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J. P. Lee, M. Barnes, F. I. Parra, E. A. Belli, and J. Candy

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The effect of diamagnetic flows on turbulent driven ion toroidal rotation\textsuperscript{a)}

J. P. Lee,\textsuperscript{1,b)} M. Barnes,\textsuperscript{2} F. I. Parra,\textsuperscript{3} E. A. Belli,\textsuperscript{4} and J. Candy\textsuperscript{4}

\textsuperscript{1}Courant Institute of Mathematical Sciences, New York University, New York, New York 10003, USA
\textsuperscript{2}Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712, USA
\textsuperscript{3}Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford OX1 3NP, United Kingdom
\textsuperscript{4}General Atomics, San Diego, California 92121, USA

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Turbulent momentum redistribution determines the radial profile of rotation in a tokamak. The momentum transport driven by diamagnetic flow effects is an important piece of the radial transport for sub-sonic rotation, which is often observed in experiments. In a non-rotating state, the diamagnetic flow and the $E \times B$ flow must cancel. The diamagnetic flow and the $E \times B$ flow have different effects on the turbulent momentum flux, and this difference in behavior induces intrinsic rotation. The momentum flux is evaluated using gyrokinetic equations that are corrected to higher order in the ratio of the poloidal Larmor radius to the minor radius, which requires evaluation of the diamagnetic corrections to Maxwellian equilibria. To study the momentum transport due to diamagnetic flow effects, three experimental observations of ion rotation are examined. First, a strong pressure gradient at the plasma edge is shown to result in a significant inward momentum transport due to the diamagnetic effect, which may explain the observed peaking of rotation in a high confinement mode. Second, the direction of momentum transport is shown to change as collisionality increases, which is qualitatively consistent with the observed reversal of intrinsic rotation by varying plasma density and current. Last, the dependence of the intrinsic momentum flux on the magnetic shear is found, and it may explain the observed rotation changes in the presence of lower hybrid current drive. © 2014 AIP Publishing LLC.

\textsuperscript{b)}Invited speaker.

I. INTRODUCTION

The ion toroidal rotation in a tokamak is beneficial because it controls many MHD phenomena (e.g., resistive wall modes)\textsuperscript{1,2} and increases plasma confinement by regulating turbulent transport.\textsuperscript{3–5} The ion toroidal rotation is determined by external momentum sources and by the turbulent radial transport of toroidal angular momentum. In the absence of external momentum sources, a significant diamagnetic rotation is found in many experiments. The size of this intrinsically generated rotation is well below the ion thermal speed (typically observed Mach number $\approx 0.1–0.2$).\textsuperscript{6} For reactor size tokamaks (plasma volume $\approx 1000$ m$^3$, e.g., ITER), external momentum sources such as neutral beams will be inefficient at rotating the plasma. Thus, it is desirable to utilize this intrinsic rotation, which requires the understanding of the mechanisms underlying the radial transport of toroidal angular momentum. This paper studies the effects of the diamagnetic flow on the radial transport of momentum. This study is accompanied by three examples illustrating the potential importance of diamagnetic effects in determining experimental rotation profiles.

The total toroidal flow in a tokamak is decomposed into two types of flows: the diamagnetic flow and the $E \times B$ flow due to the radial electric field. In this paper, we refer to both pressure gradient and temperature gradient driven neoclassical flows as diamagnetic toroidal flows.\textsuperscript{7,8} The diamagnetic toroidal flow always exists in a tokamak, and the size of the flow is of order $\rho_i (B/B_0)^{3/2}$, which is comparable to the measured intrinsic rotation in the outer radii (e.g., $r/a > 0.5$) of conventional tokamaks. Here, $B$ and $B_0$ are the magnitude of the total and poloidal magnetic fields, respectively, $v_i$ is the ion thermal speed, $r$ is the radial coordinate, $a$ is the tokamak minor radius, $\rho_i$ is the ion Larmor radius, $L_T$ is the radial scale of the temperature gradient, and $\rho_i = \rho_i/L_T$.

To explain the measured intrinsic rotation, it is important to understand momentum transport in the absence of flow and flow shear. The momentum flux in the absence of flow determines the sign of the intrinsic rotation. Consider the rotation that begins to develop from a non-rotating initial state. A positive momentum flux expels positive toroidal momentum toward the plasma-vacuum boundary, and it will result in counter-current rotation at the core. A negative momentum flux brings positive toroidal momentum to the core, and it gives co-current rotation in the core. In this paper, we evaluate the momentum transport for the case with zero net rotation; i.e., when the diamagnetic flow and the $E \times B$ flow have the same size but opposite signs. Because these two types of flows have different characteristics, the effects of two canceling flows on the turbulent momentum transport do not cancel each other, giving non-zero momentum transport even for zero net rotation.

To obtain non-zero net momentum transport, a symmetry of the turbulence needs to be broken.\textsuperscript{9,10} Previous studies in Refs. 11–13 showed that preexisting flow and flow shear can break the symmetry and result in momentum transport. However, this previous work is applicable only to the $E \times B$
flow due to the radial electric field. Recently, studies of momentum transport due to the diamagnetic flow have started.\textsuperscript{14–16} These studies show that diamagnetic flows also break the turbulence symmetry and such flows can be analyzed using gyrokinetics that retains higher order corrections in $\rho_s(B/B_0)$.

