Evaluation of the torque by Lower hybrid wave inducing ion rotation in a Tokamak

Jungpyo (J.P.) Lee¹  J.C. Wright¹  P.T. Bonoli¹  
R.R. Parker¹  Y. Petrov²  R. W. Harvey²  
D. Ernst¹  P.J. Catto¹  and C-Mod team

¹MIT-PSFC  
²CompX

Nov/17/2011 APS Salt Lake
Introduction

- LH induced ion rotation measurement
- Electron and ion momentum equations

Evaluation of torque density

- In a wave code (TORLH)
- In a Fokker-Plank code (CQL3D)
- Application to a C-MOD case

Summary
LH induced ion rotation measurement

- Time scale: electron plateau build up time ($O(1)$msec), turbulence momentum transport time ($O(10)$msec), rotation saturation time ($O(100)$msec)

- Dependency of rotation saturation: plasma current, density, magnetic configuration (i.e. high-current: counter-rotation, low-current: co-rotation)

Y. Podpaly et al. 2011 APS, and J. Rice et al. 2011 APS
Electron and ion momentum equations

- **Electrons:**
  \[
  \frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \nabla f_e + \left( \frac{e}{m_e} \nabla \Phi + \Omega_e \mathbf{v} \times \hat{\mathbf{b}} \right) \cdot \nabla f_e = C_{ee}\{f_e\} + C_{ei}\{f_e, f_i\} + \nabla_v \cdot \overline{D_{ql}} \cdot \nabla f_e
  \]

- **Ions:**
  \[
  \frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i + \left( -\frac{Ze}{m_i} \nabla \Phi + \Omega_i \mathbf{v} \times \hat{\mathbf{b}} \right) \cdot \nabla f_i = C_{ii}\{f_i\} + C_{ie}\{f_i, f_e\}
  \]

- Taking a radial current of \( e (Z\Gamma_i - \Gamma_e) \cdot \nabla \psi \),
  \[
  \frac{\partial}{\partial t} \left< m_i R_n V_{i,\psi} \right>_s = -\frac{1}{V'} \frac{\partial}{\partial \psi} \left< V' \Pi \right> + \left< \mathbf{J} \cdot \nabla \psi \right>_s + \left< \int d^3 \mathbf{v} m_e R_v \Psi \nabla_v \cdot \overline{D_{ql}} \cdot \nabla f_e \right>_s
  \]

- Ion rotation change \( \frac{\partial}{\partial t} \left< m_i R_n V_{i,\psi} \right>_s \) is determined by LH wave angular momentum source \( \left< \int d^3 \mathbf{v} m_e R_v \Psi \nabla_v \cdot \overline{D_{ql}} \cdot \nabla f_e \right>_s \) and turbulent viscosity \( \Pi = m_i \left< \int d^3 \mathbf{v} R_v \nabla_v \Psi f_i \right>_s \).
Power evaluation using the susceptibility $\chi$ of electron landau damping: $P_{\text{abs}}(\psi) \equiv \left\langle \int d^3u \gamma m_e c^2 \nabla_u \cdot \overline{D_{ql}} \cdot \nabla u_f \right\rangle_s \approx \frac{1}{\tau_s} \Re \left\{ \sum_n \sum_m \sum_{m'} \oint d\ell \frac{d}{B} e^{i(m'-m)\vartheta} E_{m'}^m \ast \text{Im}\{\chi\} E^m_{m'} \right\}$

Toroidal angular momentum input of LH wave using a new variable $\eta$: $\frac{\partial L_\phi}{\partial t}(\psi) \equiv \left\langle \int d^3u m_e R_u \nabla_u \cdot \overline{D_{ql}} \cdot \nabla u_f \right\rangle_s \approx \frac{1}{\tau_s} \Re \left\{ \sum_n \sum_m \sum_{m'} \oint d\ell \frac{d}{B} e^{i(m'-m)\vartheta} E_{m'}^m \ast \text{Im}\{\eta\} E^m_{m',n} \right\}$

