there are certain exceptions. but I haven't yet come across them' (web)

This verb helps us see a new property of embedded epistemic modals (discovered by Anand and Hacquard (2013) in the context of verbs that allow for analyses that we cannot use here). Namely, **it is odd** ('*') **to embed a universal epistemic** under a weak attitude verb like *dopuščati*.

(3) Situation as in (1).

a. ***Dopuščam****, da* ***utegne*** *biti sončno.* $D\Diamond p$
I.allow that might be sunny
'I allow for the possibility that it might be sunny.'

b. ****Dopuščam****, da* ***mora*** *biti sončno.* $*D\Box p$
I.allow that must be sunny
'I allow for the possibility that it must be sunny.'

b′. ****Dopuščam****, da* ***ne more*** *biti deževno.* $*D\neg\Diamond p$
I.allow that not can be rainy
'I allow for the possibility that it can't be rainy.'

(4) For contrast with (3b′).
Mislim*, da* ***ne more*** *biti deževno.* $B\neg\Diamond p$
I.think that not can be rainy
'I think that it can't be rainy.'

Two main questions:

1. Why is $D\Box p$ in (3b) odd? (Same for $D\neg\Diamond p$ in (3b′).)

2. Is $D\Diamond p$ in (3a) different from $B\Diamond p$ in (1a)? If yes, how?

The (constrained) **partial-collapse view** we need is one where we lose the distinction between $D\Box p$ and $B\Box p$ (=the two express the same thought) but not between $D\Diamond p$ and $B\Diamond p$ (=the two do not express the same thought). That is, **embedded boxes create a collapse, while embedded diamonds don't.** This is hard if we want to maintain duality.

A collapse between two forms gives us a chance to say why one of them is bad. No collapse between two forms opens up the question of how the two forms are different.

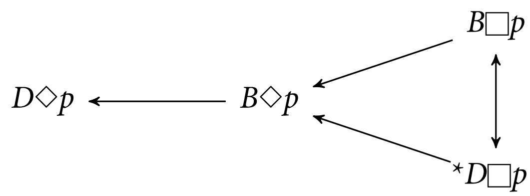


Figure 1: Partial-collapse view
($\Box$ collapses the distinction between D and B)

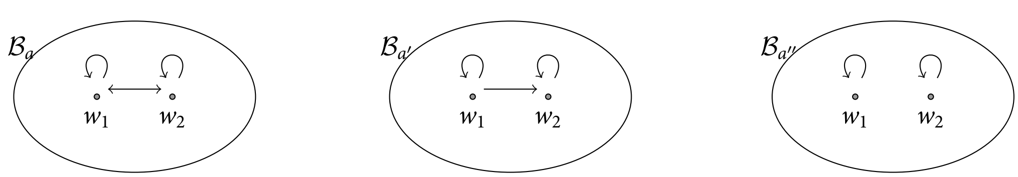
[There are two senses of the word "collapse": a collapse of nested modalities (loss of complexity) and a collapse in meaning between two formulas. I'm talking about the second one now.]

My goals:

- Create a partial-collapse view (= be between views 2 and 3): Build on Mandelkern (2019a) by adding a constraint on epistemic modal bases.
- Formulate an account where $D\Box p$ is odd because it uses a weak attitude to express a proposition ($B\Box p$) that could have been expressed with a stronger attitude. (Contextual equivalences can lead to oddness: Magri (2009, 2011).)
- When no collapse ($D\Diamond p$ vs $B\Diamond p$), explain the difference. I argue that it reflects more than just a difference in strength.

(Work in progress) Give a possible-worlds interpretation of confidence. Namely, that confidence in one's assessment of evidence can be "read-off" from the shape of the epistemic accessibility relation. Because of how attitudes and modals interact, *dopuščati* signals that the agent is not confident in their assessment of the evidence.

In modal logic terms, I propose that confidence is a property of frames. Let $\mathcal{F} = \langle \mathcal{B}_a, R_f \rangle$, where $\mathcal{B}$ is the non-empty set of a's belief worlds that constrains an embedded epistemic modal and R_f is the epistemic accessibility relation. An agent is confident in their assessment of evidence iff the frame is a universal frame ($\forall w, w' \in \mathcal{B}_s : wR_f w'$). Otherwise the agent is non-confident.



(a) Confident agent-frame

(b) Non-confident agent-frame (w_2 only sees itself)

(c) Non-confident agent-frame (two equivalence classes)

1 Some empirical points

1.1 Existential belief

Dopuščati is cross-linguistically uncommon.[4] Intuitively, it expresses something very weak, namely that the attitude holder is leaving something open.

(5) ***Dopuščam***, *da dežuje, in* ***dopuščam***, *da ne dežuje.* $D_x p \wedge D_x \neg p$
I.allow that rains and I.allow that not rains
'I allow that it's raining and I allow that it's not raining.'

The way in which *dopuščati* differs from Anand and Hacquard's fear, hope, and doubt in Romance is that it can be strengthened into a belief claim (cf. *some* to *most/all*). This significantly reduces the space of possible analyses for embedded epistemics; we also cannot use Anand and Hacquard (2013)'s diversity presupposition to account for the behaviour of *dopuščati* with epistemics.

(6) In a debate with Flat-Earthers, a scientist is asked:

Ali dopuščate, da je Zemlja okrogla?
Q you.allow that is Earth round
'Do you allow for the possibility that the Earth is round?'

The scientist replies:

Seveda ***dopuščam***, *da je – trdno verjamem, da je!* $D_x p \wedge B_x p$
of.course I.allow that is firmly I.believe that is
'Of course I allow that it is – I firmly believe that it is!'

