Reserve Markets for Power Systems Reliability

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Abstract—This paper defines a price-based decision making process for participating in a reserve market for power systems reliability. Reserve power is a fundamentally different commodity from spot market power. It is suggested that depending on the payment mechanism in place, two different types of formulae would be used by power producers and users when participating in such markets. The paper points out that despite the imminent trend to create reserve markets, several fundamental questions concerning reliable operation must be studied.

Index Terms—Deregulation, Industry, Reserve, Transmission Congestion, unit commitment.

I. INTRODUCTION

Generally, a power producer is usually capable of producing the power that it sells. Unfortunately, in the real world, mechanical devices sometimes break down, and generators are no exception. It has been remarked that “it is not a question of whether or not a particular piece of equipment will fail, but rather when it will fail” [1]. It is widely observed in electricity restructuring debates that electricity is not storable, and consequently temporary production failures can not be covered by inventory, as is the case with most other commodities. Instead, it is necessary to have generation on the system operating at less than capacity, so that reserve power is readily available in case of a generator or line failure.

Under the present regulated utility structure, reserve generation is included in the utility’s unit commitment formulation; a utility must have enough excess reserve generation available for immediate use at all times so that if one generator or one line fails, all loads may still be served without interruption. This reserve requirement is known as the \((N - 1)\) criterion [2]. However, as the industry moves into deregulation, the responsibility for reserve is no longer well defined. Maintenance of the \((N - 1)\) criterion entails a significant cost; some users would like to avoid this cost, while others depend on highly reliable service without interruption. Generators, loads, and the Independent System Operator (ISO) all have the ability to purchase and/or sell reserve generation in a deregulated marketplace; in general, the deployment of generators for spinning reserve will no longer be centrally controlled.

For a power producer, reliability poses two main questions. The first question is the provision of backup for the power that is sold to loads. The second question is to determine how much power should be sold on the spot market and how much generation should be held in reserve in order to maximize the expected profit. The exact nature of the provision of generation reserve is not clear at the time of this writing; however, we will use a generalized formulation that incorporates many possible forms. In particular, we will assume that a generator owner can trade reserve generation with other market participants.

This paper is organized as follows: Section II discusses the need for a reserve market and fundamental questions about its formation. Section III describes the general form that a market for reserve will take. Two different interpretations of the price of reserve are discussed. Section IV considers the second question above (determination of optimal selling strategy for a power producer) under both possible reserve payment methods. Loads may wish to purchase interruptible power contracts [3] as described in Section V; these contracts offer an alternative to paying the cost of reserve. Section VI returns to the first question above for producers (provision of reserve for transactions) and describes several potential solutions under deregulation. In particular, an area-wide market for reserve is examined in some detail; such a market may be expected to provide reserve generation for the system at minimal price. Section VII examines the unit commitment strategy of a power producer in the presence of a reserve market. A stochastic unit commitment formulation in [4] and [5] is used for optimal decision-making. A numerical example of unit commitment in a reserve market is shown in Section VIII to illustrate how a power producer can maximize profits while selling in both the spot and reserve markets. Section IX discusses the challenge of treating generator reliability and transmission line reliability as separate items. Finally, conclusions are summarized in Section X.

II. BASIC NEED FOR A RESERVE MARKET

In today’s industry, a significant portion of the average cost of electricity can be directly traced to the requirement to meet the \((N - 1)\) reliability criterion unconditionally. In the competitive industry, however, two striking changes are taking place which may lead to deviations from this operating concept. First, many power producers will sell power to users through bilateral contracts. To protect against failures of their own generators, these producers may purchase reserve power from other power producers. In this way, no supply/demand imbalance results if such producers experience a generator outage [6]. Second, many customers may be willing to adjust their use of electricity under unusual system conditions (partial interruption) at a price [3]. This situation creates a definite need to revisit the value of reserves and not necessarily require a uniform percentage of reserve by all power plants. This paper introduces basic decision making by power producers and users for participating in a reserve market.

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The paper describes strictly market aspects (that is, price evolution) in such a market, and contrasts it with the price process in the spot market.

