Theorem: Consider the linear control process
\[
\frac{d\hat{x}}{dt} = \hat{A}(t)\hat{x}(t) + \hat{B}u(t)
\]
where \(\hat{x}(t)\) is the state of the system, \(\hat{A}(t)\) is the transfer function matrix, and \(\hat{B}\) is the control matrix. The system is completely controllable if and only if for every \(\hat{a}\) the determinant of \(\hat{A}(t) - \hat{a}\) is nonzero for all \(t\).

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Every eigenvalue of \(\hat{A}\) may be altered by changing the characteristic equation of the companion matrix \(\hat{A}\) which is associated with it.

For example, suppose that the eigenvalues of \(\hat{A}\) are to be moved to new locations determined by the characteristic equation
\[
s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0 = 0.
\]

Each \(\hat{A}(k) = a_1, a_2, \ldots, a_n\) is a companion matrix corresponding to the eigenvalues influenced by \(b_k\). This class of systems has a unique representation except for the ordering of the \(\hat{A}\).

Assume that \(\hat{A}\) has distinct eigenvalues. In applications, the matrix \(\hat{A}\) will usually be fixed by the availability of transducers to measure the state variables. More freedom may be available in the selection of the state matrix \(\hat{B}\). Each column of \(\hat{B}\) may be considered as an activating vector for a different component of control.

Let \(\hat{b}_k\) denote the \(k\)th column of \(\hat{B}\). Let \(\hat{A}^{(k)}\) denote the set of eigenvalues of \(\hat{A}\) which may be influenced by \(\hat{b}_k\). The class of multi-input systems considered in this paper has the following properties:

\[
\hat{A}(k) = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \cdots & \hat{A}_{1n} \\ \hat{A}_{21} & \hat{A}_{22} & \cdots & \hat{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{A}_{n1} & \hat{A}_{n2} & \cdots & \hat{A}_{nn} \end{bmatrix}
\]

Condition (4) guarantees that the system is completely controllable while condition (5) indicates that each actuating vector influences a different set of eigenvalues. The class of systems satisfying conditions (4) and (5) will be referred to as systems with disjoint control.

Since we are interested in the properties of the transfer function matrix, we may consider any convenient representation of it. In terms of the canonical form for multi-input systems, the state equation of (1) admits the representation

\[
\frac{d\hat{x}}{dt} = \hat{A}(t)\hat{x}(t) + \hat{B}u(t)
\]

The characteristic equation of \(\hat{A}\) is given by
\[
\det [sI - \hat{A}] = \sum_{k=1}^{m} a_k = n.
\]

Every eigenvalue of \(\hat{A}\) may be altered by changing the characteristic equation of the companion matrix \(\hat{A}\) which is associated with it. For example, suppose that the eigenvalues of \(\hat{A}\) are to be moved to new locations determined by the characteristic equation
\[
s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0 = 0.
\]
Let the state vector be partitioned according to the companion matrices $\tilde{A}_k$ in $\tilde{A}$ and let the $k$th component of the control vector be
\[
\begin{align*}
u_k(t) &= [a_{2k}, a_{3k}, \ldots, a_{nk},
\end{align*}
\]

If all the eigenvalues are changed in this manner, the resulting system $\tilde{A}$ is still of the form (6), except that the last row of each $\tilde{A}_k$ is changed due to the shifting of the eigenvalues.

Due to the block diagonal structure of (6)
\[
\begin{align*}
[sI - \tilde{A}]^{-1} = \\
\begin{bmatrix}
\ldots \ldots \
1 \\
\ldots \ldots \\
\ldots \ldots \\
s^{n_k-1}
\end{bmatrix}
\end{align*}
\]

where a typical block $[sI - \tilde{A}_k]$ is of the form
\[
\begin{align*}
[sI - \tilde{A}_k]^{-1} = \\
\begin{bmatrix}
\ldots \ldots \
1 \\
\ldots \ldots \\
\ldots \ldots \\
s^{n_k-1}
\end{bmatrix}
\end{align*}
\]

The matrix (12) is not completely described since only the last column proves to be of importance. The transfer function matrix corresponding to the representation (6) is
\[
T(s) = \tilde{H} [sI - \tilde{A}]^{-1} \tilde{B}
\]

Identification of Process Delay Time

Abstract—A method of identification of a process delay time is suggested in conditions when the delay is varying slowly and cannot be estimated by direct measure, and in which also the process input is not available for measurement. A cross correlation function is suggested as a means of evaluating the process delay time so that the system can be adapted to the new conditions necessary for acceptable performance.

References


The type of system to be considered is that generally represented in Fig. 1. Such a system can be controllled by a classical three-term controller but not in an optimum manner, although the results may be reasonably satisfactory.

In the literature on the control of delay time processes it has often been assumed that the function $G(p)$ is of the form $1/(1 + \gamma p)$. Reference [6] discusses the simplest condition when $\gamma = 0$, [7] that of $\gamma/T_i < 1$ and [8] that of $\gamma/T_i > 1$. The system shown in Fig. 1 is that proposed by Smith [11, 22]; this system enables the output integral error to be minimized without any cost being allocated to the manipulated variable $M(p)$. For stability reasons it is necessary in this system to maintain $T_2 > T_1$. This note is concerned with a possible method whereby $T_2$, the delay time of the simulator, can be matched to $T_1$, the actual process delay time that may be varying. $F(p)$ represents the function of the process output measurement transducer.

The output–disturbance transfer function for the system in Fig. 1 is
\[
C(p) = \frac{G(p) e^{-T p}}{D(p)} = \frac{G(p) e^{-T p}}{1 - F(p)e^{-T p}(1/F(p)e^{-T p})}
\]

If $F(p) = 1$, or if $F(p)$ contains a time constant that is very small compared with $T_i$, the necessary condition for stability is that $T_2 = T_1$. If $G(p)$ is $1/(1 + \gamma p)$ and $F(p) = 1$, then the output/disturbance transfer function can be expressed in the form
\[
e^{-\beta(1 - e^{-\alpha p})}
\]

where $\alpha = \gamma/T_i$ and $\alpha = T_2/T_i$ and $T_i$ is considered as one unit of time. Then by consideration of the characteristic equation it can be shown that, if $\alpha > 0$, for stability $\alpha = 1$. If $G(p)$ were replaced, more realistically, by a function $1/(1 + \alpha p)/(1 + \beta p)$ in which $\alpha > \beta > 0$, then the same condition for stability holds as it does if the element $G(p)^{-1}$ is not present in the control system.

A possible method of obtaining an indirect estimate of $T_i$ is by use of a cross-correlation function $\Phi_{xx}(T)$ between $x(t)$ and $e(t)$, the output of the simulator and the actuating signal of the system, so that
\[
\Phi_{xx}(T) = \frac{1}{T} \int_0^T x(t) e(t + T) dt.
\]

The evaluation of this function over a range of $T$ provides an oscillating profile with clearly defined maxima and minima directly indicating values of $T_i$ and $T_2$ and also the difference $T_2 - T_1$. The presence of noise in the system should ensure the existence of these peaks even if the command $r(t)$ remains constant.

For the class of system in which $F(p) = 1$ and $G(p) = 1/(1 + \alpha T_i p)$, results for values of $\alpha = 0.15$ and $\alpha = 0.3$, $R(p) = 0$, and a unit step or sine input (period 127T) applied at $t = 0$ as a disturbance enable certain conclusions to be clearly drawn:

the first noticeable minimum of $\Phi_{xx}(T)$ occurs for $T = \xi$