Why belief and not knowledge? Knowledge cannot be false.

(7) *Dežuje, ampak Janez ne* ***dopušča***, *da dežuje.* $p \wedge \neg D_j p$
rains but John not allows that rains
'It's raining but John doesn't allow for the possibility that it's raining.'

1.2 Embedded epistemic modals

Anand and Hacquard (2013) observe for Romance that universal epistemic modals do not embed under doxastic verbs like *fear*, *hope*, or *doubt* (analysed as existential), while any force is good under *believe* and *think*. Slovenian provides an additional data point to this landscape with the existential verb *dopuščati* ('to allow for the possibility').

In (1) and (3) the attitude holder considers his own available evidence.[5] In such anchored cases, the combination of a universal epistemic ($\Box$) under an existential doxastic (D) yields oddness (*).

[4] Some (but not all) Slavic cognates behaves similarly. The only other unrelated language that I know of who has this is Koryak *ivək* (which is variable force, however). Some English speakers can use *allow* with a finite clause, as in *I'll allow that I'm wrong* or *Othello allows that Desdemona might love Cassio*. However, *allow* seems to have a discursive flavour (I'll allow x for the sake of the conversation), rather than being able to function as a mere mental state description.

[5] $D\Box p$ improves when $\Box$ echoes someone's words or is shifted with *according to*. I leave these shifts aside. Note also that other embedded flavours, e.g. deontic, are fine.

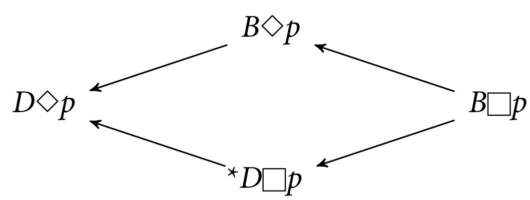


Figure 3: Semantic entailments for (1) and (3)
(non-empty restrictors assumed)

The oddness is about the **wide-scope universal meaning** ($\Box p$ as well as $\neg\Diamond p$) in the complement of *dopuščati*, rather than the force of the modal item per se, cf. (3b′). It is more difficult to find examples of $D\neg\Box$ with an epistemic interpretation since *morati* ('must') is a PPI. Here is an example with 'be necessary':

(8) Context: discussion on a forum about the borderline personality disorder (BPD). The speaker has just stated that certain feelings (e.g. isolation) were in large part responsible for him developing BPD.
Dopuščam možnost, da ni nujno, da sem prinesel "tako" hude poškodbe in bil zato lahek
I.allow possibility that is.not necessary that AUX brought such bad injuries and been therefore easy
plen MOMa, a vendar nameravam še posvetiti kopanju v to smer.
prey BPD but nevertheless I.intend still dedicate digging in this direction
'I allow for the possibility that such bad wounds did not necessarily cause me to be an easy prey to BPD, but I intend to investigate this further.' (web)

If this example does not convince you, you can take (3b) and the assumption of duality ($\Diamond p = \neg\Box\neg p$) between existential and universal epistemics, which seems an independently-desirable property to maintain.[6]

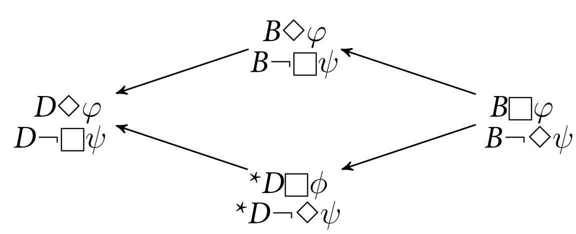


Figure 4: Embedded anchored epistemics under doxastics
($\psi = \neg\varphi$)

Intuition. An embedded universal epistemic has a contribution that makes the choice of a weak attitude inappropriate.

1.3 Negated doxastic attitudes

Anand and Hacquard (2013, fn. 27) find that main clause negation makes the embedding of a

[6] Ippolito (2017) provides an account where duality between epistemic modals is not maintained, which is problematic in light of (3b′). Her account, however, comes closest in the literature to the partial view we need for this data.

necessity modal under a doxastic attitude degraded. See also Crnič (2014) and Ippolito (2017, p. 14, fn. 9) for related comments. My example:

(9) You, me, and John see Bob go home from work early. We sit down on some couches in front of Bob's office. John has his back turned to Bob's door. He puts on some headphones and starts cheating on the latest homework. After a while, Bob, who has a secret entry to his office, which he used to come back, creeps out of his office and comes up behind John's back. John, still immersed in cheating, does not notice this. I nudge you and whisper, with both of us staring at Bob:

a. *John does not think that Bob might be behind his back.* $\neg B\Diamond p$

b.??*John does not think that Bob must be behind his back.* $??\neg B\Box p$

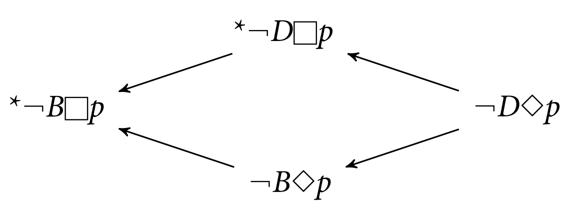
(10) a. *Janez ne misli, da je Bob mogoče za njegovim hrbtom.* $\neg B\Diamond p$
John not thinks that is Bob maybe behind his back
'John does not think that Bob might be behind his back.'

b.??*Janez ne misli, da mora biti Bob za njegovim hrbtom.* $??\neg B\Box p$
John not thinks that must be Bob behind his back
'John does not think that Bob must be behind his back.'

c. *Janez ne dopušča, da je Bob mogoče za njegovim hrbtom.* $\neg D\Diamond p$
John not allows that is Bob maybe behind his back
'John does not allow that Bob might be behind his back.'

d.??*Janez ne dopušča, da mora biti Bob za njegovim hrbtom.* $??\neg D\Box p$
John not allows that must be Bob behind his back
'John does not allow that Bob must be behind his back.'