However, if such a market-based reserve is to be pursued, as advocated by many, at least two open questions must be carefully studied:

1. The effect of market-based supply/demand reserve provision on the transmission grid flow patterns. As very large plants are being replaced by smaller, distributed plants, large power transfers may be reduced; yet, if the outage of a plant is to be replaced by a generator far away, this situation may create new flow patterns for which the grid was not designed. All issues on this topic point in the direction of separating generator reliability problems from the reliability problems created by transmission line outages.  

2. The conceptual need for uniformly dispersed reserves. It may be questioned whether this need is simply due to transmission bottlenecks, or if other problems may arise.

III. GENERAL FORM OF A RESERVE MARKET

Reserve power is a fundamentally different commodity from spot market power. While power traded on the spot market is scheduled in advance of its use, reserve power is power available on-line for immediate use should a system contingency (generator or transmission line failure) occur. A market for reserve will operate concurrently with the spot market for power, although the reserve price \( p_R \) will be different from the spot price \( p \). Like any other market, the reserve price reflects an equilibrium point between the supply and demand. The supply for reserve comes from generators, who also supply the spot market for electricity. The demand for reserve can come from any number of sources, depending on the exact nature of reliability maintenance in the market. An ISO may calculate and purchase all of the reserve needed for the system area in order to maintain a minimum standard. Alternatively, groups of generators may contract with each other to provide reserve for each other's transactions; in this case, a power producer is both supplying and demanding reserve. Loads may wish to buy reserve for their power. Reserve brokers may develop in the marketplace to purchase reserve power for their customers, who may be loads and/or generators.

The reserve price can either be higher than or lower than the spot price, depending on whether reserve payments are made for actual power delivered or for power that is merely reserved.

A. Payment for Power Delivered

In this scenario, a generator which sells power as reserve is paid the reserve price for that reserve only if the reserve power is actually used. The reserve price is therefore higher than the spot price, since excess generation capacity has a per unit cost that is higher than the spot price. (Generation with a lower marginal cost is sold for profit on the spot market.) In this case, a generator receives a profit on sales of reserve only for the time periods when the reserve actually needs to be generated. The generator receives zero payment if the reserve is not called.

B. Payment for Reserve Allocated

In this payment method, a generator receives the reserve price per unit of reserve power for every time period that the reserve is allocated and not used. If the reserve is used, then the generator receives the spot price for the reserve power that is generated. Since the reserve is not generated most of the time (hopefully!), reserve power has a very low expected cost, and hence the price of reserve will be much lower than the spot price of power. A generator receives a small profit for each time period in which the reserve is sold but not used; however, the generator will absorb a loss if the reserve is called. The reserve price \( p_R \) will be high enough such that the generator expects an overall long-term profit; otherwise, no reserve would be offered for sale.

C. Price Process for Reserve Price

Unfortunately, since there are no existing reserve markets, there is no empirical data available for building a price process model. However, it is clear that the reserve price and the spot price must be strongly correlated, since the quantity traded on the spot market largely defines the demand for reserve power.

IV. POWER PRODUCER STRATEGIES FOR SELLING RESERVE

If a power producer is able to sell power into a reserve market, then the producer's strategies for profit maximization in both the spot and reserve markets are intertwined. The producer decides to sell \( P_{G(S)} \) in the spot market and \( P_{G(R)} \) in the reserve market. The exact determination of \( P_{G(S)} \) and \( P_{G(R)} \) depends on the way reserve payments are made, although the results are very similar.

A. Payment for Power Delivered

For this payment method, \( p < p_R \). For a producer with a single generator, we model the producer's cost of generating \( P \) units of power as a quadratic function:

\[
av(P) = aP^2 + bP + c.
\]

The generation level is constrained by fixed limits:

\[
P_{G}^{\text{min}} \leq P_{G} \leq P_{G}^{\text{max}}.
\]

During a given time period with known prices, the profit for a power producer is a random function with expectation:

\[
\mathbb{E}[\pi_G] = pP_{G(S)} + rP_{G(R)}P_{G(S)} - (1-r)(aP_{G(S)}^2 + bP_{G(S)} + c) - r(aP_{G(R)}^2 + bP_{G(R)} + c).
\]

