Tensor Product Representations and Holographic Reduced Representations

Smolensky, 1990 & Plate, 1991

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Neuro-symbolic Models for NLP (6.884), October 2, 2020

Outline

Introductions (us + 3 others) (11:35 - 11:40)

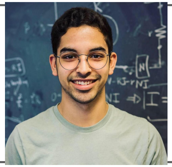
TPRs - why/what? (11:40 - 11:55)

Break out room (11:55 - 12:10 mins)

Discussion (12:10 - 12:20 mins)

[Early] Break (12:20 - 12:35)

TPR tutorial (12:35 - 12:50)



Discussion (12:50 - 12:55)

TPR Shortcomings; HRRs (12:55 - 1:10)

Discussion (1:10 - 1:25)

Outline

1. Is variable binding necessary?
2. Do humans use a TPR-like mechanism?
3. Do current models approximate *faithfulness*?
4. Small group technical questions

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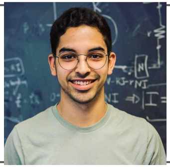

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Tensor Product Representations - *why?*

A one-sentence summary of the implications of this view for AI:

connectionist models may well offer an opportunity to ***escape the brittleness of symbolic AI*** *systems …*

… **This paper offers an example of what such a collaboration might look like**.

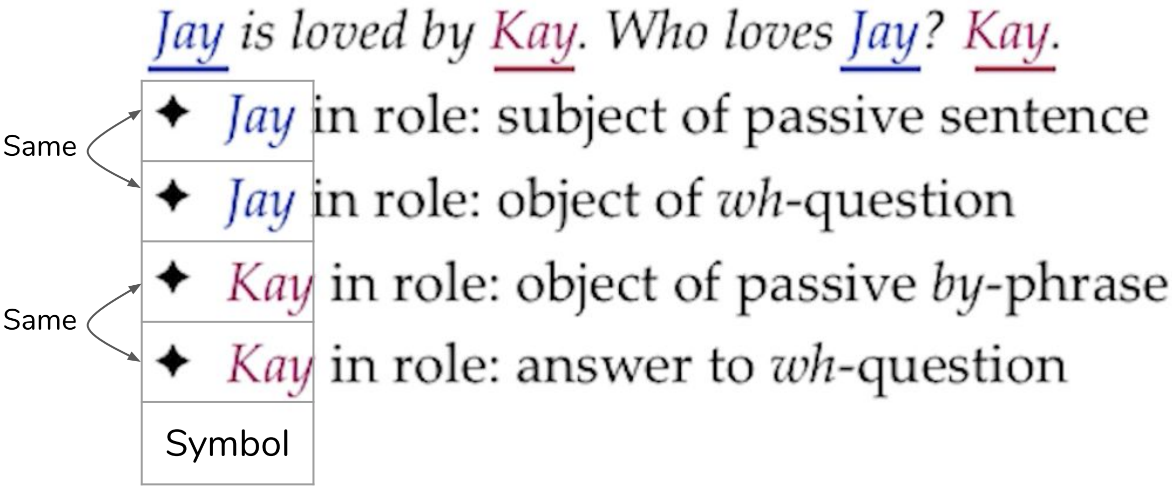
Tensor Product Representations - *why?*

Jay is loved by Kay. Who loves Jay? Kay.

- *Jay* in role: subject of passive sentence
- *Jay* in role: object of *wh*-question
- *Kay* in role: object of passive *by*-phrase
- *Kay* in role: answer to *wh*-question

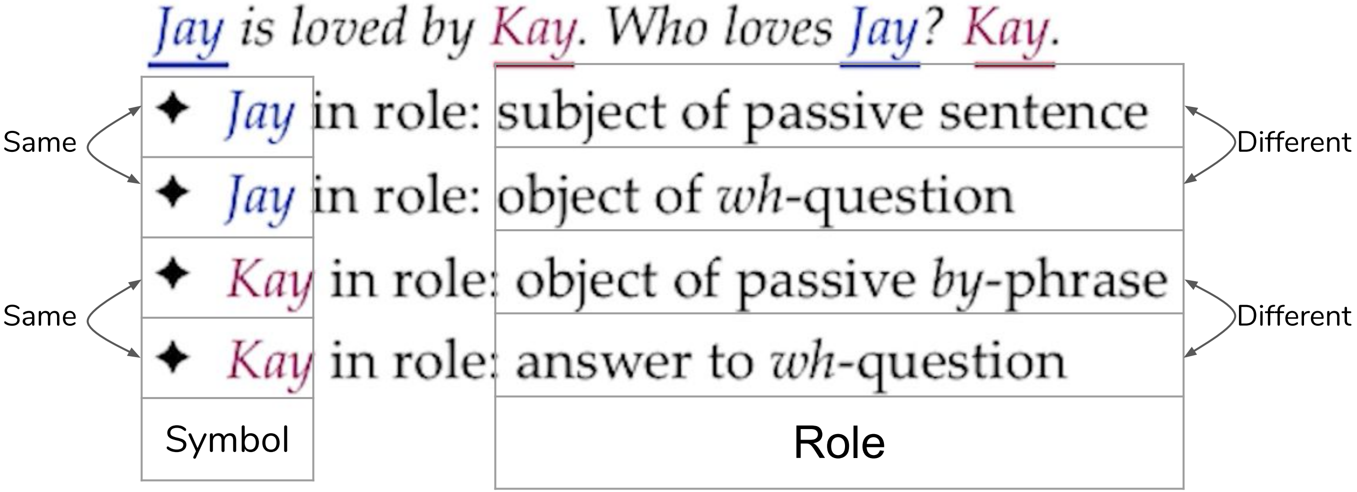
[Paul Smolensky HLAI Keynote (2019); Newell, A. (1980)]

Tensor Product Representations - *why?*



[Paul Smolensky HLAI Keynote (2019); Newell, A. (1980)]

Tensor Product Representations - *why?*



[Paul Smolensky HLAI Keynote (2019); Newell, A. (1980)]

Tensor Product Representations - *what*?

Representing Structured Objects

(1) Decomposing the structures via roles (2) representing variable/value bindings (3) representing conjunctions

[Smolensky 1990, pg. 169]

Tensor Product Representations - *what*?

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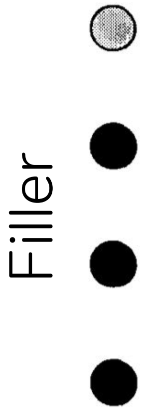
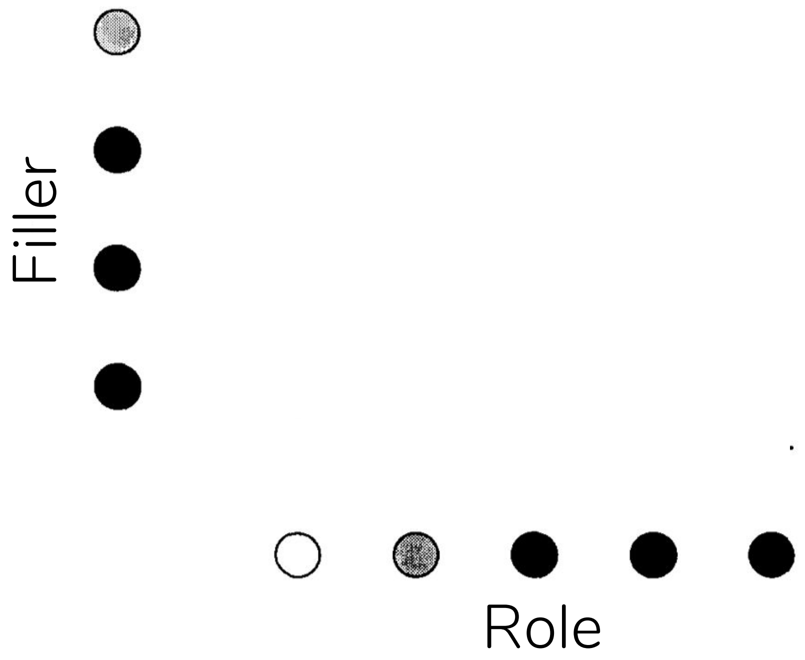


