Sampling-based Approximation Algorithms for Reachability Analysis with Provable Guarantees

Lucas Liebenwein*, Cenk Baykal*, Igor Gilitschenski, Sertac Karaman, Daniela Rus
Distributed Robotics Lab, CSAIL, MIT

* Both authors contributed equally
Motion Planning – What We Wish For


Motion Planning – What We Have

The New Nation, 06/28/2015

http://www.squirrel-project.eu/objectives.html
Reachability Analysis for Online Verification

Infeasible Plan

Feasible Plan
Objective

For given timestep $T$, initial set $\mathcal{X}$, dynamics $h(x, u)$
find reachable set $F(\mathcal{X}; T)$
Objective

For a reachable set $F(\mathcal{X}; T)$, generate a subset $\mathcal{S} \subset \mathcal{X}$ such that

$$(1 - \varepsilon)\mu(F(\mathcal{X}; T)) \leq \mu(F(\mathcal{S}; T)) \leq \mu(F(\mathcal{X}; T))$$
Main Challenge

Evaluating reachability involves reasoning about

• Initial sets and how they evolve with respect to $F(\cdot; T)$
• State space and curse of dimensionality
• Trade-off between computation time and accuracy

In general, reachable sets cannot be evaluated (exactly) within a feasible amount of time
Related Work

Liu, S.B., et al. (2017)
Erlien, S.M., et al. (2016)

Althoff, M., et al. (2014)
Method

Initial set $\mathcal{X}$

Reachable set $F(\mathcal{X}; T)$
Method

Initial set $\mathcal{X}$

Reachable set $F(\mathcal{X}; T)$
Method

Initial set $\mathcal{X}$  

Reachable set $F(\mathcal{X}; T)$
Method

Initial set $\mathcal{X}$

Reachable set $F(\mathcal{X}; T)$
Method

Initial set $X$

Reachable set $F(X; T)$

$X \rightarrow F(\cdot; T)$
Method

Initial set $\mathcal{X}$

Reachable set $F(\mathcal{X}; T)$
$\delta$-Packing
\(\delta\)-Packing
$\delta$-Packing
$\delta$-Packing
Reachable Set from $\delta$-Packing

Initial set $\mathcal{X}$

Reachable set $F(\mathcal{X}; T)$
Reachable Set from $\delta$-Packing

For a given $\delta$-packing, there is an upper bound on volume of $F(x_1; T) \cap F(x_2; T)$

Guaranteed underapproximation for user-specified $\varepsilon$ (see paper for formal proof)
Results: Dubin’s Car with Various Initial Sets

Dumbbell
Lollipop
Hedgehog
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