There are several other mechanisms that also break the turbulence symmetry in the absence of flow and cause turbulent momentum transport. The slow variation of radial density and temperature gradients across turbulent eddies\textsuperscript{17–19} and the slow poloidal variation of the turbulence\textsuperscript{20} can break the symmetry. These two slow variations give formally small corrections to the turbulence whose size can be comparable to the diamagnetic effect.\textsuperscript{21} An up-down asymmetric magnetic equilibrium can also result in momentum transport. This effect can in principle be larger than other symmetry breaking mechanisms, but the intrinsic rotation observed in an extremely up-down asymmetric tokamak was found to be insignificant (Mach number $< 0.05$)\textsuperscript{22} and up-down asymmetry driven momentum transport is also found to be small in numerical analysis.\textsuperscript{23} This paper focuses on the diamagnetic flow effects, showing that they can explain some experimental observations using the three examples in Sec. IV. Our treatment of the effect of the diamagnetic flows on the turbulence is based on the fact that neoclassical effects and turbulent effects are separable and additive to lowest order in $\rho_s$.\textsuperscript{14,24–27}

The rest of the paper is organized as follows. Section II explains the role of momentum transport in the absence of flow, and Sec. III describes the diamagnetic flow effects on momentum transport using gyrokinetics. In Sec. IV, we evaluate the diamagnetic effects and their dependence on three parameters: (A) pressure gradient, (B) collisionality, and (C) magnetic shear. These parameters are related to three experimental phenomena: (A) rotation peaking in L-H transition, (B) rotation direction reversal due to a density increase and a plasma current decrease, and (C) rotation change due to lower hybrid wave momentum injection. Finally, a discussion is given in Sec. V.

\textbf{II. ROTATION RADIAL PROFILE}

The steady-state ion toroidal rotation is achieved by balancing the external torque and the radial momentum redistribution

\begin{equation}
-\frac{1}{V'} \frac{\partial}{\partial \psi} (V' \Pi) + T_\psi = 0,
\end{equation}

where $V' = \int d\theta d\phi (B \cdot \nabla \theta)^{-1}$ is the flux surface area, $\Pi$ is the radial transport of ion toroidal angular momentum, and $T_\psi$ is the external torque. Here, we have used the static magnetic field $B = I\nabla \phi + \nabla \phi \times \nabla \psi$, where $I = B_0 R$, $B_0$ is the toroidal magnetic field, $R$ is the major radius, $\phi$ is the poloidal coordinate, $\theta$ is the poloidal coordinate, and $\psi$ is the poloidal magnetic flux, which is the radial coordinate.

A linear model for the dependence of the momentum flux on the radial rotation profile is obtained using the momentum advection and diffusion

\begin{equation}
\Pi = \Pi_{\text{int}} - P_{\phi} n_i m_i (R^2) \Omega_{\phi} - \chi_{\phi} n_i m_i (R^2) \frac{\partial \Omega_{\phi}}{\partial \psi},
\end{equation}

where $n_i$, $m_i$, and $\Omega_{\phi}$ are the ion density, mass, and toroidal angular frequency, respectively. The flux surface average is defined as $\langle X \rangle_{\psi} = (1/V') \int d\theta d\phi (X \cdot \nabla \theta)^{-1}$. This model is valid in the low flow regime (Mach $\ll 1$) in which the effects of the toroidal flow and flow shear on the momentum flux can be linearized about zero flow and flow shear. Here, $\Pi_{\text{int}}$ is the intrinsic momentum transport, which is the momentum flux generated even for zero flow and flow shear (i.e., $\Omega_{\phi} = 0$ and $\partial \Omega_{\phi}/\partial \psi = 0$). The advective term has the coefficient $P_{\phi}$ that is called the momentum pinch coefficient, and the diffusive term is proportional to the momentum diffusivity $\chi_{\phi}$. The momentum diffusion and advection for the $E \times B$ toroidal flow have been theoretically investigated in previous work.\textsuperscript{12,13,28}

Using Eqs. (1) and (2), the radial profile of toroidal rotation can be reconstructed

\begin{equation}
\Omega_{\phi}(\psi) = \Omega_{\phi}(\psi_0) \exp \left( \int_{\psi}^{\psi_0} d\psi' \frac{P_{\phi}}{\chi_{\phi}} \right) - \int_{\psi_0}^{\psi_0} d\psi' \left\{ \frac{\Pi_{\text{int}}(\psi')}{\chi_{\phi} n_i m_i (R^2) \psi'_0} \exp \left( \int_{\psi}^{\psi'} d\psi'' \frac{P_{\phi}}{\chi_{\phi}} \right) \right\}
+ \int_{\psi_0}^{\psi} d\psi' \left\{ \int_{\psi_0}^{\psi'} d\psi'' V' \langle \psi'' \rangle T_\phi(\psi'') \frac{\Pi_{\text{int}}(\psi_0)}{V' \chi_{\phi} n_i m_i (R^2) \psi'} \exp \left( \int_{\psi}^{\psi'} d\psi'' \frac{P_{\phi}}{\chi_{\phi}} \right) \right\},
\end{equation}

where the boundary condition for the toroidal velocity at the last closed flux surface, $\Omega_{\phi}(\psi_0)$, and zero momentum flux at the magnetic axis $\Pi(\psi = 0) = 0$ are used. Here, we assume $\psi = 0$ at the magnetic axis and $\psi = \psi_0 > 0$ at the last closed flux surface without loss of generality.