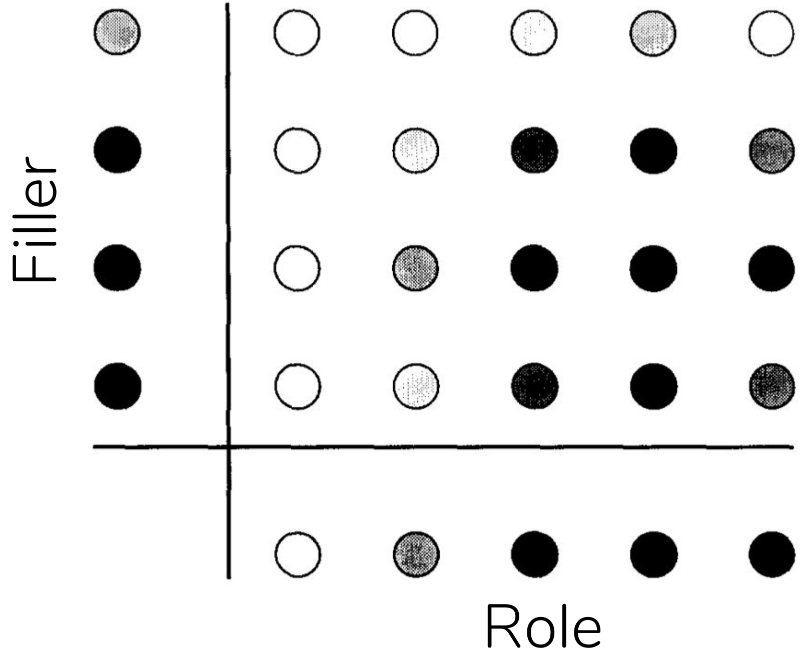
(1) Decomposing the structures via roles

Tensor Product Representations - *what?*



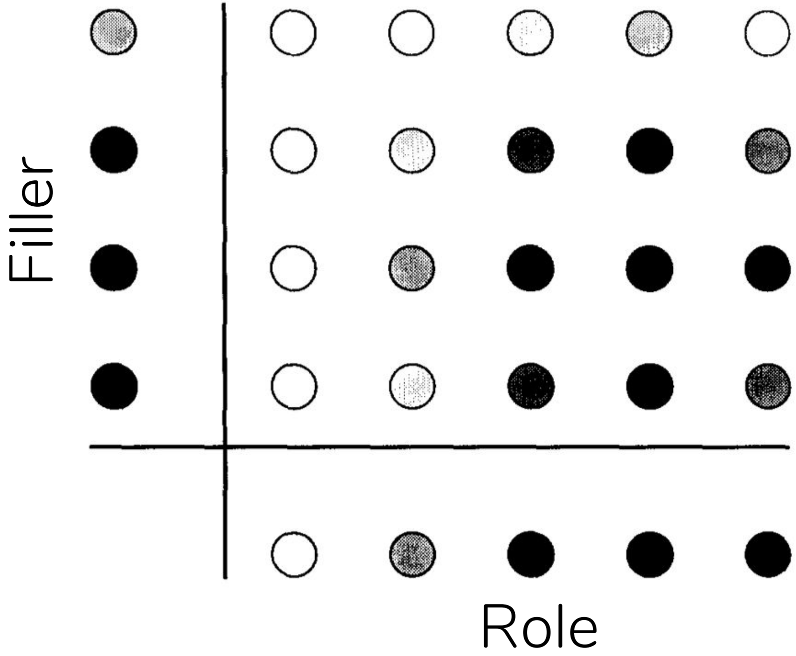
(1) Decomposing the structures via roles

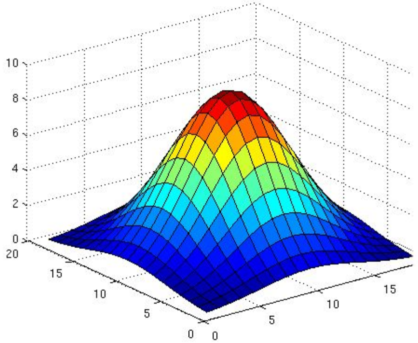
Tensor Product Representations - *what*?



(2) representing variable/value bindings

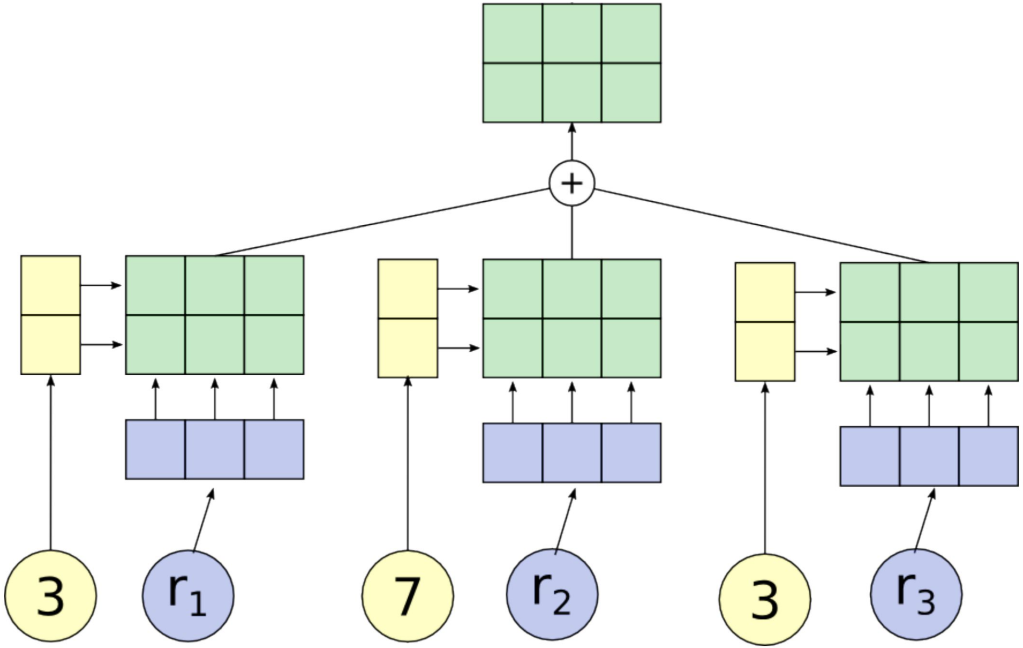
Tensor Product Representations - *what*?





(2) representing variable/value bindings

Tensor Product Representations - *what*?