The relation between $\eta$ and $\chi$: $\text{Im}\{\eta\} = \frac{k_{||}}{\omega} R(\hat{e}_{||} \cdot \hat{e}_\varphi) \text{Im}\{\chi_{||||}\} = \frac{n_{||}}{c} R \cos \Theta \text{Im}\{\chi_{||||}\}$, corresponding to $\frac{\partial L_\phi}{\partial t} = \int d\vec{A} \cdot \frac{R}{\omega} \tilde{s} \vec{k} \cdot \hat{\varphi} = \frac{n_\phi R}{c} \int dA \left\langle W \right\rangle \vec{v}_g \cdot \hat{\varphi} \approx \frac{n_\phi R_0}{c} P_{\text{abs}}$
New bounce-averaged constants needed since $m_e Ru \varphi$ is not invariant along the orbit

Flux surface averaged angular momentum:

\[
\langle \int \, d^3 u (m_0 u \varphi R) Q(f_e) \rangle_s = \\
\langle \int \, d^3 u (u^2 \sin \theta) (m_0 u \varphi R) \left[ \frac{1}{u^2} \frac{\partial}{\partial u} \left\{ B \frac{\partial}{\partial u} + C \frac{\partial}{\partial \theta} \right\} f_e + \frac{1}{u^2 \sin \theta} \frac{\partial}{\partial \theta} \left\{ E \frac{\partial}{\partial u} + F \frac{\partial}{\partial \theta} \right\} f_e \right] \rangle_s \approx \\
\langle \int \, d^3 u (- \sin \theta) (m_0 \hat{e} \varphi \cdot \hat{u} R) \left\{ B \frac{\partial}{\partial u} + C \frac{\partial}{\partial \theta} \right\} f_e - \frac{\partial}{\partial \theta} (m_0 u \hat{e} \varphi \cdot \hat{u} R) \left\{ E \frac{\partial}{\partial u} + F \frac{\partial}{\partial \theta} \right\} f_e \rangle_s = \\
- \langle \int \, d^3 u \left[ \frac{1}{u^2} (m_0 \hat{e} \varphi \cdot \hat{u} R) \left\{ B \frac{\partial}{\partial u} + C \frac{\partial}{\partial \theta} \right\} f_e + \frac{1}{u^2 \sin \theta} \frac{\partial}{\partial \theta} (m_0 u \hat{e} \varphi \cdot \hat{u} R) \left\{ E \frac{\partial}{\partial u} + F \frac{\partial}{\partial \theta} \right\} f_e \right] \rangle_s
\]

Bounce-averaged angular momentum:

\[
\int \, d^3 u_0 \lambda \langle (m_0 u \varphi R) Q(f) \rangle_b = \\
- \int \, d^3 u_0 \lambda \left[ \frac{1}{u^2} (m_0 \hat{e} \varphi \cdot \hat{u} R) \left\{ B \frac{\partial}{\partial u_0} + C \frac{\partial}{\partial \theta_0} \right\} f_e + \frac{1}{u^2 \sin \theta} \frac{\partial}{\partial \theta_0} (m_0 u \hat{e} \varphi \cdot \hat{u} R) \left\{ E \frac{\partial}{\partial u_0} + F \frac{\partial}{\partial \theta_0} \right\} f_e \right] \rangle_b \\
\equiv \int \, du_0 d\theta_0 (-m_0 \sin \theta_0) \left[ \left\{ B_{0M} \frac{\partial}{\partial u_0} + C_{0M} \frac{\partial}{\partial \theta_0} \right\} F_e + \left\{ E_{0M} \frac{\partial}{\partial u_0} + F_{0M} \frac{\partial}{\partial \theta_0} \right\} F_e \right].
\]
Application to a C-MOD case

