Errata

"On the power of (even a little) resource pooling"

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This note provides some corrections to the original paper, to be referred to as [TX]. The reason for the correction is the following. The original paper worked with sets of vectors $v$ that can be obtained from vectors $s$, through the relation

$$v_i = \sum_{j=i}^{\infty} s_j, \quad i = 0, 1, \ldots$$

However, the set of vectors $v$ that admit such a representation is not closed, and as a consequence the set $\overline{V}^M$ defined in Eq. (9) of the original paper is not compact.\(^1\)

It turns out that all of the results in the paper remain valid as long as $\overline{V}^M$ is redefined in a way that makes it compact. In what follows, we list the necessary changes.

1. Modified Definitions of Certain Sets, on Page 12 of [TX].

Replace the definitions of the sets $\overline{V}^\infty$ and $\overline{V}^M$ in Eqs. (9)-(10) of [TX], respectively, by

$$\overline{V}^\infty = \{v \in \mathbb{R}^\infty_+: 1 = v_0 - v_1 \geq v_2 \geq \cdots \geq 0\},$$

and

$$\overline{V}^M = \{v \in \overline{V}^\infty : v_1 \leq M\}.$$ 

Furthermore, let

$$\overline{V}^\infty_0 = \{v \in \overline{V}^\infty : \lim_{i \to \infty} v_i = 0\}.$$ 

Wherever the notation $\overline{V}^\infty$ or $\overline{V}^M$ is encountered in [TX], it will have the meaning defined in this note, unless a change is indicated in the next section. It turns out that all proofs remain unchanged.

Note that under the weighted $L_2$ norm defined in Eq. (12) of [TX], the sets $\overline{V}^M$ are compact, and their union is equal to $\overline{V}^\infty$. Furthermore, $\overline{V}^\infty_0$ is the set of vectors in $\overline{V}^\infty$ that can be represented as in Eq. (1) above, and there is a one-to-one correspondence between elements of $\overline{V}^\infty_0$ and $\overline{S}^\infty$, where the latter set is defined in Eq. (8) of [TX]. In particular, any vectors $V^N_i(t)$ associated with the actual stochastic model automatically belong to $\overline{V}^\infty_0$.

2. Modifications in the Statements of Certain Results or their Proofs.

1. Pages 13. In Definition 1, replace “a function $v(t)$...” with “a continuous function $v(t)$...”.

2. Page 15. In the second line of the statement of Theorem 2, replace “a state that satisfies...” by “a state in $\overline{V}^\infty$ that satisfies...”.

\(^1\)The authors are grateful to Xiaohan Kang at the Arizona State University for pointing out this error.
3. **Page 19.** Replace the statement of Theorem 5 by the following, which asserts convergence of $s(t)$, rather than the original stronger statement about convergence of $v(t)$.

"Given any initial condition $v^0 \in \overline{V}_0^\infty$, and with $v(v^0, t)$ the unique solution to the fluid model, consider the vector $s(t)$ associated with $v(v^0, t)$. Then,

$$\lim_{t \to \infty} \|s(t) - s^I\|_w = 0,$$

where $s^I$ is the invariant state of the fluid model given in Theorem 2."

4. **Page 19.** Add the following after the proof of Theorem 5.

"**Note.** With some additional work, it can be shown that $v(v^0, t)$ also converges to $v^I$, and that this remains true even for initial conditions in $V^\infty$ that are outside $V_0^\infty$."

5. **Page 33.** Replace the first line of the proof of Theorem 2 by the following.

"Let $v^I$ be an invariant state. If $p = 0$, then $g(v^I) = 0$ and $v^I$ obeys a system of linear equations. It is easily verified that if we set the right-hand side of Eq. (15) in [TX] to zero, and use the boundary condition in Eq. (13) of [TX], we obtain a unique solution. Suppose now that $p > 0$. Since $v^I_i$ is nonincreasing in $i$, and bounded below by 0, it follows that $v^I_i - v^I_{i+1}$ converges to zero. If the sequence $v^I_i$ does not have finite support, then $g_i(v^I) = p > 0$ for all $i$, which implies that for large enough $i$, the right-hand side of the drift equation (15) in [TX] is nonzero and $v^I$ is not an invariant state. We conclude that $v^I$ has finite support and therefore admits a representation of the form (1). Thus, for the rest of the proof, we can work with both $v^I$ . . ."  

6. **Page 33, line -4.** Replace "... at all $t > 0$." by "... at all $t > 0$, as long as $v(0) \in \overline{V}_0^\infty$.

7. **Page 43, line 1.** Replace “closed and bounded” by “compact”.

8. **Page 45, line 14.** Right before “Eq. (105) will imply...” insert: “in light of Theorem 5”.

9. **Pages 62-63.** Replace $\overline{V}^\infty$ by $\overline{V}_0^\infty$ in the three places: (i) in the first line of Proposition 33; (ii) in the first line of the proof of Proposition 33; (iii) in the second line of p. 63. Furthermore, Eq. (164) of [TX] now follows automatically from the definition of $\overline{V}_0^\infty$; the sentence that includes Eq. (165) can be removed.