If there is no intrinsic momentum transport and no momentum source ($\Pi_{\text{int}} = 0$ and $T_\psi = 0$), the rotation profile is determined by the first term on the right hand side of Eq. (3), which is due to momentum diffusion and pinch. Then, the sign of the rotation is given by the velocity at the boundary. The intrinsic momentum transport (the second term on the right hand side of Eq. (3)) can change the sign, which is necessary to explain many experimentally observed intrinsic rotation phenomena. Inward intrinsic momentum transport ($\Pi_{\text{int}} < 0$) results in rotation in the co-current direction, and outward intrinsic momentum
transport \((\Pi_{int} > 0)\) results in rotation in the counter-current direction, as mentioned in the introduction. Notice that \(\Pi_{int}\) at outer radii contributes more to the core rotation than \(\Pi_{int}\) at inner radii because of the exponential factor \(\exp\left(\int_{\rho}^{\rho_{1}} d\rho_{1} / (\rho_{1} / \rho_{0})\right)\) in the integral in the second term on the right hand side of Eq. (3). The exponential factor arises due to the inward momentum pinch \((P_{o} / \rho_{0})\) is observed to be positive in many different turbulent simulations.\(^{13,28}\) As a result, the effects from the diamagnetic flow, whose size is typically larger at outer-radii than at inner-radii, becomes important in determining the core rotation. Also, note that the rotation profile is determined by the ratio of the momentum flux to the momentum diffusivity, \(\Pi_{int} / \rho_{0} n m_{i} R^{2} \phi\), rather than by the magnitude of the momentum flux \(\Pi_{int}\). In Secs. III and IV, we examine the effect of the diamagnetic flow on the intrinsic momentum transport \(\Pi_{int}\).

### III. TURBULENT MOMENTUM TRANSPORT OF DIAMAGNETIC FLOW

The turbulent momentum flux can be evaluated using gyrokinetics, which assumes \(\rho_{e} \ll 1\), turbulence perpendicular wavelengths of order the ion gyroradius, and turbulence parallel wavelengths comparable to the size of the tokamak.\(^{29}\) Gyrokinetics averages the particle motion over the fast gyration, reducing a kinetic description to two velocity space variables (the kinetic energy \(E = v^{2} / 2\) and magnetic moment \(\mu = v_{\perp}^{2} / 2B\)) and the position of the gyration center. Here, \(v_{\perp} = \sqrt{v^{2} - v_{\parallel}^{2}}\) and \(v_{\parallel}\) are the velocities perpendicular and parallel to the static magnetic field, respectively. We split the distribution function into different pieces, \(f = f_{0} + f_{1} + f_{tb}\), where the background piece \(f_{0} = F_{M}(v - \Omega_{p}E\hat{R})\) is the shifted Maxwellian due to \(E \times B\) toroidal flow, \(F_{M}(v)\) is the stationary Maxwellian, \(\Omega_{p,E} = -c / (\partial \phi_{e} / \partial \psi)\) is the \(E \times B\) rotation frequency, \(\phi_{e}(\psi)\) is the lowest order electrostatic potential, \(f_{1}\) is the higher order equilibrium that describes diamagnetic flows, and the fluctuating piece is much smaller than the background piece, \(f_{tb} \sim \rho_{e} F_{M}\). Similarly, the electrostatic potential can also be divided into three pieces, \(\phi = \phi_{0} + \phi_{1} + \phi_{tb}\), where \(\phi_{1}\) is the non-fluctuating higher order potential. For electrostatic turbulence, which tends to dominate in low \(\beta\) tokamaks, the ion gyrokinetic equation for \(f_{tb}\) to lowest order in \(\rho_{e}\), without any flow or flow shear, is

\[
\frac{\partial f_{tb}}{\partial t} + \left( (v_{\parallel}) \cdot \hat{b} + v_{M} + (v_{E0} + v_{E1} + v_{E}) \right) \cdot \nabla f_{tb} - \frac{Ze}{m_{i}} (v_{\parallel}) \cdot \hat{b} + v_{M} \cdot \nabla (\phi_{0} + \phi_{1}) \frac{\partial f_{tb}}{\partial \phi}
\]

\[
= -v_{E1} \cdot \nabla (0 + F_{0}) + \frac{Ze}{m_{i} v_{\parallel}} (v_{\parallel}) \cdot \hat{b} + v_{M} \cdot \nabla (\phi_{0} + \phi_{1}) \frac{\partial (F_{0} + F_{1})}{\partial \phi} + (C(f)),
\]