What is relevant to notice is that epistemic modals behave differently under negated attitudes. There are now two odd combinations and if we "pushed in" the negation, we would not get the oddness in the same place as before:



Finally, notice also the contrast between (9b)/(10b) and (11).

(11) Situation as before.
It's not the case that John thinks that Bob must be behind his back.

In sum. The data is somewhat messy, as observed already by Ippolito (2017). I will take it that embedding an anchored universal epistemic under a negated attitude verb, (9b)/(10b) and (10d), leads to oddness. However, we need a theory of oddness that is somewhat flexible, given (11).

2 Higher-order confidence

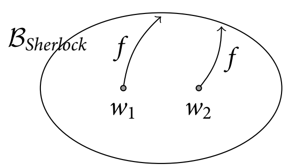
(12) **Case study 1: Sherlock and Watson**
A murder investigation in which Sherlock and Watson are given access to identical evidence. They are asked to state their personal opinion on the murder.

a. *Mislim, da utegne biti Janez morilec.* ('I think that John might be the murderer')

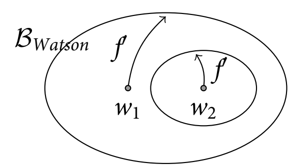
b. *Dopuščam, da utegne biti Janez morilec.* ('I allow that John might be the murderer')

The intuition is that Sherlock, who has a quick mind, can make a quick and accurate assessment of the evidence. Therefore, he can more appropriately respond with (12a). Watson, on the other hand, might be more cautious and say (12b).

Sherlock and Watson both make the claim that is consistent with their evidence that John is the murderer ($\Diamond p$). The difference is in how confident they are in their assessment of the evidence.



(a) Confident agent (to be shown: $D\Diamond p \Leftrightarrow B\Diamond p$)



(b) Non-confident agent (to be shown: $D\Diamond p \not\Leftrightarrow B\Diamond p$)

Figure 5: Confidence in one's capacity
to conclude epistemic statements ($\Diamond p$, $\Box p$, etc.)

Confidence is reflected in the shape of the epistemic modal base function. When an agent, such as Sherlock, masters the evidence, his evidence doesn't change in his belief worlds – the worlds compatible with the evidence are always just his beliefs worlds (Figure 5a). By contrast, when the agent has a lesser grasp of the evidence at hand, the worlds compatible with his evidence can differ per doxastic world. In particular, there are doxastic worlds (w_2 in Figure 5b maps to a **proper subset**) at which the agent learns more from his evidence than at others.[7]

(13) **Case study 2: The idiot**
Suppose that John is president. John has many false beliefs but he is very confident.

a. *Mislim, da utegne biti Janez morilec.* ('I think that John might be the murderer')

b. **Dopuščam, da utegne biti Janez morilec.* ('I allow that John might be the murderer')

The idiot *takes himself to* fully master the evidence. His epistemic modal base is like Sherlock's in Figure 5a.

[7]While the traditional (Hintikka-Kratzer) analysis permits configurations like Figure 5b, Yalcin's (2007) semantics is essentially as what we see in Figure 5a.

Notice that a way to *become* confident is to learn more propositions, in this case to form more beliefs. Intuitively, if the idiot gains more beliefs, even if they are all false, he is more likely to be confident. There is perhaps a common knowledge understanding that there is a correlation between how much you believe and how confident you are.

I'm taking evidence to be propositional. In particular, the propositions (p, q,…) we believe are also pieces of evidence. So confidence correlates with increasing your evidence.

(14) **Case study 3: Othello**

Context: Othello is asked whether he thinks that Desdemona is cheating on him. He replies:

a. *Dopuščam, da me (mogoče) vara.* $D\Diamond p$
   I.allow that me (maybe) cheats.on
   'I allow for the possibility that she is (perhaps) cheating on me.'

b. *Mislim, da me mogoče vara.* $B\Diamond p$
   I.think that me maybe cheats.on
   'I think she might be cheating on me.'

Speakers report Othello to have perhaps some reason for suspecting Desdemona of cheating in (14b), whereas in (14a) Othello's thought is that in principle she might be.

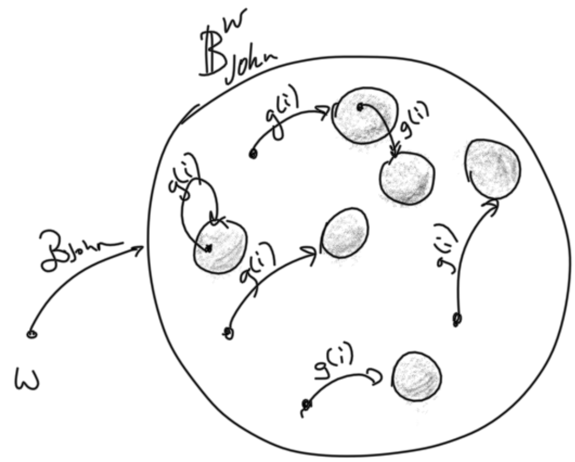
How do we explain the native speaker observation? We will set up a semantics where (14a)–(14b) differ in their truth-conditions when the agent is non-confident, as in Figure 5b (but they collapse on a confident frame). In order for (14a) to be felicitous, Othello needs to be non-confident. On the other hand, if the speaker uses (14b), Othello can in principle be either (because the formula can be true in either). If (14a)–(14b) are, however, contrasted and (14a) is felicitous only on a non-confident frame, then it's reasonable to infer that Othello is confident in (14b).