- Power profiles from TORLH and CQL3D converges by nonlinear iterations
- Torque density evaluation by two methods from TORLH and CQL3D almost agree
- The ratio of torque to power indicates the effective refractive index of the flux surface \( \frac{\partial L_\varphi}{\partial t} \approx \frac{n_\varphi R}{c} P_{\text{abs}} \)

\[ B_\varphi = 5.3 T, \quad I_p = 700 kA, \quad T_e(0) = 3.5 K eV, \quad n_e(0) = 1.2 \times 10^{20}, \quad n_\parallel = -1.6 \quad \text{and} \quad P_{\text{abs}} = 0.8 MW. \]
(a) Poloidal cross section of parallel electric field simulated by TORLH and CQL3D for C-Mod shot 1080320017 (Lower single null divertor, $B_T = 6T$, $I_p = 850kA$, $T_e(0) = 3.5KeV$, $n_e(0) = 0.7 \times 10^{20}$, $n_{\parallel} = -1.9$, LH power= 850$kW$) (b) Comparison between the simulated momentum source term (blue solid) by TORIC-LH and the initial counter-current direction increase rate of ion angular rotation by X-ray spectroscopy (yellow solid) for the same shot 1080320017.
Summary

- **Torque density** is evaluated either by new bounce-averaged constants in Fokker-Plank equation or directly from the wave momentum input computed by a full-wave code.

- The computed torque density is comparable to the size of initial change of ion rotation when LH wave is injected, but turbulence viscosity can be already effective within the instrument resolution time, O(10) msec.
Torque comparison with experiment

Comparison between the simulated momentum source term (blue solid), and the initial counter-current direction increase rate of ion angular rotation by X-ray spectroscopy (yellow solid) (a) C-Mod shot 1110128026 (Lower single null divertor, $B_T = 5.3T$, $I_p = 700kA$, $T_e(0) = 3.5KeV$, $n_e(0) = 1.2 \times 10^{20}$, $n_\parallel = -1.6$, LH power= 700kW) (b) C-Mod shot 1110128034 (Lower single null divertor, $B_T = 5.3T$, $I_p = 370kA$, $T_e(0) = 2.5KeV$, $n_e(0) = 1.6 \times 10^{20}$, $n_\parallel = -1.6$, LH power= 700kW)
$\frac{\partial \theta_0}{\partial \theta} = \frac{\cos \theta}{\sin \theta} \frac{\sin \theta_0}{\cos \theta_0}$

Use the conserved magnetic moment and the resonance condition $\omega = kv \cos \theta + m\Omega$.

Definition of new bounced averaged constants for the angular momentum:

\[
B_{0M} = \lambda \left\langle (\hat{e}_\phi \cdot \hat{u}R)B \right\rangle_b = \\
= \lambda \left\langle (\hat{e}_\phi \cdot \hat{e}_\parallel \cos \theta + |\hat{e}_\phi \cdot \hat{e}_\perp| \sin \theta \times \text{sgn}(\cos \theta))RB \right\rangle_b
\]

\[
(\sin \theta_0)E_{0M} = \lambda \left\langle -\frac{\partial (\hat{e}_\phi \cdot \hat{u})}{\partial \theta} uR \frac{\sin \theta_0}{\sin \theta} E \right\rangle_b =
\]

\[
= \lambda \left\langle ((\hat{e}_\phi \cdot \hat{e}_\parallel) \sin \theta - |\hat{e}_\phi \cdot \hat{e}_\perp| \cos \theta \times \text{sgn}(\cos \theta)) R \frac{\sin \theta_0}{\sin \theta} E \right\rangle_b
\]

\[
= \lambda \left\langle ((\hat{e}_\phi \cdot \hat{e}_\parallel) \frac{\sin^2 \theta}{\cos \theta} - |\hat{e}_\phi \cdot \hat{e}_\perp| \sin \theta \times \text{sgn}(\cos \theta)) RB \right\rangle_b
\]

\[
C_{0M}/B_{0M} = F_{0M}/E_{0M} = C_{0ql}/B_{0ql}
\]
Ware pinch vs. LH induced pinch

- **DC Electric field induced pinch (Ware Pinch)**
  - Trapped electrons: accelerated for \((+v_\parallel)\) region and decelerated for \((-v_\parallel)\) region \(\Rightarrow \langle m_e R v_\varphi C_{ei}(f_e, f_i) \rangle_b = 0\) and only EXB drift drives the inward pinch.
  - Passing electrons: Remains from the imbalance of \(\langle m_e R v_\varphi (C_{ei}(f_e, f_i) - \frac{e}{m_e} \nabla \Phi \cdot \nabla v f_e) \rangle_b\) drives the small amount of inward pinch.