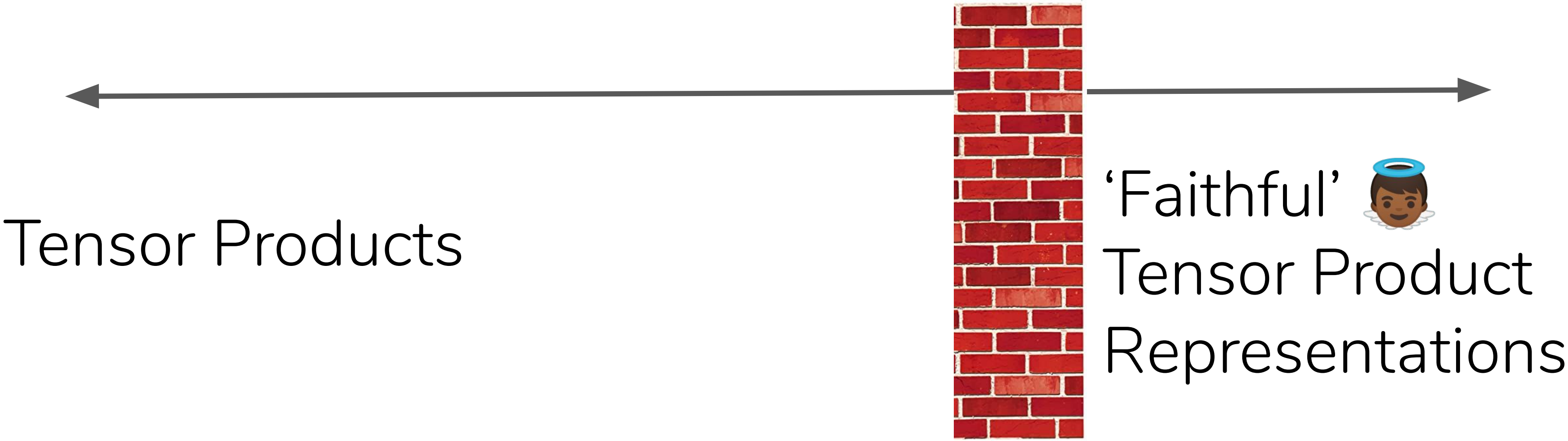


[Soulos et al. 2019]

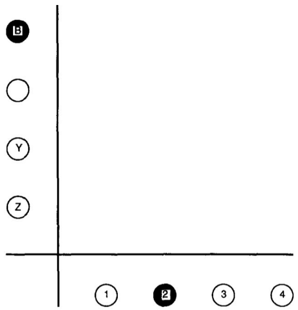
(3) representing conjunctions

Tensor Products

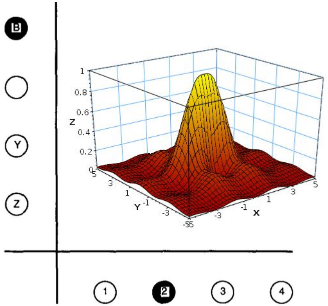
‘Faithful’ 👼
Tensor Product
Representations


Tensor Products
‘Faithful’ 👼🏾
Tensor Product
Representations

Faithfulness



Faithfulness



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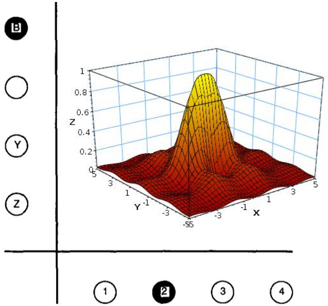
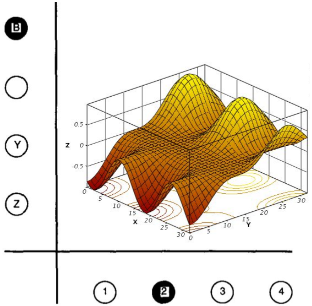
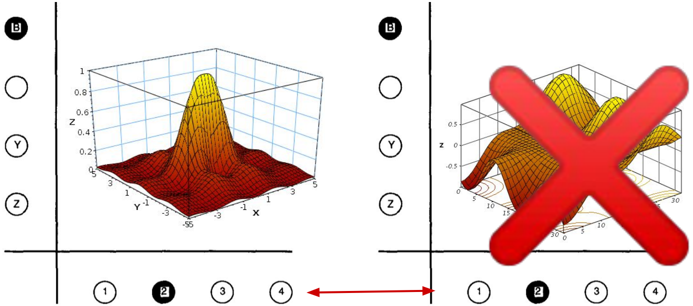




Faithfulness



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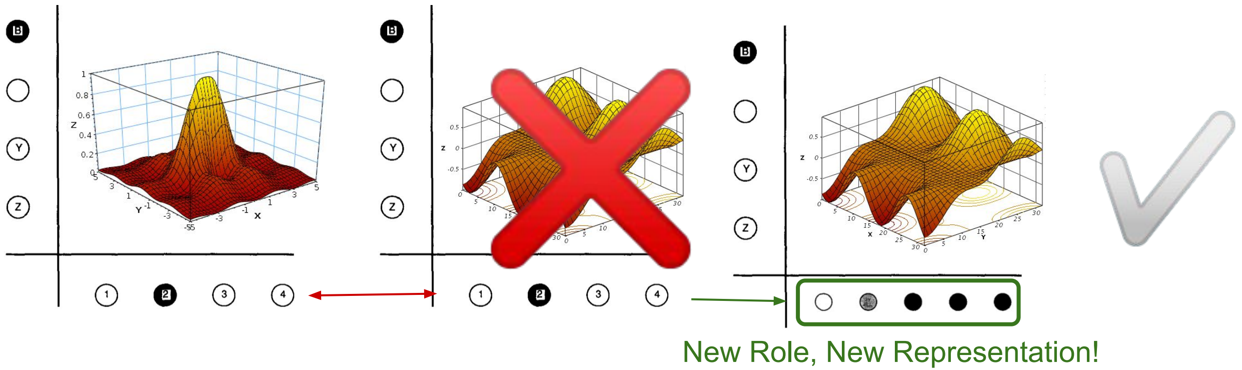
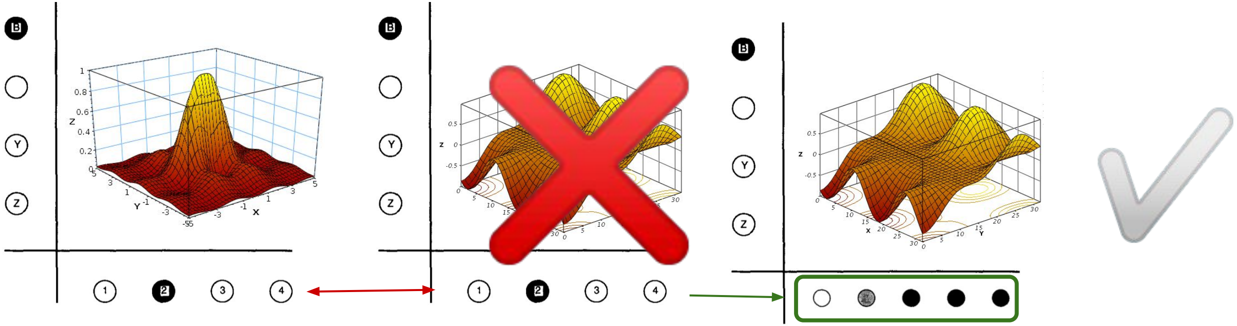
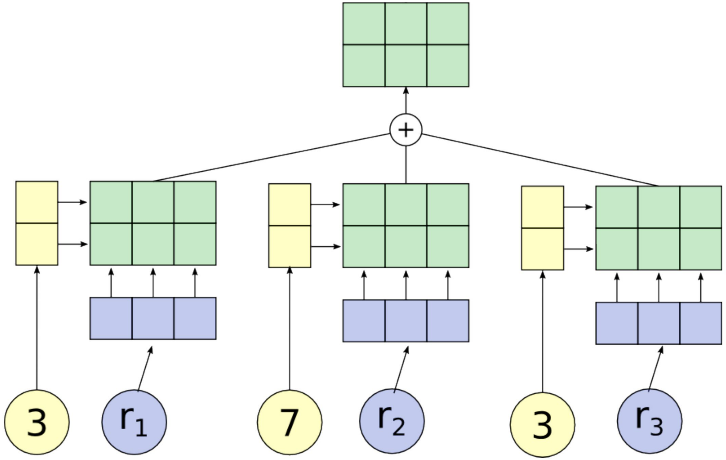