So, in (14b) Othello is understood to be more confident in his assessment (that the evidence is consistent with Desdemona being unfaithful). How could he have become confident? Since learning more evidence is a passage to confidence, one thing we can infer from (14b) is that he simply gathered more evidence, so he must have "some reason for suspecting Desdemona of cheating".

[Important: I do not claim that attitude verbs literally encode confidence. I claim that the choice of verb helps bring out a confidence distinction (the latter being something that is embodied in the shape of the epistemic accessibility relation over a set of worlds).]

3 Two constraints on epistemic modals

1. **Locality** (Mandelkern 2019a). By default (i.e. unless shifted) epistemic modals are sensitive to the information that is locally provided to them. When embedded under belief, epistemic modals quantify over the attitude holder's belief worlds.



Locality under belief: $\forall v \in \mathcal{B}_J^w$: $MB(v) \subseteq \mathcal{B}_J^w$

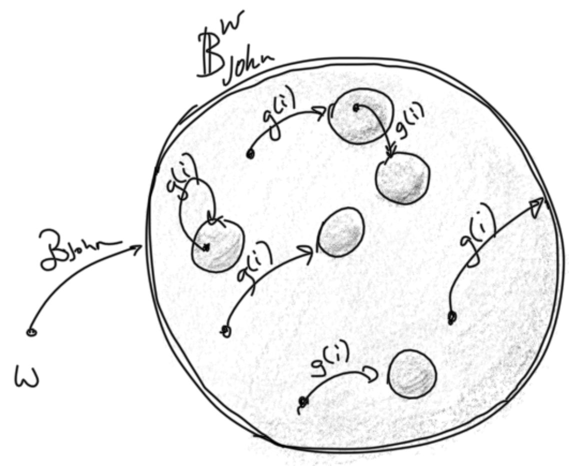
For any belief world v: the modal base from v forms a subset of the doxastic set.

($g(i)$ in the picture is MB, i.e. the modal base function)

• I propose to interpret this as follows: our individual beliefs (p, q, ...) **serve as pieces of evidence** (e.g. if I believe it's raining, I can use this as evidence for John being in the house) and an epistemic modal under a belief predicate is restricted by **this body of** (established) **evidence** (=the attitude holder's beliefs).

• How to interpret when the modal base forms a proper subset? Non-confident agents make hypothetical updates: they check whether the prejacent would hold **if they learned a piece of evidence** that is at the moment merely consistent with their beliefs (e.g. 'the weather report is accurate').

2. **Totality**. When evaluating an epistemic modal, we also consider the locally established evidence in its totality. The epistemic modal base function doesn't perform only hypothetical updates but also looks at the local set as a whole.



$\boxed{\text{Totality under belief: } \exists v \in \mathcal{B}^w_J : \mathcal{B}^w_J = MB(v)}$
(to be revised)

There is a world v: the modal base from v is the doxastic set.

If Locality is assumed, Totality can be weakened: $\exists v \in \mathcal{B}^w_J : \mathcal{B}^w_J \subseteq MB(v)$. The weaker version can be understood as making reference to some salient piece(s) of established evidence (rather than the worlds where all of the established evidence holds).

Totality as stated above runs into the **Binding Problem** (see Potts (2007) for overview):

(15) *Somebody managed to succeed George V on the throne of England.* (Karttunen and Peters 1979)[8]
   a. presupposes that someone had a hard time trying to succeed George V
   b. asserts that someone eventually succeeded George V
   intuition: the presupposition and the assertion relate to one and the same individual

I will introduce a salience parameter to deal with this (and will locally accommodate Totality into the restrictor of the attitude verb for the attitude's existential to bind into it).