Schematic of the trapped electron radial pinch (a) by DC electric field and (b) by LH wave.
LH induced pinch by trapped electrons: one direction acceleration $\Rightarrow Q_{LH}^{\text{even}} \sim Q_{LH}(f_e\{+v_\parallel\})/2$ induces the orbit shape change, and $Q_{LH}^{\text{odd}} \sim Q_{LH}(f_e\{+v_\parallel\})/2$ induces inward pinch. But, the inward pinch is very small due to small population of resonant trapped electron ($< 1/100$ of ware pinch).

Passing electrons: Remaining from the imbalance of $\langle m_eRv_\varphi(C_{ei}(f_e, f_i) + Q_{LH}(f_e))\rangle_b \Rightarrow$ contribute more than trapped electron for the inward pinch.
Flux surface averaging vs. Bounce averaging

- **Flux surface averaging:** spatial averaging on a flux surface
  \[ \langle X \rangle_s = \frac{1}{\tau_s} \oint d\vartheta \frac{X}{B \cdot \nabla \vartheta} = \frac{1}{\tau_s} \oint d\ell \frac{X}{B} , \]
  because the jacobian is
  \[ J = \frac{1}{\nabla \vartheta \times \nabla \psi \cdot \nabla \varphi} = \frac{1}{B \cdot \nabla \vartheta} \]

- **Bounce averaging:** temporal averaging along the orbits
  \[ \langle X \rangle_b = \frac{1}{\tau_b} \oint d\ell \frac{X}{v_{\parallel}} = \frac{1}{\tau_b} \oint d\vartheta \frac{X}{v_{\parallel} \cdot \nabla \vartheta} \]
  where \( \tau_b(E, \mu) = \oint d\ell (E, \mu, l) \)

  - Advantage 1: remove the free streaming term along field line (e.g. \( v_{\parallel} \nabla_{\parallel} f_e = C_e(f_e) + Q_{LH}(f_e) \implies \langle C_e(f_e) + Q_{LH}(f_e) \rangle_b \))
  - Advantage 2: reduce a dimension in terms of poloidal angle \( \vartheta \), using a distribution function in the outer midplane
    \[ F_e(u_0, \theta_0(\theta, \vartheta), \psi, t) = f_e(u, \theta, \vartheta, \psi, t) \]

  - **Relation 1:** \( \langle X \rangle_b \tau_b = \frac{\langle X \rangle_s}{\tau_s} \tau_s \) for passing electron
  - **Relation 2:**
    \[ \int d^3 v X \langle X \rangle_s \tau_s = \int d^3 v_0 \frac{v_{\parallel 0}}{B(0)} \langle X \rangle_b \tau_b = \int d^3 v_0 \frac{v_{\parallel 0}}{B(0)} \langle X \rangle_b \tau_b = \]
    \[ \frac{1}{B(0)} \int d^3 v_0 \lambda \langle X \rangle_b \]
Application to the C-MOD case

- Introduction of new bounce-averaged constants for torque evaluation by collision and DC electric field in CQL3D
- Power profile balanced between LH wave, DC electric field and collisions
- The torque density dominated by DC electric field and its associated collisions

![Graphs showing power and torque density](image-url)
Results from only LH wave effect

- Removing the effect of DC electric field and temperature difference by subtracting the result without LH wave from the result with LH wave.