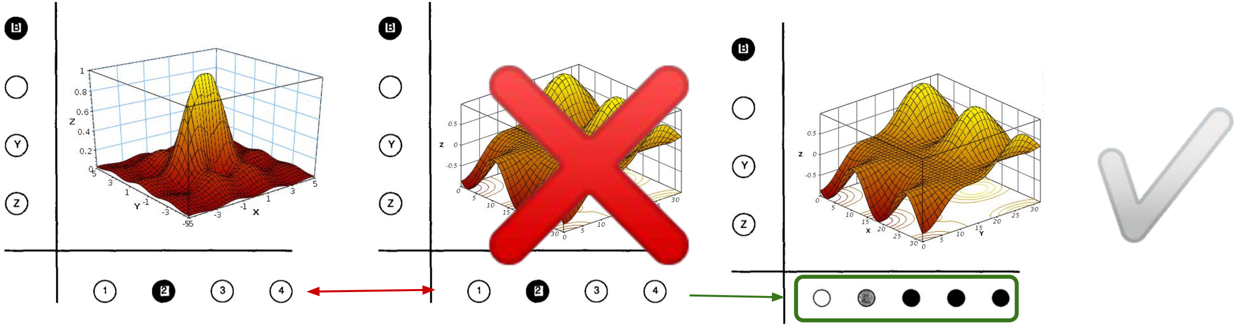
Faithfulness



New Role, New Representation!

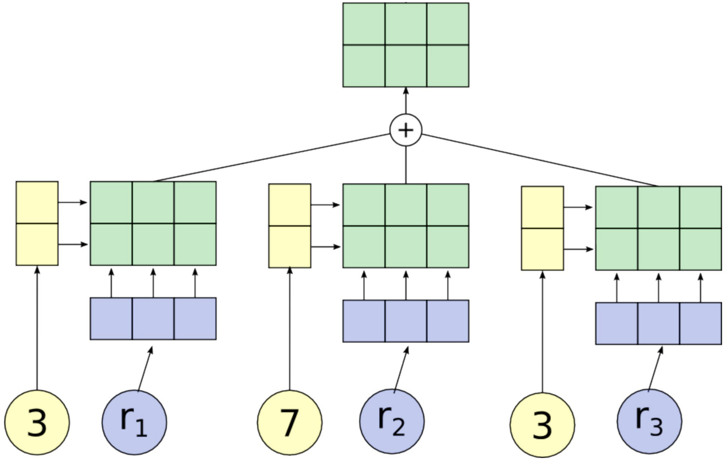


Faithfulness

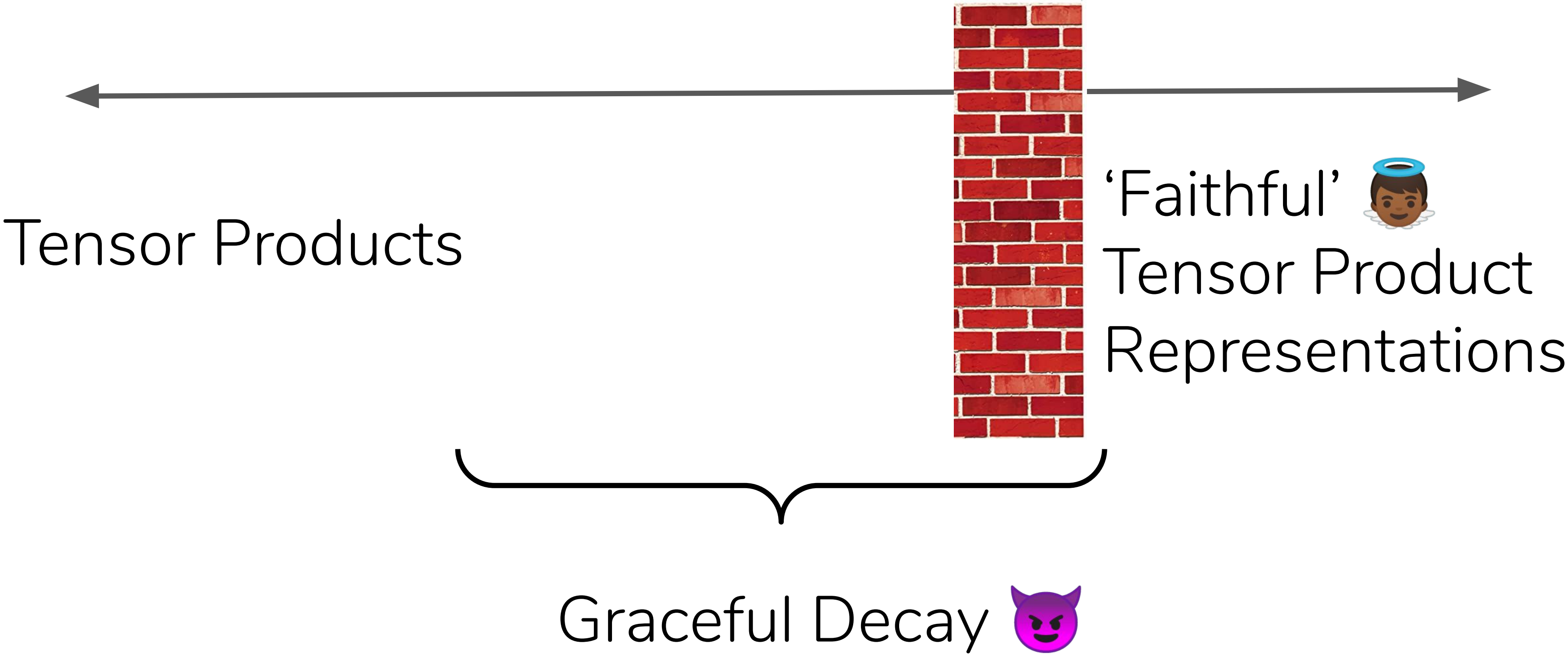


New Role, New Representation!

Orthogonality
Theorem 3.3, Section 3.2



Linear Independence
Definition 2.8, Section 2.2.2


Tensor Products
'Faithful' 👼🏾
Tensor Product
Representations
Graceful Decay 😈

Variable Binding

1. Is Variable Binding Necessary?
2. Do humans use a TPR-like mechanism?
3. Do current models approximate faithfulness?
4. Small group technical questions

Gary Marcus @GaryMarcus · Feb 6, 2018

completely agreed, @tdietterich! lots of cases where **variable binding** is absolutely necessary. no **binding**, no AGI.