where \(v_{E0}, v_{E1}\), and \(v_{E}\) are the \(E \times B\) drift due to \(\phi_{tb}, \phi_{0}\), and \(\phi_{1}\), respectively. Here, we assume \(v_{E0} \sim \rho_{e} v_{th}\) and \(v_{E1} \sim \rho_{e}^{2} (B/B_{0}) v_{th}\). Equation (7) includes the diamagnetic flow corrections based on the derivation in Refs. 14 and 30 that assumes \(B/B_{0} \gg 1\) and turbulence with characteristic perpendicular wavelengths of the order of the ion gyroradius.
For turbulent wavelengths of the order of \((B/B_0)\rho_i\) terms arising from other effects (e.g., due to slow radial\(^{17,19}\) and poloidal\(^{20}\) variation of turbulence) can be of comparable size.\(^{21}\) We neglected these other effects in Eq. (7) to focus on the diamagnetic flows. The resulting equations are consistent with turbulence with perpendicular wavelengths of the order of the ion gyroradius. Note that Eq. (7) is written in the laboratory frame, but it can also be written in the frame rotating with angular velocity \(\Omega_{\phi,E}\) (see Eq. (1) of Ref. 15). The drift kinetic equation determines the neoclassical correction \(F_1\) and \(\phi_1\) (Refs. 7 and 8), giving

\[
F_1 = \frac{mv_||}{T_i} \frac{\Omega_{\phi,d}}{B} F_M + \rho_* F_\text{other} \approx \rho_* \frac{B}{B_0} F_M, \tag{8}
\]

where the first term on the right hand side results in a shift of the Maxwellian distribution function due to the parallel particle diamagnetic flow \(\Omega_{\phi,d}\). Here, \(T_i\) is the ion temperature. The other term \(F_\text{other}\) gives no net toroidal angular momentum, but it also breaks the symmetry of turbulence (e.g., due to the parallel heat flow).

Using the solutions for \(j^{th}\) and \(\phi^{th}\) from Eq. (7), we can calculate

\[
\Delta \pi^{th} = \pi^{th} - \pi^{th}_0 \sim \rho_* \frac{B}{B_0} \pi^{th}, \tag{9}
\]

giving the non-zero momentum flux

\[
\Pi \simeq \sum_{k_\theta} \int_{-\infty}^{\infty} d\theta d\nu_|| \Delta \pi^{th} = \sum_{k_\theta} \int_{-\infty}^{\infty} d\theta d\nu_|| \Delta \pi^{th} \neq 0. \tag{10}
\]

If the size of the flows are sufficiently low to linearize their contribution to the moment flux, the deviation has the symmetry property \(\Delta \pi^{th}(\theta, \nu_||, k_\phi) = \Delta \pi^{th}(\theta, -\nu_||, -k_\phi)\), as shown in Figs. 1(b) and 1(c).

Both the diamagnetic flow and the \(E \times B\) flow cause the same shift of the Maxwellian distribution on the right hand side of Eq. (7). However, there are terms in Eq. (7) that are different depending on the type of the flow. The drift and the acceleration due to the radial electric field on the left hand side, \(v_E \cdot \nabla j^{th}\) and \((Ze/m_i)(v_M \cdot \nabla \phi_0)(\partial j^{th}/\partial \psi)\), are only due to the \(E \times B\) flow, and the additional equilibrium correction \(F_1^{\text{other}}\) on the right hand side is only due to the diamagnetic flow. Because of these different terms for the diamagnetic flow and the \(E \times B\) flow, they contribute to the momentum transport differently. Consequently, finite momentum flux occurs for a non-rotating state in which two types of flows cancel each other (\(\Omega_{\phi,d} + \Omega_{\phi,E} = 0\)), but their effect on the momentum flux does not. We can interpret the momentum flux for canceling flows by expanding the linear model in Eq. (2) using different pinch coefficients and different momentum diffusivities for the diamagnetic flow and the \(E \times B\) flow

\[
\Pi \equiv \Pi_\text{other} - n_i m_i \langle R^2 \rangle_v (P_{\phi,d} \partial \Omega_{\phi,d} / \partial \psi + P_{\phi,E} \partial \Omega_{\phi,E} / \partial \psi)
\]

\[
- n_i m_i \langle R^2 \rangle_v \left( \Gamma_{\phi,d} \frac{\partial \Omega_{\phi,d}}{\partial \psi} + \Gamma_{\phi,E} \frac{\partial \Omega_{\phi,E}}{\partial \psi} \right), \tag{11}
\]