Here \( P_{G(S)} = P_{G(S)} \) and \( P_{G(R)} = P_{G(R)} \) and \( r \) is the probability that the reserve power is called and generated. A producer will choose \( P_{G(S)} \) and \( P_{G(R)} \) to maximize (3); these values are:

\[
p - r p_R = 2 a P_{G(S)} + b \quad \text{and} \quad p_R = 2 a P_{G(R)} + b.
\]

1As the generation and transmission business unbundle, this becomes imminent.
These equations are easy to interpret. Equation (4) indicates that power is sold on the spot market until the marginal cost of power equals an adjusted version of the spot price \( p \). The adjustment reflects the fact that the marginal units of power have very little profit and would be more profitable on average if they are sold in the reserve market at the higher reserve price \( p_R \). Since \( r \) is typically very small, the adjustment to \( p \) will also be very small. Equation (5) means that the power producer will sell reserve until the marginal cost reaches the price of reserve.

Both \( P_G(S) \) and \( P_G(T) \) must fall between the upper and lower generation limits. Since the derivatives of profit are monotonically decreasing, if (4) yields a value of \( P_G(S) \) that is less than \( P_G^{\text{min}} \), the optimal choice is \( P_G(S) = P_G^{\text{min}} \). Similarly, if \( P_G(S) \) is calculated to be greater than \( P_G^{\text{max}} \), then \( P_G(S) = P_G^{\text{max}} \). The same is true for \( P_G(T) \). Mathematically, this relationship may be written using the marginal cost limits defined in [4] and [5]:

\[
P_G^{\text{min}} = 2aP_G^{\text{min}} + b
\]

\[
P_G^{\text{max}} = 2aP_G^{\text{max}} + b.
\]

By further defining \( p_{\text{eff}} \), the “effective” price for spot market sales:

\[
p_{\text{eff}} = \frac{p - p_R}{1 - r}
\]

the prices \( p_{\text{eff}} \) and \( p_R \) may be written as truncated random variables [4], [5]:

\[
P(MG_{\text{eff}}) = \begin{cases} 
  P_{\text{min}}^{\text{MC}}, & \text{if } p_{\text{eff}} \leq P_G^{\text{min}} \\
  \frac{P_{\text{eff}}}{P_{\text{eff}}^\text{MC}}, & \text{if } P_G^{\text{min}} < p_{\text{eff}} < P_G^{\text{max}} \\
  P_{\text{max}}^{\text{MC}}, & \text{if } P_{\text{eff}} \geq P_G^{\text{max}}
\end{cases}
\]

\[
P(MG_{\text{R}}) = \begin{cases} 
  P_{\text{min}}^{\text{MC}}, & \text{if } P_R \leq P_G^{\text{min}} \\
  \frac{P_R}{P_R^\text{MC}}, & \text{if } P_G^{\text{min}} < P_R < P_G^{\text{max}} \\
  P_{\text{max}}^{\text{MC}}, & \text{if } P_R \geq P_G^{\text{max}}
\end{cases}
\]

Using this notation, the optimal \( P_G(S) \) and \( P_G(T) \) are:

\[
P_G(S) = \frac{P(MG_{\text{eff}}) - b}{2a}
\]

\[
P_G(T) = \frac{P(MG_{\text{R}}) - b}{2a}
\]

The expected profit in stage \( k \) is given by:

\[
E\{\pi_G\} = (1 - r)(p_{\text{eff}}P_G(S) - (aP_G^2 + bP_G + c))
+ r(p_RP_G(T) - (aP_G^2 + bP_G + c)).
\]

\[
E\{\pi_G\} = p_PG(S) + (1 - r)p_R(G(T) - P_G(S))
- (1 - r)(aP_G^2 + bP_G + c)
- r(aP_G^2 + bP_G + c).
\]

\[
E\{\pi_G\} = p_P(G(S) + (1 - r)p_R(G(T) - P_G(S))
- (1 - r)(aP_G^2 + bP_G + c)
- r(aP_G^2 + bP_G + c).
\]

The strategy for maximizing (14) is:

\[
p - p_R = 2aP_G(S) + b
\]

\[
p + (r^{-1} - 1)p_R = 2aP_G(T) + b.
\]

Equation (15) indicates that the marginal cost of power sold on the spot market should be reduced by the reserve price, in comparison to the original formulation without a reserve market. One disadvantage of this payment method can be observed from the optimal strategy for a power producer given by (16). The amount of reserve offered for sale is highly sensitive to \( r \), because of the dependence on \( r^{-1} \).