- Bounce averaging effect for torque: Comparison between

\[
m_e R_0 v_0 \langle (C_{ei}(f_e, f_i) + Q_{LH}(f_e)) \rangle_b \quad \text{and} \quad \langle m_e R v_\varphi (C_{ei}(f_e, f_i) + Q_{LH}(f_e)) \rangle_b
\]
Bounce averaging effect for torque

- Increased LH wave torque from bounce averaging:
  - $R$ in electron Landau damping region $> R_0$ (mostly low field side)

- Decreased torque by $u$ direction velocity flux (determined by A,B,C term) by bounce averaging effect
  - $\cos \theta \frac{\partial}{\partial u} A f_e$ is smaller than $\cos \theta_0 \frac{\partial}{\partial u} A_0 F_e$
  - due to $\frac{\sin^2 \theta}{\sin^2 \theta_0} = \frac{B}{B_0}$

- Decreased torque by pitch angle scattering collision term by bouncing averaging (determined by F term)
  - $\cos \theta \frac{\partial}{\partial \theta} F \frac{\partial f_e}{\partial \theta}$ is smaller than $\cos \theta_0 \frac{\partial}{\partial \theta_0} F_0 \frac{\partial F_e}{\partial \theta_0}$
Bounce averaging effect for radial pinch

- The radial pinch due to the LH wave torque interacted with trapped electron is tiny, about 1% of total LH wave torque (∼1 mm/s)

- LH induced pinch is dominant by imbalance of passing electrons, about 30% of total LH wave torque

- $|\text{LH wave torque}| \uparrow$ and $|\text{torque by collisions}| \downarrow$ by bounce averaging effect, so the pinch is negative (inward) (∼several cm/s)

- Is it always inward?

- LH induced pinch is non-ambipolar. What is its effect to the neoclassical toroidal viscosity? It may be another momentum transfer mechanism from electron to ion.
Electron and ion momentum equations

- For example, if the remaining torque for the radial pinch is 20%:
  
  Electrons:
  \[
  \langle \Gamma_e \cdot \nabla \psi \rangle_s = \left[ \frac{R}{e} \int d^3v \ m_e v_\varphi C_{ei} \{f_e, f_i\} + m_e v_\varphi \nabla_v \cdot D_{ql} \cdot \nabla_v f_e \right]^{20}_{-80}^{100}
  \]

  Ions: \[\langle \Gamma_i \cdot \nabla \psi \rangle_s = \left[ -\frac{R}{Ze} \int d^3v \ m_i v_\varphi \frac{\partial f_i}{\partial t} + m_i v_\varphi \nabla f_i + (-m_i v_\varphi C_{ie} \{f_i, f_e\}) \right]^{100}_{-80}^{20}
  \]

- Taking the radial current of \[e(2Z\Gamma_i - \Gamma_e) \cdot \nabla \psi,\]

  \[
  \frac{\partial}{\partial t} \langle m_i RnV_{i\varphi} \rangle_s + \frac{1}{V'} \frac{\partial}{\partial \psi} (V' \Pi) = 
  \]

  \[
  \langle J \cdot \nabla \psi \rangle_s + \left[ \int d^3v m_e Rv_\varphi \nabla_v \cdot D_{ql} \cdot \nabla_v f_e \right]^{100}_{0}
  \]

  where \[\Pi = m_i \langle \int d^3v Rv_\varphi v_\varphi f_i \rangle_s\]
Issues of new bounce-averaged constants

- Integration by parts
  - discontinuity of derivative function in terms of $\theta_0$
  - asymmetric boundary values in $\theta_0 = 0$ and $\pi$. It turns out to be smaller than integration value
  - passing-trapped boundary

- Singularity arose from the very small parallel velocity of the barely passing electrons

- Possibility to extend to ICRF or ECCD induced rotation case: use the fact that

\[
\langle m_e R v_\varphi (C_{ei}(f_e, f_i) + Q_{ECCD}(f_e)) \rangle_b \neq 0. \text{(c.f. Fisch-Boozer current drive mechanism)}
\]