> **Thomas G. Dietterich** @tdietterich · Feb 6, 2018
>
> Replying to @tdietterich @jahendler and @GaryMarcus
>
> There are lots of cases where binding appears to be necessary. Ex 1: If you put X into your pocket and then walk to work, you will be able to take X out of your pocket at work. Ex 2: If I ask you query X and you know X, you will tell me X, forall X.

1 5 9

Break

TPR Tutorial

TPR Tutorial

(1) Symbolic Structure

TPR Tutorial

(1) Symbolic Structure
(2) Encoding w/ TPRs

TPR Tutorial

(1) Symbolic Structure
(2) Encoding w/ TPRs
(3) Representation Proofs

(1) Symbolic Structure : *Give*, the programming language

<u>(1) Symbolic Structure</u> : *Give*, the programming language

Syntax

p ::= (Give p) | $\square$

(1) Symbolic Structure : *Give*, the programming language

Syntax

p ::= (Give p) | ⬜

Examples

(Give ⬜)

(Give (Give ⬜))

(Give (Give (Give ⬜))

(1) Symbolic Structure : *Give*, the programming language

Syntax

p ::= (Give p) | □

Examples

(Give □)

(Give (Give □))

(Give (Give (Give □))

Semantics

$$\frac{}{(\text{Give } p) \longrightarrow p}$$

$$\frac{p \longrightarrow p'}{(\text{Give } p) \longrightarrow (\text{Give } p')}$$

(1) Symbolic Structure : *Give*, the programming language

Syntax

p ::= (Give p) | □

Examples

(Give □) → □

(Give (Give □)) → (Give □)

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Semantics

$$\frac{}{(\text{Give } p) \longrightarrow p}$$

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(2) Encoding w/ TPRs : *Give*, the programming language

A TPR is a mapping, $[\![p]\!] : Give_4 \mapsto R^{4\times3}$, from a set of symbols to a vector space via filler/role decompositions. Here, $Give_4$ denotes the set of all Give programs up to length 4.

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$$Give_4 = \{ \square, (Give\ \square), (Give\ (Give\ \square)), (Give\ (Give\ (Give\ \square)))\}$$

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$$R^{4x3} = \left\{ \begin{matrix}[0\ 0\ 0]\\ [1\ 1\ 1]\\ [1\ 1\ 1]\\ [0\ 0\ 2]\end{matrix}, \begin{matrix}[1\ 2\ 0]\\ [0\ 1\ 0]\\ [1\ 0\ 1]\\ [2\ 0\ 1]\end{matrix}, \begin{matrix}[2\ 2\ 1]\\ [1\ 1\ 3]\\ [1\ 0\ 1]\\ [2\ 0\ 0]\end{matrix}, \ldots\ldots \right\}$$

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Fillers

$$f = \{ i_1, i_2, i_3, i_4 \}$$

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Fillers

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Roles

$$r = \{ Give, \square, \varepsilon \}$$

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Fillers

$f = \{ i_1 , i_2 , i_3 , i_4 \}$

Roles

r = { Give , □ , ε }

(Give (Give (Give □))) = (i_1:Give) $\wedge$ (i_2:Give) $\wedge$ (i_3:Give) $\wedge$ (i_4: □)

(2) Encoding w/ TPRs : *Give*, the programming language

A TPR is a mapping, $[\![p]\!] : \text{Give}_4 \mapsto R^{4\times3}$, from a set of symbols to a vector space via filler/role decompositions. Here, Give_4 denotes the set of all Give programs up to length 4.

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Fillers

$f = \{ i_1 , i_2 , i_3 , i_4 \}$

Roles

$r = \{ \text{Give} , \square , \varepsilon \}$

$$(\text{Give } (\text{Give } (\text{Give } \square))) = (i_1\text{:Give}) \wedge (i_2\text{:Give}) \wedge (i_3\text{:Give}) \wedge (i_4\text{:} \square)$$

$$(\text{Give } (\text{Give } \square)) = (i_1\text{: } \varepsilon\ ) \wedge (i_2\text{:Give}) \wedge (i_3\text{:Give}) \wedge (i_4\text{:} \square)$$

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Fillers

$$f = \{ i_1 , i_2 , i_3 , i_4 \}$$

Roles

$$r = \{ \text{Give} ,\ \square\ , \varepsilon \}$$

$$(\text{Give}\ (\text{Give}\ (\text{Give}\ \square\ ))) = (i_1{:}\text{Give}) \land (i_2{:}\text{Give}) \land (i_3{:}\text{Give}) \land (i_4{:}\ \square\ )$$

$$(\text{Give}\ (\text{Give}\ \square\ )) = (i_1{:}\ \varepsilon\ ) \land (i_2{:}\text{Give}) \land (i_3{:}\text{Give}) \land (i_4{:}\ \square\ )$$

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Fillers

$$f = \left\{ \underset{\mathbf{i_1}}{\begin{matrix}[1]\\ [0]\\ [0]\\ [0]\end{matrix}},\ \underset{\mathbf{i_2}}{\begin{matrix}[0]\\ [1]\\ [0]\\ [0]\end{matrix}},\ \underset{\mathbf{i_3}}{\begin{matrix}[0]\\ [0]\\ [1]\\ [0]\end{matrix}},\ \underset{\mathbf{i_4}}{\begin{matrix}[0]\\ [0]\\ [0]\\ [1]\end{matrix}} \right\}$$

Roles

$$r = \left\{ \overset{[1\ 0\ 0]}{\mathbf{Give}},\ \overset{[0\ 1\ 0]}{\square},\ \overset{[0\ 0\ 1]}{\boldsymbol{\varepsilon}} \right\}$$

$$[\![(Give\ (Give\ (Give\ \square)))]\!] = [\![(i_1{:}Give) \wedge (i_2{:}Give) \wedge (i_3{:}Give) \wedge (i_4{:}\square)]\!]$$

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Fillers

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Roles

$$r = \{ \overset{[1\ 0\ 0]}{\mathbf{Give}},\ \overset{[0\ 1\ 0]}{\square},\ \overset{[0\ 0\ 1]}{\boldsymbol{\varepsilon}} \}$$

$$[\![(\text{Give}\ (\text{Give}\ (\text{Give}\ \square)))]\!] = [\![(i_1{:}\text{Give}) \wedge (i_2{:}\text{Give}) \wedge (i_3{:}\text{Give}) \wedge (i_4{:}\ \square)]\!]$$

$$= (\mathbf{i_1} \otimes \mathbf{Give}) + (\mathbf{i_2} \otimes \mathbf{Give}) + (\mathbf{i_3} \otimes \mathbf{Give}) + (\mathbf{i_4} \otimes \square)$$

(2) Encoding w/ TPRs : *Give*, the programming language

A TPR is a mapping, $[\![p]\!] : \text{Give}_4 \mapsto R^{4\text{x}3}$, from a set of symbols to a vector space via filler/role decompositions. Here, Give_4 denotes the set of all Give programs up to length 4.

$$\text{Give}_4 = \{ \square, (\text{Give } \square), (\text{Give } (\text{Give } \square)), (\text{Give } (\text{Give } (\text{Give } \square)))\}$$

$$R^{4\text{x}3} = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix}, \ldots\ldots \right\}$$