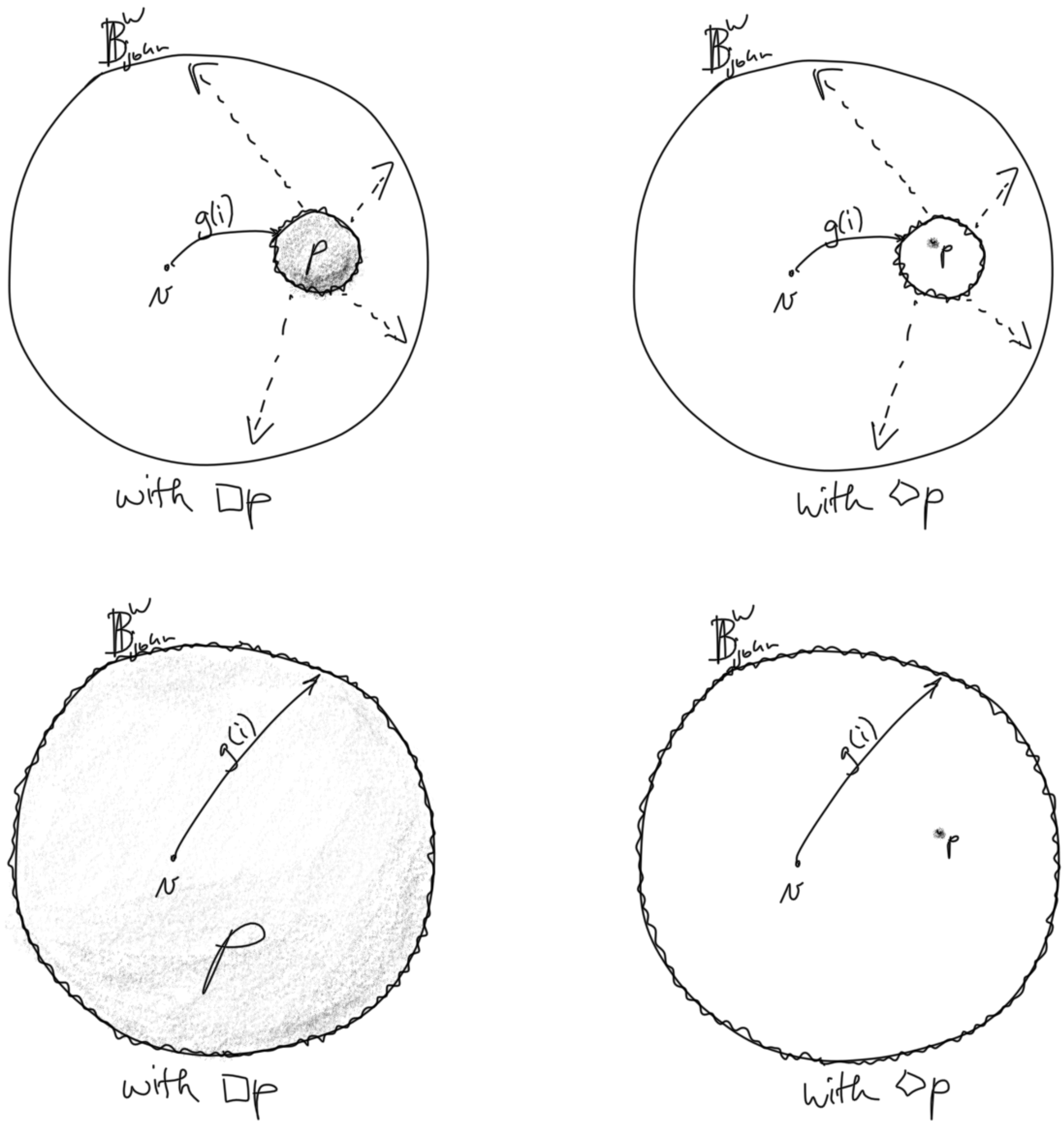
$\boxed{\text{Totality under belief: } \exists v \in c(\mathcal{B}^w_J) : \mathcal{B}^w_J = MB(v)}$

There is a world v in the chosen part of the doxastic state such that the modal base from v is the doxastic set.

(more details in the lexical entries to follow)

[8]Example summary from Dekker (2002).

What do we gain by adding Totality on top of Locality?



A universal statement gets strengthened, while an existential one does not (in fact, it becomes weaker). This is the source of **collapse with embedded boxes** and the lack of it with embedded diamonds: □ makes $D\Box p$ so strong that it becomes equivalent to $B\Box p$.

Yalcin (2007) interprets sentences with respect to a point of evaluation containing a world and an **information state** s (a set of worlds). Attitude verbs shift the information state to the attitudinal state. I extend this by adding another parameter s' (also a set of worlds) that specifies which worlds in the information state are salient. The motivation for this is for now technical (Binding Problem). The intuitive idea, however, is that *dopuščati* (D) makes salient the witnesses to its existential statement, while *misliti* (B) does not make salient anything in particular.

(16) a. $[\![B_{John}\ \varphi]\!]^{g,s,s',w} = 1$ iff $\forall w' \in \mathcal{B}_J^w : [\![\varphi]\!]^{g,\mathcal{B}_J^w,\mathcal{B}_J^w,w'} = 1$ *misliti* ('think')

b. $[\![D_{John}\ \varphi]\!]^{g,s,s',w} = 1$ iff $\exists w' \in \mathcal{B}_J^w : [\![\varphi]\!]^{g,\mathcal{B}_J^w,\{w'\},w'} = 1$ *dopuščati* ('allow for the p.')

(17) a. $[\![\Box_i\ \varphi]\!]^{g,s,s',w'}$ is defined when
$\forall v \in s[g(i)(v) \subseteq s]$ (Locality from Mandelkern) and
$\exists v \in s'[s = g(i)(v)]$ (Totality), and,
when defined, is true iff $\forall w'' \in g(i)(w') : [\![\varphi]\!]^{g,s,s',w''}{=}1$

b. $[\![\Diamond_i\ \varphi]\!]^{g,s,s',w'}$ is defined when
$\forall v \in s[g(i)(v) \subseteq s]$ (Locality from Mandelkern) and
$\exists v \in s'[s = g(i)(v)]$ (Totality), and,
when defined, is true iff $\exists w'' \in g(i)(w') : [\![\varphi]\!]^{g,s,s',w''}{=}1$

Some truth-conditions (see Appendix for more). The boxes represents what was replaced as s' (the choice of $c(s)$ on p. 10). Totality is defined with respect to this parameter s', which gets bound by the outermost quantifier under *dopuščati*, due to the shift in (16b). While Locality could be projected out, I will keep them together, accommodated into the restrictor of the attitude predicate (underlined below). Restrictors are assumed to be non-empty.