For cancelling flows, \(\Omega_{\phi,E} = -\Omega_{\phi,d}\) and \(\partial \Omega_{\phi,E} / \partial \psi = -\partial \Omega_{\phi,d} / \partial \psi\), Eq. (11) results in

\[
\Pi_{\text{int}} = \Pi_\text{other} + \Pi_{\Omega_{\phi,d}}^{\Delta \pi} + \Pi_{\Omega_{\phi,E}}^{\Delta \pi}, \tag{12}
\]

where \(\Pi_{\Omega_{\phi,d}}^{\Delta \pi} \simeq -n_i m_i \langle R^2 \rangle_v \left( \chi_{\phi,d} - \chi_{\phi,E} \right) \partial \Omega_{\phi,d} / \partial \psi\) is due to the different pinch coefficients for the two types of flow and \(\Pi_{\Omega_{\phi,E}}^{\Delta \pi} \simeq -n_i m_i \langle R^2 \rangle_v \left( \chi_{\phi,d} - \chi_{\phi,E} \right) \partial \Omega_{\phi,d} / \partial \psi\) is due to the different momentum diffusivities. Here, \(\Pi_{\text{other}}\) contains the contribution of \(F_1^{\text{other}}\) and \(\phi_1\), which are diamagnetic corrections other than the flow. Because the diamagnetic flow depends on some plasma parameters such as the radial pressure and temperature gradients, collisionality, and safety factor, the momentum flux \(\Pi_{\text{int}}\) depends on these parameters and their radial derivatives (through the \(\nabla F_1\) term appearing in Eq. (7)). In Sec. IV, we examine three examples showing the primary dependences of the momentum flux on (a) the pressure gradient, (b) the collisionality, and (c) the magnetic shear (the radial derivative of the safety factor).

IV. EXAMPLES

A. Rotation peaking during the L-H transition

When the plasma confinement mode changes from L-mode to H-mode, a significant increase of the core intrinsic
where plasma parameters in Ref. 44 and a reasonable range of flow \( \Delta W \), during the transition, and with the inverse of the plasma current, \( 1/I_p \). Fig. 1 in Ref. 6 shows the experimental data supporting this correlation and the different coefficients, \( C_V \equiv \Delta V_\phi/(\Delta W/I_p) \), for different tokamaks. For example, the coefficient for Alcator C-Mod is \( C_V \simeq 7 \times 10^3 \text{(m/s)/(A/J)} \) from the line drawn through the data in Fig. 1 of Ref. 6.

To try to explain this rotation peaking, we estimate the momentum transport due to a strong diamagnetic flow in the pedestal. A strong pressure gradient is established in the pedestal in the L-H transition, and it gives co-current diamagnetic toroidal flow with a size comparable to the ion thermal speed. However, the size of the net toroidal rotation at the pedestal is measured to be much smaller than the size of the diamagnetic flow. It means that there is a strong \( E \times B \) flow in the counter-current direction due to a negative radial electric field balancing the diamagnetic flow. To simulate this situation, we study a non-rotating state in which the diamagnetic flow and the \( E \times B \) flow cancel each other. In the intrinsic momentum transport \( \Pi_{\text{int}} \) of Eq. (12), the contribution of the different momentum pinches for the two types of the flows \( \Pi_{\text{int}}^{\Delta \pi_\theta} \) was reported in our previous paper. In this paper, we revisit the simulation results and we analyze them. Using the second term on the right hand side of Eq. (3), we estimate the rotation peaking by

\[
\Delta V_\phi \simeq - \frac{\Pi_{\text{int}} \Delta \psi_\theta}{\chi_{\text{int}}^n m R_0} \exp \left( \int_0^{\psi_0} d\psi_\theta \frac{P_\phi}{\chi_{\text{int}}} \right) \left( \frac{\Delta W}{P_\phi} \right)_0 \cdot \Delta \theta \tag{13}
\]

where \( R_0 \simeq (R)_\psi \), \( \chi_{\text{int}}^n = \chi_{\theta} \phi/\psi_0 \), \( \Delta \psi_\theta \) and \( \Delta \theta \) are the pressure drop and the poloidal flux increase within the pedestal, respectively, and the pressure gradient \( \Delta P/\Delta \psi_\theta \) is used to estimate the diamagnetic flow size at the pedestal. For simplicity, the stored energy and the poloidal magnetic field in Eq. (13) were estimated by assuming an elliptical poloidal cross section, giving \( \Delta W \sim 2\pi^2 k^2 \rho R_0 \Delta \psi_\theta \) and \( B_\theta \sim 2\sqrt{2} I_p/\text{ca} \sqrt{1 + \kappa^2} \), where \( \kappa \) is the elongation. Then, \( C_S \simeq \sqrt{1 + \kappa^2}/4\sqrt{2} \pi \text{en}_a R_0^2 \) can significantly differ for different tokamaks. Eq. (13), \( \Pi_{\text{int}} \simeq \Pi_{\text{int}}^{\Delta \pi_\theta} \) is used by neglecting \( \Pi_{\text{int}}^{\Delta \pi_\theta} \) for the plasma parameters in Ref. 44 and a reasonable range of flow shear (e.g., \( \Pi_{\text{int}}^{\Delta \pi_\theta}/\chi_{\text{int}}^n m R_0 \partial \phi/\partial \Omega_{\text{d},d} R_0^2 \simeq 0.018 \) for \( \partial \Omega_{\text{d},d}/\partial \phi \)) \( \simeq -1.0 \). The experimentally observed scaling of the rotation peaking is recovered in Eq. (13) due to the inherent scaling of the diamagnetic flow, whose size is proportional to the radial pressure gradient and the inverse of the poloidal magnetic field.