Fillers

$$f = \left\{ \underset{\mathbf{i_1}}{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}, \underset{\mathbf{i_2}}{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}, \underset{\mathbf{i_3}}{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}, \underset{\mathbf{i_4}}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}} \right\}$$

Roles

$$r = \left\{ \overset{[1\ 0\ 0]}{\textbf{Give}}, \overset{[0\ 1\ 0]}{\square}, \overset{[0\ 0\ 1]}{\boldsymbol{\varepsilon}} \right\}$$

$$[\![(\text{Give } (\text{Give } (\text{Give } \square)))]\!] = [\![(i_1{:}\text{Give}) \wedge (i_2{:}\text{Give}) \wedge (i_3{:}\text{Give}) \wedge (i_4{:}\square)]\!]$$

$$= (\mathbf{i_1} \otimes \textbf{Give}) + (\mathbf{i_2} \otimes \textbf{Give}) + (\mathbf{i_3} \otimes \textbf{Give}) + (\mathbf{i_4} \otimes \square)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(3) Representation Proofs : *Give*, the programming language

Theorem. The following linear transformation is a representation of the instruction Give.

$$\mathbf{Give} : \sum_i \mathbf{f}_i \otimes \mathbf{r}_i \mapsto \sum_i \mathbf{f}_i \otimes \mathbf{r}_i$$

(3) Representation Proofs : *Give*, the programming language

Theorem. *The following linear transformation is a representation of the instruction Give.*

$$\mathbf{Give} : \textstyle\sum_i \mathbf{f}_i \otimes \mathbf{r}_i \mapsto \sum_i \mathbf{f}_i \otimes \mathbf{r}_i$$

Proof.
Recall that w/ TPRs we encode *Give* programs as conjunctions of filler/role decompositions, i.e. $[\![p]\!] = \sum_i \mathbf{f}_i \otimes \mathbf{r}_i$. Additionally, recall that: (Give p) $\rightarrow$ p

$$\begin{aligned} [\![(\text{Give } p)]\!] &= [\![p]\!] \\ &= \textstyle\sum_i \mathbf{f}_i \otimes \mathbf{r}_i \\ &= \mathbf{Give} \textstyle\sum_i \mathbf{f}_i \otimes \mathbf{r}_i \\ &= \mathbf{Give}\ [\![p]\!] \end{aligned}$$

$\square$

Discussion

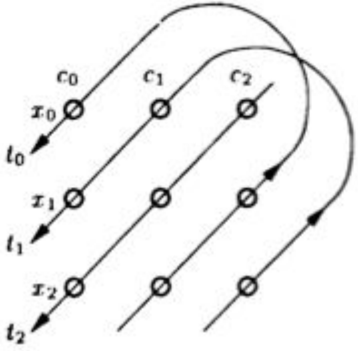
- How does this scale to larger programs in Give?
- What if our programming language was more complicated?
- Other thoughts…

Benefits & Shortcomings of Tensor Decomposition

- \+ No impositions on structure
- \+ Faithful
- \+ Variable binding
- \- Scaling up can be memory and compute demanding
    - Using ConceptNet as an example, ~4M nodes, ~40 relations might need to play around with pretty large tensors

Holographic Reduced Representations

- Use Circular Convolutions and Correlations to associate/disassociate vectors that represent structures
- Requires a reconstruction system to sort through the noise
- Circular Conv. and Circular Corr. can be manipulated to query structure



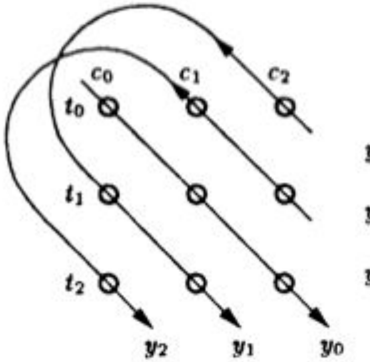
$$\mathbf{t} = \mathbf{c} \circledast \mathbf{x}$$

$$t_0 = c_0 x_0 + c_2 x_1 + c_1 x_2$$

$$t_1 = c_1 x_0 + c_0 x_1 + c_2 x_2$$

$$t_2 = c_2 x_0 + c_1 x_1 + c_0 x_2$$

Circular Convolution



$$\mathbf{y} = \mathbf{c} \bullet \mathbf{t}$$

$$y_0 = c_0 t_0 + c_1 t_1 + c_2 t_2$$

$$y_1 = c_2 t_0 + c_0 t_1 + c_1 t_2$$

$$y_2 = c_1 t_0 + c_2 t_1 + c_0 t_2$$

Circular Correlation (Inverse)

Representations with HRR

Sequences

$$\mathbf{s}_{abc} = \mathbf{a} + \mathbf{a} \circledast \mathbf{b} + \mathbf{a} \circledast \mathbf{b} \circledast \mathbf{c}$$

$$\mathbf{s}_{de} = \mathbf{d} + \mathbf{d} \circledast \mathbf{e}$$

$$\mathbf{s}_{fgh} = \mathbf{f} + \mathbf{f} \circledast \mathbf{g} + \mathbf{f} \circledast \mathbf{g} \circledast \mathbf{h}$$

S(abcdefgh)= $\mathbf{s}_{abc} + \mathbf{s}_{abc} \circledast \mathbf{s}_{de} + \mathbf{s}_{abc} \circledast \mathbf{s}_{de} \circledast \mathbf{s}_{fgh}$

Representations with HRR

Sequences

Variable Binding

$$\tilde{\mathbf{t}} = \tilde{\mathbf{x}} \circledast \tilde{\mathbf{a}} + \tilde{\mathbf{y}} \circledast \tilde{\mathbf{b}}.$$

Binding a to X and b to Y

Representations with HRR

Sequences

Variable Binding

Frame-Slots

Running frame: Spot runs

$$\mathbf{t}_{running} = \mathbf{l}_{run} + \mathbf{r}_{agent} \circledast \mathbf{f}_{spot}$$

Seeing frame: Dick saw Spot run

$$\begin{aligned}\mathbf{t}_{seeing} &= \mathbf{l}_{see} + \mathbf{r}_{agent} \circledast \mathbf{f}_{dick} + \mathbf{r}_{object} \circledast \mathbf{t}_{running} \\ &= \mathbf{l}_{see} + \mathbf{r}_{agent} \circledast \mathbf{f}_{dick} \\ &\quad + \mathbf{r}_{object} \circledast (\mathbf{l}_{run} + \mathbf{r}_{agent} \circledast \mathbf{f}_{spot})\end{aligned}$$

Example: Filling a frame

Frame:

Job Application:

- Name
- Date

Filler

- September 1, 2020

Example: Filling a frame

Frame:	Job Application	0.35	0.28	0.11
	Name	0.19	-0.14	0.02
	Date	-0.22	0.04	0.10
Filler:	September 1, 2020	0.06	0.05	-0.16