$$[\![B_x\Box_i p]\!]^{w,s,g} = \forall w' \in \mathcal{B}_x^w[(\underline{\forall v \in \mathcal{B}_x^w[g(i)(v) \subseteq \mathcal{B}_x^w]\ \&\ \exists v \in \boxed{\mathcal{B}_x^w}[\mathcal{B}_x^w = g(i)(v)])} \to \forall w'' \in g(i)(w')[p(w'')]]$$

$$\begin{aligned}[\![D_x\Box_i p]\!]^{w,s,g} &= \exists w' \in \mathcal{B}_x^w[\underline{\forall v \in \mathcal{B}_x^w[g(i)(v) \subseteq \mathcal{B}_x^w]\ \&\ \exists v \in \boxed{\{w'\}}[\mathcal{B}_x^w = g(i)(v)]}\&\ \forall w'' \in g(i)(w')[p(w'')]]\\ &= \exists w' \in \mathcal{B}_x^w[\underline{\forall v \in \mathcal{B}_x^w[g(i)(v) \subseteq \mathcal{B}_x^w]\ \&\ \mathcal{B}_x^w = g(i)(w')]}\&\ \forall w'' \in g(i)(w')[p(w'')]]\end{aligned}$$

$$[\![B_x\Diamond_i p]\!]^{g,s,w} = \forall w' \in \mathcal{B}_x^w[(\underline{\forall v \in \mathcal{B}_x^w[g(i)(v) \subseteq \mathcal{B}_x^w]\&\exists v \in \boxed{\mathcal{B}_x^w}[\mathcal{B}_x^w = g(i)(v)])} \to \exists w'' \in g(i)(w')[p(w'')]]$$

$$\begin{aligned}[\![D_x\Diamond_i p]\!]^{g,s,s',w} &= \exists w' \in \mathcal{B}_x^w[(\underline{\forall v \in \mathcal{B}_x^w[g(i)(v) \subseteq \mathcal{B}_x^w]\ \&\ \exists v \in \boxed{\{w'\}}[\mathcal{B}_x^w = g(i)(v)])}\ \&\ \exists w'' \in g(i)(w')[p(w'')]]\\ &= \exists w' \in \mathcal{B}_x^w[(\underline{\forall v \in \mathcal{B}_x^w[g(i)(v) \subseteq \mathcal{B}_x^w]\ \&\ \mathcal{B}_x^w = g(i)(w')])}\ \&\ \exists w'' \in g(i)(w')[p(w'')]]\end{aligned}$$

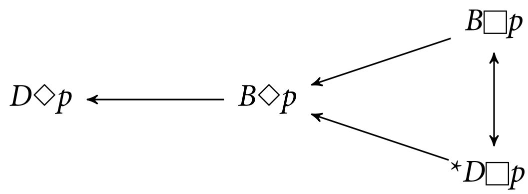
(a) $[\![D_x\Box_i p]\!]^{g,s,w} = 1$

(b) $[\![D_x\Diamond_i p]\!]^{g,s,w} = 1$
$[\![B_x\Diamond_i p]\!]^{g,s,w} = 0$

Figure 6: Why we get only a partial collapse

4 Wrapping up

- I argued for a **partial-collapse** view, in between Mandelkern (2019a) and Yalcin (2007)



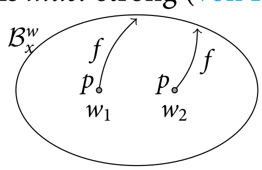
- I showed how this can be achieved by having epistemic modal bases obey two constraints: **Locality** (Mandelkern 2019a) (epistemic accessibility function maps to subsets of the local information state, e.g. belief state) and **Totality** (epistemic accessibility function doesn't map only to proper subsets of the local context)

- I proposed a possible-worlds interpretation of confidence: confidence is a property of frames (information state with an epistemic accessibility relation),
embedded epistemic modals interact with this property and thus enable us to see it

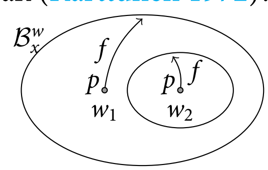
I didn't say much about how to derive oddness ($^*D\Box p$), see Močnik (2019b) for the exhaustification approach of Magri (2009, 2011). Future work: resolve the binding problem without accommodating into the main content, so that it can interface well with Magri (2009, 2011). Maybe the two-dimensional approach of Sudo (2014)?

Future work on confidence:

(18) Is *must* strong (von Fintel and Gillies 2010) or weak (Karttunen 1972)?



must is strong and the agent is confident



must is strong but the agent is non-confident
=? illusion of weakness?

(19) *definitely* imposing confidence (requiring the frame to universal)?

a. *John might definitely be at home.*

b. *John must definitely be at home.*

(20) What about when there is no (at least overt) epistemic modal?

a. *John is definitely at home.*

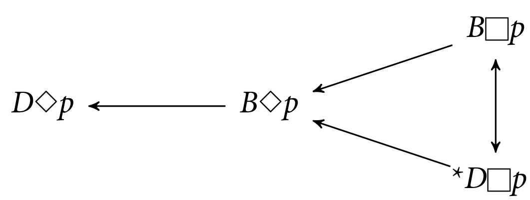
b. *I'm confident that John is at home.*