Generation limits are handled by the same procedure used in the preceding section. First, we define the following “effective” prices, which determine the optimal marginal cost:

\[
p_{\text{eff}}(S) = p - p_R
\]

\[
p_{\text{eff}}(R) = p + (r^{-1} - 1)p_R.
\]

The corresponding truncated random variables, using the limits of (6) and (7), are:

\[
P(MG_{\text{eff}}) = \begin{cases} 
  P_{\text{min}}^{\text{MC}}, & \text{if } p_{\text{eff}} \leq P_G^{\text{min}} \\
  \frac{p_{\text{eff}}}{P_{\text{eff}}^\text{MC}}, & \text{if } P_G^{\text{min}} < p_{\text{eff}} < P_G^{\text{max}} \\
  P_{\text{max}}^{\text{MC}}, & \text{if } p_{\text{eff}} \geq P_G^{\text{max}}
\end{cases}
\]

\[
P(MG_{\text{R}}) = \begin{cases} 
  P_{\text{min}}^{\text{MC}}, & \text{if } P_R \leq P_G^{\text{min}} \\
  \frac{P_R}{P_R^\text{MC}}, & \text{if } P_G^{\text{min}} < P_R < P_G^{\text{max}} \\
  P_{\text{max}}^{\text{MC}}, & \text{if } P_R \geq P_G^{\text{max}}
\end{cases}
\]

Using this notation, the optimal \( P_G(S) \) and \( P_G(T) \) are:

\[
P_G(S) = \frac{P(MG_{\text{eff}}) - b}{2a}
\]

\[
P_G(T) = \frac{P(MG_{\text{R}}) - b}{2a}
\]

The expected profit in stage \( k \) is given by:

\[
E\{\pi_G\} = (1 - r)(p_{\text{eff}}P_G(S) - (aP_G^2 + bP_G + c))
+ r(p_{\text{eff}}P_G(T) - (aP_G^2 + bP_G + c)).
\]

\[
E\{\pi_G\} = p_PG(S) + (1 - r)(p_R(G(T) - P_G(S))
- (1 - r)(aP_G^2 + bP_G + c)
- r(aP_G^2 + bP_G + c).
\]

V. INTERRUPTIBLE SERVICE CONTRACTS

At present, reserve requirements are based on the \((N - 1)\) criterion, which means that there must be sufficient reserve on the system such that no load will lose service if any one line or any one generator fails [2]. Our formulation allows for the possibility of customer choice of interruptible service for a reduced rate. In this scenario, a customer chooses service with a given reliability \( p \) for a given price \( p \) [3]. A discrete number of contracts are available, including an option for maximum reliability. If all customers choose the maximum reliability, the problem will be the same as the current \((N - 1)\) criterion; otherwise, the utility
will be allowed to drop some loads in the event of a component failure. The rationing of a load is associated with a contingency that is at least as severe as a minimum contingency level specified in the service contract. To compensate the customer for loss of service, it is assumed that the utility makes an insurance payment \( f \) to the customer for loss of service, for which the customer regularly pays a premium \( (1 - p) f \) \cite{3}.

The formulation of reliability levels in \cite{3} presumes that the total supply available takes on discrete values with known probabilities. Given a set of generators with maximum generating limits and failure probabilities, a set of contracts with known reliabilities can be obtained. One method is to consider every possible combination of at most \( I \) generator failures. The probabilities of each such combination may be used to devise a set of contracts.