Fig. 2 shows the momentum flux divided by the diamagnetic flow size, \( \Pi_{\text{int}}^{\Delta \pi_\theta}/\chi_{\text{int}}^n m \Omega_{\text{d},d} R_0 \). It is evaluated by solving Eq. (7) in the gyrokinetic code GS2 (Ref. 31) using the plasma parameters in Ref. 44 chosen to model the plasma edge. For low diamagnetic flow (Mach number \( \lesssim 0.2 \), the momentum flux \( \Pi_{\text{int}}^{\Delta \pi_\theta} \) of Eq. (13) does not vary with the diamagnetic flow size because the momentum flux is proportional to the flow size. However, for flows with Mach number \( \gtrsim 0.2 \), \( \Pi_{\text{int}}^{\Delta \pi_\theta} \) of Eq. (13) changes due to nonlinear effects. Using typical plasma parameters for Alcator C-Mod (\( \kappa = 1.7 \), \( n_0 = 1 \times 10^{20} \text{m}^{-3} \), \( a = 0.22 \text{m} \), \( R_0 = 0.22 \text{m} \)), we obtain \( P_\phi/\chi_{\text{int}} \simeq (3/R_0) (\alpha/\psi_0) \) from gyrokinetic simulations, giving \( \chi_{\text{int}}^n = 1 \). Then, the results of Fig. 2 and Eq. (13) give the coefficient \( C_V \simeq 4.0 - 6.5 \times 10^3 \text{(m/s)/(A/J)} \). This is in fairly good agreement with the coefficient obtained from the experimental results in Ref. 6. Thus, the intrinsic momentum transport due to the different momentum pinches for the two types of flow in the pedestal can be one of the main mechanisms leading to rotation peaking in H-mode plasmas.

The momentum flux \( \Pi_{\text{int}}^{\Delta \pi_\theta} \) originates from the different symmetry breaking mechanisms of the diamagnetic flow and the \( E \times B \) flow, as represented by the different contours of \( \Delta \pi_\theta \) in Figs. 1(b) and 1(c). The results in Fig. 1 are also obtained with GS2 with the parameters in Ref. 44. For \( \Omega_{\text{d},d} R_0/v_\text{ti} \sim 0.3 \), we can see \( \Delta \pi_\theta /\pi_\theta \sim 0.2 \) based on the magnitude of the contours in Figs. 1(a) and 1(b). Also, Fig. 1(c) shows \( \Pi_{\text{int}}^{\Delta \pi_\theta} \) is due to the about 30% difference between the momentum pinches for the diamagnetic flow and the \( E \times B \) flow (compare the magnitude of the contours between Figs. 1(b) and 1(c)). Because the difference occurs mainly due to the acceleration term, \( (Ze/m)_i (\nu_\text{m} \cdot \nabla \psi_0)(\partial \theta/\partial \Omega_{\text{d},d}) \) of Eq. (7), the contour shape of \( \Delta \pi_\theta \) in Fig. 1(c) is different from that in Fig. 1(b). For example, the acceleration term is odd in \( \theta \) unlike the other symmetry breaking terms, so it gives additional \( \theta \) variation in Fig. 1(c).

**B. Direction reversal due to collisionality**

In Ohmic L-mode discharges on Alcator C-Mod32 and TCV,33 it is found that the core intrinsic rotation changes its direction from co-current to counter-current as the plasma density increases or the plasma current decreases. Because the collisionality \( (\nu_\text{s}) \) is proportional to the density and the inverse of the plasma current, this is a likely control parameter for the reversals. Here, the collisionality is defined to be the ratio of the effective collision frequency for pitch angle scattering to the bounce frequency \( \nu_\text{s} = (r/R)^{-1/2} \nu_R Q R/v_\text{ti} \), where
where $\nu_s$ is the ion collision frequency and $q$ is the safety factor. In this section, we present the dependence of $\Pi_{\text{int}}$ on the collisionality, first reported in Ref. 15, and analyze the results.

As shown in the black curve of Fig. 2 in Ref. 15, the intrinsic momentum flux for the canceling diamagnetic flow and $E \times B$ flow changes its sign from negative to positive as $\nu_s$ increases. The results are obtained numerically for the Cyclone plasma parameters using GS2 and the neoclassical code NEO, which solves the drift-kinetic equation for $F_1$ and $q_1$. The change of sign occurs around $\nu_s \sim 1$ in which the neoclassical transport changes from the banana to the plateau regime. As the collisionality increases, the pieces of the diamagnetic flow that is proportional to the temperature gradient contributes more to the co-current diamagnetic flow, giving an increase in the diamagnetic particle flow and flow shear. For the Cyclone based plasma parameters with a diamagnetic flow $\Omega_{\text{eq}} dR_0/v_{i0} \sim -0.09$ and a flow shear $\left(\partial \Omega_{\text{eq}} / \partial \psi \right) \left(\psi_0 R_0/v_{i0} \right) \sim -0.1$, it is found that $\Pi_{\text{int}} > 0$ and $\left[\Pi_{\text{int}}^\Delta \right] / \left[\Pi_{\text{int}}^\Delta \right]$ (this is opposite to what happened with the plasma parameters of Sec. IV A). The amplified positive flux of $\Pi_{\text{int}}^\Delta + \Pi_{\text{int}}^\Delta$ can give a reversal of the momentum flux when the collisionality is increased because $\Pi_{\text{int}}^\Delta + \Pi_{\text{int}}^\Delta$ can beat the negative value of $\Pi_{\text{other}}$ (see the green curve of Fig. 2 in Ref. 15).