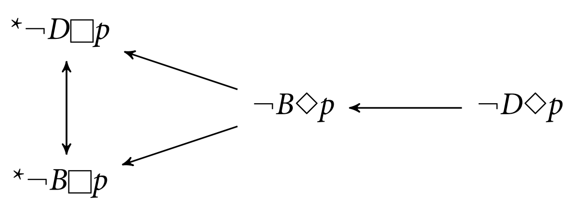
Thanks

This has been a slow-cooking project. Special thanks to Kai von Fintel, Danny Fox, Irene Heim, and Roger Schwarzschild. Thanks also to the following people, who have seen this material at some stage or other: Rafael Abramovitz, Moshe Bar-Lev, Christopher Baron, Rajesh Bhatt, David Beaver, David Boylan, Nate Charlow, Simon Charlow, Gennaro Chierchia, Cleo Condoravdi, Luka Crnič, Milica Denić, Jon Gajewski, Valentine Hacquard, Martin Hackl, Sabine Iatridou, Roni Katzir, Justin Khoo, Daniel Lassiter, Giorgio Magri, Matt Mandelkern, Lisa Matthewson, Mitya Privoznov, Jessica Rett, Floris Roelofsen, Daniel Rothschild, Viola Schmitt, Benjamin Spector, Frank Staniszewski, anonymous reviewers, and participants in 24.991, ESSLLI29, FASL27, Vienna, Crete, SuB23, and NELS49.

Appendix

Derivations are from Močnik (2019a). For oddness, see Močnik (2019b).





$$[\![\textit{John thinks it must}_i \textit{ be raining}]\!]^{g,\langle s,s',w\rangle} = 1 \text{ iff} \qquad (B\Box p)$$
$$[\![\textit{thinks}]\!]^{g,\langle s,s',w\rangle}([\![\textit{it must}_i \textit{ be raining}]\!]^g_{¢})(\text{John}) = 1 \text{ iff}$$
$$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![\textit{it must}_i \textit{ be raining}]\!]^g_{¢}(\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle) = 1] \text{ iff}$$
$$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![\textit{it must}_i \textit{ be raining}]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle} = 1] \text{ iff}$$
$$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![\textit{must}]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle}([\![i]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle})([\![\textit{it is raining}]\!]^g_{¢}) = 1] \text{ iff}$$
$$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![\textit{must}]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle}(g(i))([\![\textit{it is raining}]\!]^g_{¢}) = 1] \text{ iff}$$
$$\forall w' \in \mathcal{B}^w_{\text{John}}[[\lambda R\text{: } \forall v \in s[R(v) \subseteq s] \& \exists v \in s'[s = R(v)].\lambda p.\forall w'' \in R(w')[p(\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w''\rangle)=1]](g(i))([\![\textit{it's r.}]\!]^g_{¢})=1] \text{ iff}$$
$$\forall w' \in \mathcal{B}^w_{\text{John}}[(\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}] \ \& \ \exists v \in \mathcal{B}^w_{\text{John}}[\mathcal{B}^w_{\text{John}} = g(i)(v)]) \rightarrow \forall w'' \in g(i)(w')[\text{it is raining at } w'']]$$

$$[\![\textit{Janez dopušča, da mora}_i \textit{ deževati}]\!]^{g,\langle s,s',w\rangle} = 1 \text{ iff} \qquad (D\Box p)$$
$$\exists w' \in \mathcal{B}^w_{\text{John}}[[\![\textit{mora}]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \{w'\}, w'\rangle}(g(i))([\![\textit{dežuje}]\!]^g_{¢}) = 1] \text{ iff}$$
$$\exists w' \in \mathcal{B}^w_{\text{John}}[\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}] \ \& \ \exists v \in \{w'\}[\mathcal{B}^w_{\text{John}} = g(i)(v)] \ \& \ \forall w'' \in g(i)(w')[\text{ it's r. at } w'']] \text{ iff}$$
$$\exists w' \in \mathcal{B}^w_{\text{John}}[\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}] \ \& \ [\mathcal{B}^w_{\text{John}} = g(i)(w')] \& \ \forall w'' \in g(i)(w') : \text{it is raining at } w'']$$

$[\![D_J \neg \Diamond_i \neg p]\!]^{g,s,s',w} = 1$ iff
$\exists w' \in \mathcal{B}^w_{\text{John}}[[\![\Diamond]\!]^{g,\langle \mathcal{B}^w_{\text{John}},\{w'\},w'\rangle}(g(i))([\![\neg p]\!]^g_{\text{¢}}) = 0]$ iff
$\exists w' \in \mathcal{B}^w_{\text{John}}[\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}]\ \&\ [\mathcal{B}^w_{\text{John}} = g(i)(w')]\ \&\ \neg\exists w'' \in g(i)(w')[p(w'') = 0]]$

$[\![B_J \Diamond_i p]\!]^{g,s,s',w} = 1$ iff
$\forall w' \in \mathcal{B}^w_J[(\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \& \exists v \in \mathcal{B}^w_J[\mathcal{B}^w_J = g(i)(v)]) \rightarrow \exists w'' \in g(i)(w')[p(w'') = 1]]$

$[\![D_J \Diamond_i p]\!]^{g,s,s',w} = 1$ iff
$\exists w' \in \mathcal{B}^w_J[(\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J]\ \&\ \mathcal{B}^w_J = g(i)(w')])\ \&\ \exists w'' \in g(i)(w')[p(w'') = 1]]$

