
The aforementioned paper contains some technical errors concerning some of the lower bounds on the performance of multiclass Markovian queueing networks. The corrections are listed below.

1 Statement and proof of Proposition 2

The statement and the proof of Proposition 2 of the paper contain errors. The claimed statement with values of $p_{\text{min}}$ and $\nu_{\text{min}}$ as defined in the paper is not correct, and $p_{\text{min}}$ and $\nu_{\text{min}}$ have to be redefined. Specifically, the correct value of $p_{\text{min}}$ should be as follows. For every station $\sigma_j$, let $I_j$ be the set of classes $i = 1, \ldots, I$ such that $(i, k) \in \sigma_j$ for some stage $k$. Namely, $I_j$ is the set of types that are eventually served by server $\sigma_j$. Instead of letting $p_{\text{min}}$ to be $\sum_i \lambda_i$, as was done in the paper, we define

$$p_{\text{min}} = \sum_{i \in I_j} \lambda_i.$$  

Similarly, instead of defining $\nu_{\text{min}} = \rho_{\sigma_j}/\lambda_{\text{max}}$, we let

$$\nu_{\text{min}} = \min_{i \in I_j} \frac{\rho_{\sigma_j}^+}{\lambda_i}.$$  

We claim that Proposition 2 is valid with these modified definitions of $p_{\text{min}}$ and $\nu_{\text{min}}$. The proof of the proposition is corrected as follows. The value of the Lyapunov function increases when an arrival into class $i \in I_j$ occurs, (as opposed to any arrival into the network, as was incorrectly stated in the proof of the proposition in the paper). In particular, an arrival into class $i$ for which no stages correspond to station $\sigma_j$ does not change the value of the Lyapunov function. An arrival into type $I$ occurs with probability $\lambda_i$ and therefore $p_{\text{min}}$ is as stated. The derivation of the correct value of $\nu_{\text{min}}$ is similar.

2 Statement and proof of Proposition 4

Similarly, the statement and the proof of Proposition 4 of the paper contain errors. In the statement, the correct value of $p_{\text{min}}$ should be as follows. For every $K$-virtual station $V$, let $I_V$ be the set of classes $i = 1, \ldots, I$ such that $(i, k) \in V$ for some stage $k$. Then define

$$p_{\text{min}} = \sum_{i \in V} \lambda_i.$$  

Similarly, the definition of $\nu_{\text{min}}$ is incorrect. The correct definition is

$$\nu_{\text{min}} = \min_{i \in V} \frac{\rho_{\sigma_j}^+}{\lambda_i}.$$  

The changes in the argument are similar to the ones for Proposition 2.
3 Implications for other statements

In light of these changes, the lower bounds appear in Theorem 2 should be corrected as follows.

\[
\mathbb{P} \left( \sum_{i,j} \frac{\rho_{i,k}^{\sigma_j^+}}{\lambda_i} Q_{i,k}(t) \geq \frac{1}{2} \left( \min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j^+}}{\lambda_i} \right) m \right) \geq \frac{\left( \frac{1}{2} \left( \sum_{i \in I_j} \lambda_i \right) \left( \min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j^+}}{\lambda_i} \right) \right)^m}{\left( \frac{\left( 1 - \rho_{\sigma_j} \right)}{4} \right)}.
\]

\[
\mathbb{E} \left[ \sum_{i,j} \frac{\rho_{i,k}^{\sigma_j^+}}{\lambda_i} Q_{i,k}(t) \right] \geq \frac{\left( \sum_{i \in I_j} \lambda_i \right) \left( \min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j^+}}{\lambda_i} \right)^2}{4 \left( 1 - \rho_{\sigma_j} \right)}.
\]

Similarly, the lower bounds appear in Theorem 3 should be corrected as follows.

\[
\mathbb{P} \left( \sum_{i,j} \frac{\rho_{i,k}^{V^+}}{\lambda_i} Q_{i,k}(t) \geq \frac{1}{2} \left( \min_{i \in V} \frac{\rho_{i,1}^{V^+}}{\lambda_i} \right) m \right) \geq \frac{\left( \frac{1}{2} \left( \sum_{i \in V} \lambda_i \right) \left( \min_{i \in V} \frac{\rho_{i,1}^{V^+}}{\lambda_i} \right) \right)^m}{\left( \frac{\left( 1 - \rho_{\sigma_j} \right)}{4} \right)}.
\]

\[
\mathbb{E} \left[ \sum_{i,j} \frac{\rho_{i,k}^{V^+}}{\lambda_i} Q_{i,k}(t) \right] \geq \frac{\left( \sum_{i \in V} \lambda_i \right) \left( \min_{i \in V} \frac{\rho_{i,1}^{V^+}}{\lambda_i} \right)^2}{4 \left( K - 1 - \rho(V) \right)}.
\]

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