VI. PROVISION OF RESERVE FOR TRANSACTIONS

We now return to the first question regarding reserve posed earlier: How is reserve provided for power sold on the open market from generators to loads? The responsibility for provision of generation reserve can rest either with the loads or with the generators, although the end result (price paid by loads) will likely be the same in either case. Reserve can also be provided by the ISO as a system service, with its cost included in the charge for system use. If the load is responsible for reserve, the load has flexibility to precisely determine a desired tradeoff between reliability and price. A group of loads may collectively purchase a block of reserve under a joint agreement; purchases through a reserve broker have a similar net result. A load may also choose to be fully or partially interruptible and thus avoid or reduce the cost of reserve.

If generators are responsible for providing reserve, then they can form collective agreements in a similar fashion to loads as described above, either through negotiation or through a reserve broker. The price paid by loads for power will be somewhat higher than if loads are responsible for reserve, with the difference reflecting the cost of reserve. Loads that choose interruptible power will pay a lower price.

The development of an area-wide market for reserve has an advantage of offering a lower price than if bilateral reserve agreements are made. This concept is best illustrated by an example in which reserve payments are for power that is generated. If generator \( A \) has a 500 MW contract and needs backup for this contract, then in order to induce generator \( B \) to sell reserve, \( A \) would need to offer a reserve price equal to \( B \)'s marginal generation cost at the generation level of both spot power for \( B \)'s sales and all 500 MW of \( A \)'s reserve power. However, in a reserve market, the 500 MW reserve can be spread across all generators in the area, which means that \( B \) might only offer 100 MW in the reserve market. In this case, the marginal cost of generating the 100 MW of reserve is clearly less than the cost of generating all 500 MW of reserve, leading to a lower price for reserve. If the total reserve offered among all generators in the area exceeds the largest amount of power sold on the spot market by any one generator, then the \( (N - 1) \) contingency criterion will be satisfied, assuming that transmission constraints are not a factor.

VII. EFFECT OF RESERVE MARKET ON UNIT COMMITMENT

Unit commitment is the process of selecting whether a given generator on the system should be running at a given hour. Since most generators on the grid will have different owners in the deregulated utility industry, unit commitment strategies will be done independently by different power producers instead of being coordinated on the system level. A generator owner will make unit commitment decisions in response to price so that the expected profit over many hours is maximized. Unit commitment therefore becomes a stochastic optimization problem, which may be formulated and solved as in \cite{4} and \cite{5}.

The inclusion of a reserve market has two principal effects on the unit commitment algorithm. First, the ability to sell reserve power affects the profit maximization strategy, as shown earlier in Section IV, and therefore the expected one-stage profit is also changed. The correlation between reserve calls and prices may need to be included in the expected cost; a higher price may imply a higher value of \( r \). Second, the responsibility for reserve and the possibility of generator failure (a major reason for having reserve in the first place) should also be included in the expected profit. Note that both factors do not change the available unit commitment options; therefore, the unit commitment algorithm can be modified to account for the reserve market by adjusting the expected one-stage cost, and if necessary, adding another continuous state variable, which is the price of reserve.

Since reserve market sales are added to the problem formulation, the reliability of the producer's own generator should also be considered. If a single generator, with failure probability \( f \), experiences a failure and is unable to produce the power it sold, it must buy that power from the reserve market at the reserve price \( p_R \) (assuming that reserve payments are for power delivered). In this case, the expected profit per stage should be adjusted according to:

\[
E\{\pi_G\} = (1 - f)E\{\pi_G\} + f(p_R - p_R)P_{G(S)} \tag{24}
\]

where \( a_G \) is a zero-one variable indicating whether the generator has failed. The optimal \( P_{G(S)} \) also needs to be adjusted, according to:

\[
\frac{\partial E\{\pi_G\}}{\partial P_{G(S)}} = \frac{f(p_R - p)}{1 - f} \tag{25}
\]

If \( f \) is small, then the effects of a power producer's generator failure are also small and may be neglected. The assumption that a reserve call is generated if any other generator in the system area goes out implies \( f \ll r \). The exact form of (24) for a specific situation may have many possibilities, depending on the form of the reserve market.