$[\![\neg D_J \Box_i p]\!]^{g,s,s',w} = 1$ iff
$[\![D_J \Box_i p]\!]^{g,s,s',w} = 0$ iff
$[\![D]\!]^{g,\langle s,s',w\rangle}([\![\Box_i p]\!]^g_{\text{¢}})(J) = 0$ iff
$\neg(\exists w' \in \mathcal{B}^w_J[[\![\Box_i p]\!]^g_{\text{¢}}(\langle \mathcal{B}^w_J, \{w'\}, w'\rangle) = 1]$
$\neg(\exists w' \in \mathcal{B}^w_J[[\![\Box_i p]\!]^{g,\langle \mathcal{B}^w_J,\{w'\},w'\rangle} = 1])$ iff (shortcutted from "$D\Box p$" above)
$\neg(\exists w' \in \mathcal{B}^w_J[\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \& [\mathcal{B}^w_J = g(i)(w')] \& \forall w'' \in g(i)(w')[p(w'') = 1]])$ iff
$\forall w' \in \mathcal{B}^w_J[\neg\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \vee \neg[\mathcal{B}^w_J = g(i)(w')] \vee \neg\forall w'' \in g(i)(w')[p(w'') = 1]]$ iff
$\forall w' \in \mathcal{B}^w_J[(\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \& [\mathcal{B}^w_J = g(i)(w')]) \rightarrow \neg\forall w'' \in g(i)(w')[p(w'') = 1]])$ iff
$\forall w' \in \mathcal{B}^w_J[(\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \& [\mathcal{B}^w_J = g(i)(w')]) \rightarrow \exists w'' \in g(i)(w')[p(w'') = 0]])$

$[\![\neg B_J \Box_i p]\!]^{g,s,s',w} = 1$ iff
$\exists w' \in \mathcal{B}^w_J[\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \& \exists v \in \mathcal{B}^w_J[\mathcal{B}^w_J = g(i)(v)]\ \&\ \exists w'' \in g(i)(w')[p(w'') = 0]]$

$[\![\neg B_J \Diamond_i p]\!]^{g,s,s',w} = 1$ iff
$\exists w' \in \mathcal{B}^w_J[\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J] \& \exists v \in \mathcal{B}^w_J[\mathcal{B}^w_J = g(i)(v)]\ \&\ \forall w'' \in g(i)(w')[p(w'') = 0]]$

$[\![\neg D_J \Diamond_i p]\!]^{g,s,s',w} = 1$ iff
$\forall w' \in \mathcal{B}^w_J[(\forall v \in \mathcal{B}^w_J[g(i)(v) \subseteq \mathcal{B}^w_J]\ \&\ \mathcal{B}^w_J = g(i)(w')) \rightarrow \forall w'' \in g(i)(w')[p(w'') = 0]]$

References

Anand, P. and Hacquard, V. (2013). Epistemics and attitudes. Semantics and Pragmatics, 6:1–59.

Crnič, L. (2014). Pragmatics of epistemics in attitudes: A reply to Anand & Hacquard (2013). Manuscript (`http://semanticsarchive.net/Archive/WNiZTVkM/crnic-modals.pdf`, last accessed February 2017).

Dekker, P. (2002). A proper architecture for presupposition and quantification. Manuscript, University of Amsterdam.

von Fintel, K. and Gillies, A. (2010). Must...stay...strong. Natural Language Semantics, 18(4):351–383.

von Fintel, K. and Heim, I. (2016). Intensional semantics. Lecture notes.

Hacquard, V. (2006). Aspects of modality. PhD thesis, Massachusetts Institute of Technology.

Hacquard, V. (2010). On the event relativity of modal auxiliaries. Natural Language Semantics, 18(1):79–114.

Hintikka, J. (1962). Knowledge and belief. Cornell University Press.

Ippolito, M. (2017). Constraints on the embeddability of epistemic modals. In Sinn und Bedeutung 21 Preproceedings. (available online since 20 April).

Karttunen, L. (1972). *Possible* and *must*. In Kimball, J., editor, Syntax and semantics Vol. 1, pages 1–20. Academic Press, New York.

Karttunen, L. and Peters, S. (1979). Conventional implicature. In Oh, C.-K. and Dinneen, D., editors, Presupposition, Syntax and Semantics 11, pages 1–56, New York. Academic Press.

Kratzer, A. (1981). The notional category of modality. In Eikmeyer, H.-J. and Rieser, H., editors, Words, Worlds, and Contexts: New Approaches in Word Semantics, pages 38–74. de Gruyter, Berlin.

Magri, G. (2009). A theory of individual-level predicates based on blind mandatory scalar implicatures. Natural Language Semantics, 17(3):245–297.

Magri, G. (2011). Another argument for embedded scalar implicatures based on oddness in downward entailing contexts. In Proceedings of Semantics and Linguistic Theory (SALT) XX, pages 564–581, Ithaca. CLC Publications.

Mandelkern, M. (2019a). Bounded modality. Philosophical Review, 128(1):1–61.

Mandelkern, M. (2019b). Modality and expressibility. The Review of Symbolic Logic, pages 1–38.

Močnik, M. (2019a). Slovenian 'dopuščati' and the semantics of epistemic modals. In Formal Approaches to Slavic Linguistics (FASL) 27, Ann Arbor. Michigan Slavic Publications.

Močnik, M. (2019b). Where force matters: Embedding epistemic modals (and attitudes). In Proceedings of Sinn und Bedeutung 23.

Potts, C. (2007). Into the conventional-implicature dimension. In Philosophy Compass, volume 2, pages 665–679. Blackwell Publishing Ltd.

Stephenson, T. (2007). Judge dependence, epistemic modals, and predicates of personal taste. Linguistics and Philosophy, 30:487–525.

Sudo, Y. (2014). Presupposition projection in quantified sentences and cross-dimensional anaphora. Manuscript.

Willer, M. (2013). Dynamics of epistemic modality. Philosophical Review, 122(1):45–92.

Yalcin, S. (2007). Epistemic modals. Mind, 116:983–1026.

Yalcin, S. (2012). Context probabilism. In Proceedings of the 18th Amsterdam Colloquium, pages 12–21.