This numerical result is consistent with the key features of the experimentally observed intrinsic rotation reversal at Alcator C-Mod and TCV because the negative flux causes co-current rotation and the positive flux gives counter-current rotation. The collisionality can be evaluated using the parameters measured in Alcator C-Mod with some non-negligible uncertainties. In Ref. 32, it is found that the rotation reversal occurs inside of the $q = 3/2$ flux surface, which corresponds to $\nu_s^* \sim 0.7$ using $v^*/d = 0.6, n_0 \sim 0.8 \times 10^{20} \text{m}^{-3}, T_i \sim 0.6 \text{KEV}$, and the effective charge $Z_{eff} \sim 2.5$. However, a numerical analysis using experimental parameters over all radii may be required for more rigorous and quantitative comparisons with the rotation profiles in Ref. 32. The direction of the rotation cannot be simply determined by whether the turbulence type is ITG (ion temperature gradient driven turbulence) or TEM (trapped electron mode driven turbulence) as shown in the experimental measurements in Alcator C-Mod. If turbulence characteristics change significantly, the coefficients determining the rotation profile in Eq. (3) would change accordingly. This may explain why the rotation direction can change depending on the magnetic field configuration as found in TCV.

C. Rotation change due to the lower hybrid waves

Significant toroidal rotation changes with lower hybrid wave momentum injection have been observed in many tokamaks. The counter-current momentum of the wave accelerates the ions in the counter-current direction right after the wave injection (see $t < t_1$ in Fig. 3), and the acceleration decreases and almost saturates in $O(100)$ ms (see $t_1 < t < t_3$ in Fig. 3). The final rotation change is determined by the momentum input and the turbulent radial transport of the toroidal angular momentum, and it is given by Eq. (3).

![Figure 3](image-url)

**FIG. 3.** A schematic of typical core rotation measurements during lower hybrid wave injection at Alcator C-Mod for a high plasma current discharge ($I_p \approx 700 \text{kA}$) and a low plasma current discharge ($I_p \approx 350 \text{kA}$). Time history after the wave injection can be divided into three phases: acceleration in $t \approx t_1$, direction reversal in $t \approx t_2$, and saturation in $t \approx t_3$. Here, $t_1 \approx O(10)$ ms is the momentum transport time scale, and $t_2 \approx O(100)$ ms corresponds to the current resistive relaxation time scale.

In Alcator C-Mod, different responses to lower hybrid injection are observed depending on the size of the plasma current, as shown in Fig. 3. For high plasma current ($I_p \approx 700 \text{kA}$), the change in steady state rotation due to wave injection is in the counter-current direction ($\Delta V_\varphi < 0$), which is the same direction as the wave momentum. The size of the change is correlated with the size of the internal inductance drop, which is a measure of the lower hybrid wave power absorption off-axis. It means that the size of the change is likely to be proportional to the size of the external torque. This counter-current acceleration can be explained theoretically by the third term on the right hand side of Eq. (3). The rotation change is due to the balance between the external torque and the momentum pinch and diffusion. The observed change agrees well with the third term evaluated numerically using gyrokinetic analysis (see Fig. 5-3 in Ref. 42).

For a discharge with low plasma current ($I_p \approx 350 \text{kA}$), the change in steady state rotation due to wave injection is in the co-current direction, which is opposite to the wave momentum input. The initial acceleration after lower hybrid injection is in the counter-current direction, but it changes direction to co-current at about 150 ms after the wave injection (see $t \sim t_3$ in Fig. 3). Eventually, the rotation change is saturated in another several hundred milliseconds, and it is in the co-current direction ($\Delta V_\varphi > 0$). This reversal of the rotation change due to lower hybrid waves requires a negative change of the intrinsic momentum flux in the second term on the right hand side of Eq. (3).

We postulate that this reversal of the rotation change is related to the momentum transport due to the diamagnetic flows and its dependence on the magnetic shear. The change in the radial profile of the safety factor is more significant for the low current case than for the high current case. The efficiency of the lower hybrid current drive in Alcator C-Mod is $(n_e R_{d\text{LH}}) / P_{\text{LH}} \approx 2.3 \times 10^{19} \text{AW}^{-1} \text{m}^{-2}$, where $n_e$ is the electron density, $R_{d\text{LH}}$ is the lower hybrid wave driven current, and $P_{\text{LH}}$ is the wave power absorption, giving $I_{\text{LH}} \approx 130 \text{kA}$ for $P_{\text{LH}} \approx 850 \text{kW}$. For the high current case with $I_p = 700 \text{kA}$, the lower hybrid driven current is only about

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The normalized intrinsic momentum flux in Eq. (12), $\Pi_{\text{int}}/\rho_i n_i m_i v_i$, as a function of the magnetic shear $s = (r/q)(dq/dr)$. The error bars show the standard deviation due to the turbulent fluctuations around the time averaged momentum flux.