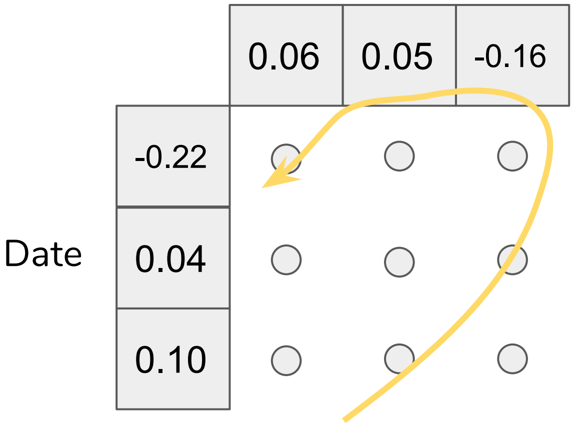
Example: Filling a frame

Binding Date & Filler

September 1, 2020

C[0]=0.1*0.05+-0.16*0.04+-0.22*0.06+=-0.0146

C: -0.0146



Example: Filling a frame

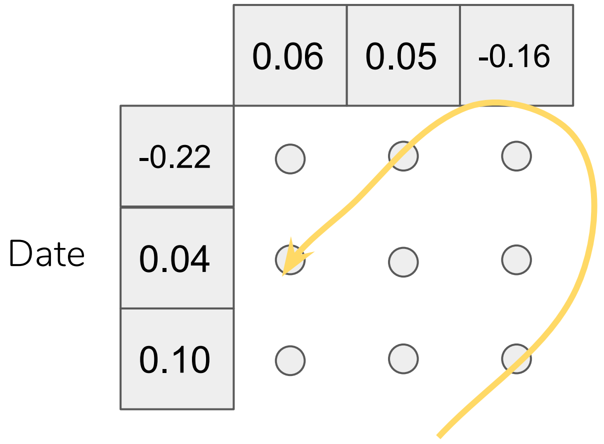
Binding Date & Filler

September 1, 2020

C[1]=0.1*-0.16+0.05*-0.22+0.04*0.06=-0.0246

C:

-0.0146	-0.0246

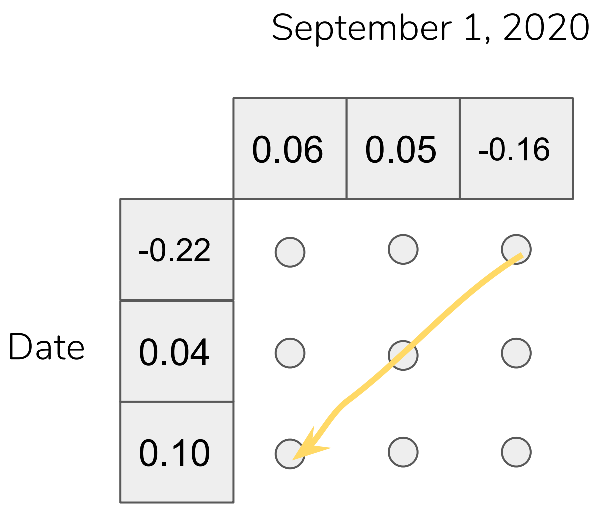


Example: Filling a frame

Pairing Job Application + Date Field

C[2]=0.1*0.06+0.05*0.04+-0.16*-0.22=0.0432

C:	-0.0146	-0.0246	0.0432

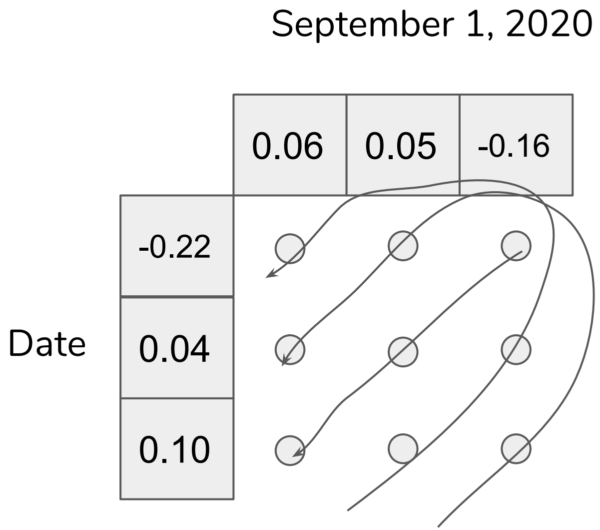


Example: Filling a frame

C:

-0.0146	-0.0246	0.0432

C: {Date:September 1,2020}



Example: Filling a frame

September 1, 2020

16

C:

C: {Jo

Add Name:
C: {Date: September 1, 2020}+ Name
C’:{Date: September 1, 2020, Name}

Example: Filling a frame

C’: {Date: September 1, 2020, Name}

Example: Filling a frame

C’: {Jo

Add in Frame Label

C’: {Date: September 1, 2020, Name}+ Job Application

C’’:{Job Application: Date: September 1, 2020, Name}

Example: Filling a frame

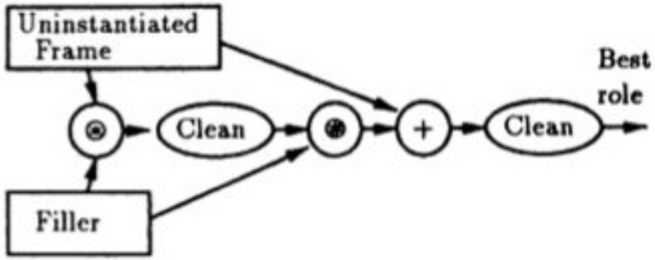
C'': {Job Application: Date:September 1,2020, Name}

Keep in mind representations are stored in a distributed manner
We used the "decoder" implicitly to clean the noise
Our representations are the result of an FFT

Example Application for Holographic Representation

Best role finder:

Job Application: Name, Date



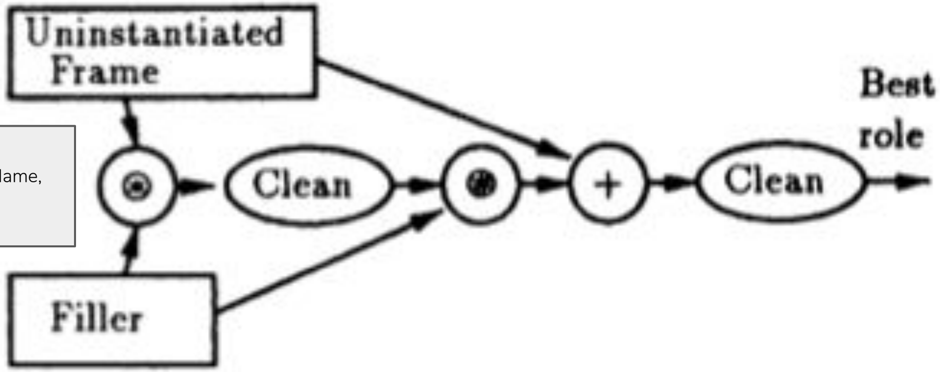
January, 1, 2021

Example Application for Holographic Representation

Best role finder:

Job Application: Name, Date

Job Application: Name, Date:January 1,

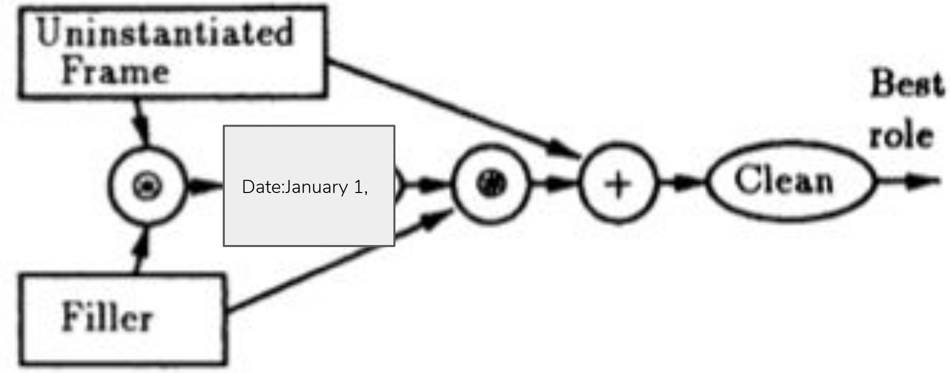


January, 1, 2021

Example Application for Holographic Representation

Best role finder:

Job Application: Name, Date

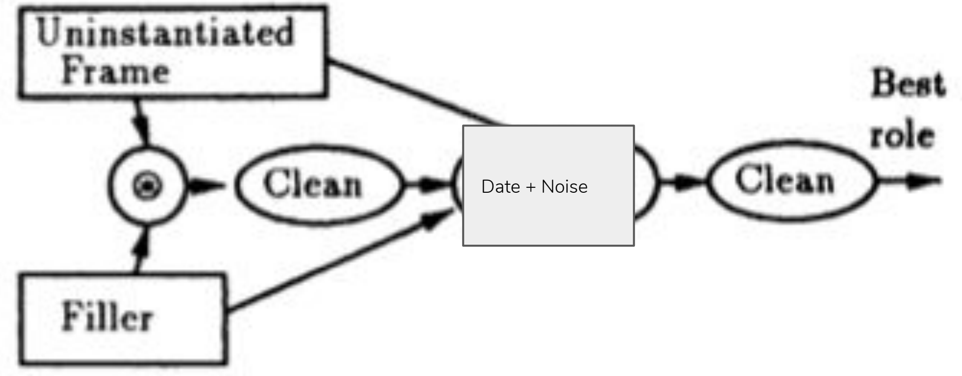


January, 1, 2021

Example Application for Holographic Representation

Best role finder:

Job Application: Name, Date

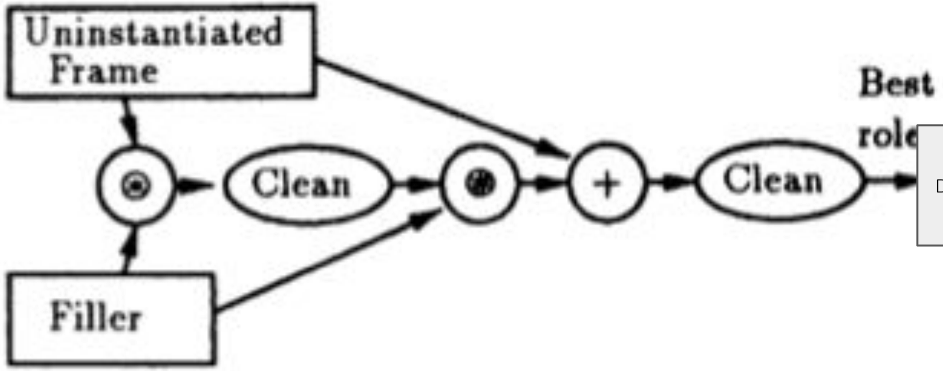


January, 1, 2021

Example Application for Holographic Representation

Best role finder:

Job Application: Name, Date



Date

January, 1, 2021

Benefits of Holographic Representations

- Format for the two input vectors is not specified, only independently distributed
- **Space Efficiency:** you just need the 2 vectors rather than the whole Tensor, result is the same size as the input
- Can be calculated in O(n log n) with FFT
- HRRs could retain ambiguity while processing ambiguous input (New York as City and as Name)
- Easy analysis of capacity, scaling and generalization

Shortcomings of Holographic Representations

- Decoder/cleaner must store all the possible outputs. If it knows everything, then why not find a way to exploit it?
- Is the decoder static? How would you add some new domain?
- Elements of each vector must be independently distributed, but have meaningful features
- Hit until you decode the correct thing?
- Some operations to decode require additional machinery (recursive)

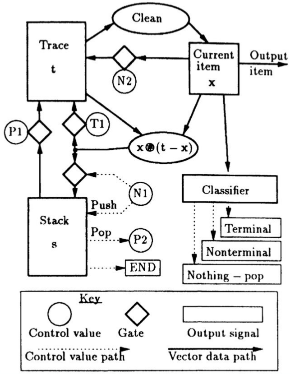
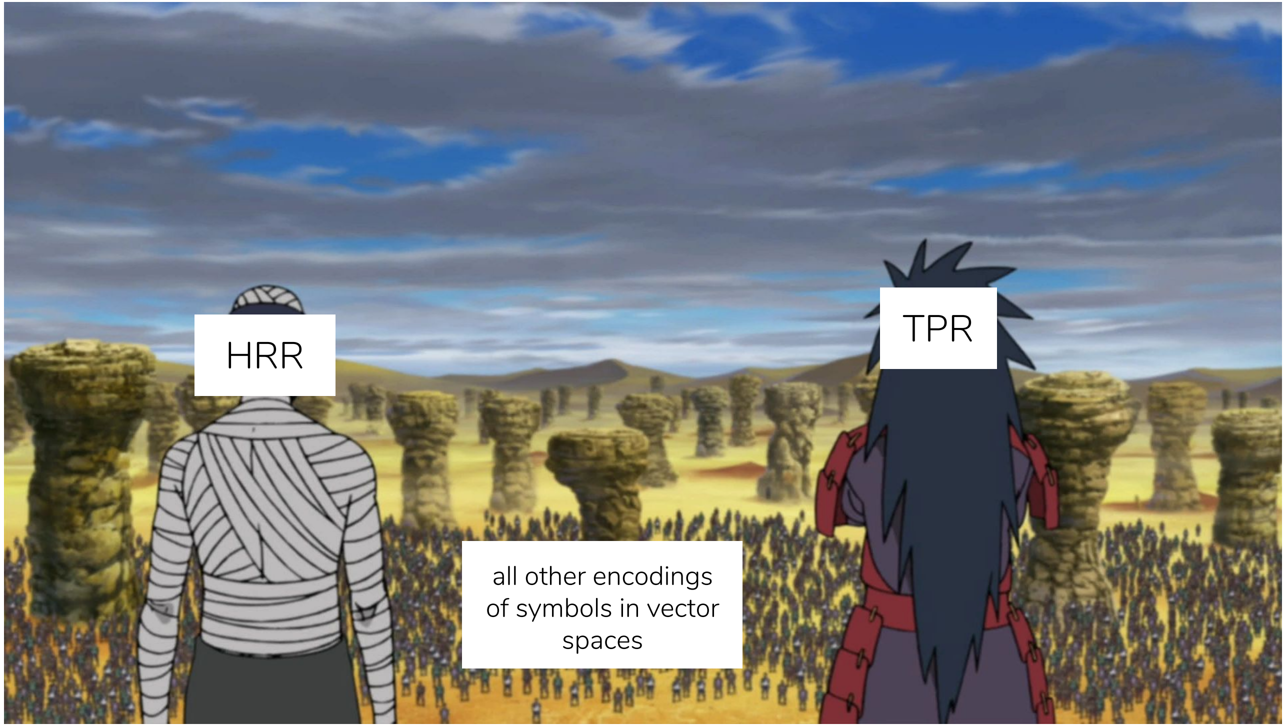


Figure 5: A chunked sequence readout machine.

Encoding Methods?

Encoding Methods?



[END]