18% of the total current. However, for the low current case with $I_p = 350$ kA, the lower hybrid driven current is about 36% of the total current, resulting in a significant change in the radial profile of the plasma current. Consequently, only for the low current, the magnetic shear changes sensitively. The change in the magnetic shear depends on the position of the lower hybrid wave power deposition.

The measured magnetic shear for the low current case is found in Ref. 39. The reduction of the magnetic shear in the inner radii is due to the lower hybrid wave driving current off-axis $r/a > 0.5$, which gives a relatively flat profile of the safety factor inside the driven current location. Fig. 4 shows that the reduced magnetic shear results in reduced positive intrinsic momentum flux for a non-rotating state with canceling diamagnetic and $E \times B$ flow. The results are obtained with GS2 and NEO using Alcator C-Mod plasma parameters.45 The change of the magnetic shear results in the change of the radial shear of the diamagnetic flow and, as a consequence, a change of the momentum flux $\Pi_{\text{int}}$. This reduced positive momentum flux corresponds to a decrease of the absolute value of the counter-current rotation, which is consistent with the measurements in the low current case. To have a net increase in the steady state rotation due to the wave injection, $\Delta V_{\phi} > 0$, the negative change in $\Pi_{\text{int}}$ needs to be large enough to overcome the counter-current external momentum. In other words, significant reduction of the intrinsic momentum flux is required to have the change in the second term of Eq. (3) larger than the third term. For a momentum source with $P_{\text{LH}} \approx 850$ kW, the change of the normalized intrinsic momentum flux $\Delta \Pi_{\text{int}}/\rho_i n_i m_i v_i < -0.6$ is needed.42 Based on Fig. 4, this momentum flux reduction corresponds to a significant reduction of the magnetic shear (e.g., $s = 2.2 \rightarrow 0.5$), which may be larger than the observed reduction of the magnetic shear $|\Delta s| \approx 1.0$ in Ref. 39.

Nevertheless, the momentum flux due to the diamagnetic flows depends on the magnetic shear, and this dependence is consistent with many features of the rotation saturation in the lower hybrid wave injection. The co-current direction saturation only happens for the low plasma current in which the diamagnetic flow size is large and the change of the magnetic shear is significant. Also, this magnetic shear effect is consistent with the observed time scale for the reversal of the rotation change. The change in the safety factor profile takes a resistive current relaxation time (O(100) ms), which is also the observed time scale for the rotation changing its direction at low currents ($t_2$ in Fig. 3).

V. DISCUSSION

This paper investigates the radial transport of toroidal angular momentum $\Pi_{\text{int}}$ for a non-rotating state in which the diamagnetic flow and the $E \times B$ flow cancel each other. To demonstrate that this piece of the momentum transport is helpful to explain low Mach number rotation in a tokamak, we examine three examples. In the first example, the experimentally observed rotation peaking in H-mode is quantitatively compared with the estimated value using the numerical results of momentum transport due to different momentum pinchers for the diamagnetic flow and the $E \times B$ flow. The observed scaling of the rotation peaking with the stored energy and the plasma current is consistent with the diamagnetic flow effects, and the coefficient of the observed scaling agrees well with the results for Alcator C-Mod. The second example shows the qualitative agreement of the intrinsic rotation reversal observed in Alcator C-Mod and TCV with the dependence of the momentum transport on collisionality. The direction of the rotation reversal is consistent with the sign change of momentum transport around collisionality $\nu_\ast \approx 1.0$, which is also in the range of the observed collisionality for the reversal. In the last example, the dependence of the momentum transport on the magnetic shear may partially explain the co-current rotation change with the counter-current lower hybrid wave momentum injection for low current shots in Alcator C-Mod. Numerical simulations using Alcator C-Mod parameters result in reduced positive momentum flux due to a decrease in the magnetic shear, which is consistent with the direction and the time scale of the rotation change in the low current case.

In each example, we use one of the plasma parameters that determine the diamagnetic flow and flow shear as an independent variable and leave many other plasma parameters fixed. Thus, the examples investigate a small set of plasma parameters in the full configuration space. If a different set of control variables is given, the effects of the independent variable on the results may change. For instance, the control parameters, which determine the characteristics of the turbulence, are different in the above examples, and the contributions of $\Pi_{\text{int}}^\nu_{\phi}$ and $\Pi_{\text{int}}^{\nu_{\phi}}$ to the momentum transport $\Pi_{\text{int}}$ in Eq. (12) vary accordingly (compare Sec. IV A and Sec. IV B). The sign of these contributions, however, does not seem to depend on the direction of propagation of the instability that drives the turbulence. In general, we must say that the dependences of the momentum transport on pressure gradient, collisionality, and magnetic shear in this paper may not be universal in the full plasma parameter space. More work and comparisons with experiments are needed.

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