VIII. EXAMPLE

The following hypothetical example, derived from the example in \cite{4}, illustrates a possible reserve market unit commitment strategy. The reserve price will be modeled as:

\[
p_{Rk} = K_R + e_{ik} + \ln p_k \tag{26}
\]

\( p_k \) and \( p_{Rk} \) are the spot and reserve prices, respectively, at hour \( k \). \( K_R \) is a constant, while \( e_{ik} \) is normally distributed with zero
TABLE I
OPTIMAL DECISION AND EXPECTED PROFIT FOR EACH STATE AT 10:00 PM
WITH RESERVE MARKET

<table>
<thead>
<tr>
<th>Status</th>
<th>Expected Profit</th>
<th>Optimal Decision</th>
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<tr>
<td>Off since 8:00 PM</td>
<td>404.62</td>
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</tr>
</tbody>
</table>

mean for this numerical example, \( K_R = 0.7 \) and \( \text{var}(e_R) = 0.0625 \). Reserve payments are made for power delivered. The probability of reserve calls will be taken as \( r = 5 \times 10^{-3} \), independent of the price. The failure probability of the generator is \( f = 1 \times 10^{-4} \). To simplify the problem, we use the approximation \( p_{\text{eff}} \approx p_k \). With this approximation, the expected profit for hour \( k \), if the generator is running, is:

\[
E \{ \pi_k \} = (1 - f) \left[ (1 - r) E_{p_k, p_{\text{eff}}} \{ \pi_k(n) \} + r \sum_{p_k, p_{\text{eff}}} E \{ \pi_k(r) \} \right] + f \sum_{p_k, p_{\text{eff}}} E \{ \pi_k(f) \} \quad (27)
\]

\[
\pi_k(n) = \frac{p_k p_k(MC_{\text{eff}})k - b p_k}{2a} - \frac{p_k^2 (MC_{\text{eff}})k}{4a} - c \\
\pi_k(r) = \frac{p_k p_k(MC_{\text{eff}})k - b p_{\text{eff}}}{2a} - \frac{p_k^2 (MC_{\text{eff}})k}{4a} - c \\
\pi_k(f) = (p_k - p_{\text{eff}}) \frac{p_k(MC_{\text{eff}})k - b}{2a} \quad (30)
\]

Since \( p_k \) and \( e_R \) are independent, the expectation of the product \( p_k p_k(MC_{\text{eff}})k \) may be determined by:

\[
E_{p_k, p_{\text{eff}}} \{ p_k p_k(MC_{\text{eff}})k \} = e^{K_R} E_{p_k} \{ p_k p_k(MC_{\text{eff}})k \} E_{e_R} \{ e_R \} \quad (31)
\]

TABLE II
OPTIMAL DECISION AND EXPECTED PROFIT FOR EACH STATE AT 11:00 PM
WITH RESERVE MARKET

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TABLE III
OPTIMAL DECISION AND EXPECTED PROFIT FOR EACH STATE AT 10:00 PM
WITHOUT RESERVE MARKET

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TABLE IV
OPTIMAL DECISION AND EXPECTED PROFIT FOR EACH STATE AT 11:00 PM
WITHOUT RESERVE MARKET

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IX. CHALLENGE TO THE TRANSMISSION SERVICE PROVIDER

Swapping responsibilities to meet supply and demand has a potentially serious effect on the capabilities of the grid. Transmission bottlenecks in a system with a required percentage of reserve by all suppliers have completely different effects when compared to transmission bottlenecks in a system with an active reserve market. Supply and demand reliability described in this paper is at a price and not unconditional. The transmission grid still has to unconditionally remain intact and be a fully connected system. This paper suggests separating supply reliability from transmission reliability. More work is needed in this area.

X. CONCLUSIONS

This paper examines various means for the provision of reserve generation in deregulated electricity markets. It is shown that reserve may be bought and sold, and it is a separate commodity from power in the spot market. As the number of generators participating in the reserve market increases in a given area, the price of reserve in that area drops. Interruptible loads have the same net effect as reserve generation; they provide a means of maintaining the balance of supply and demand in the event of a failure in the system. Finally, an example is presented illustrating how power producers make decisions when having the option of selling in both the spot power market and the reserve market. The price of reserve at equilibrium will balance the supply and demand of reserve, thus setting the most economical quantity of backup generation for the system.

ACKNOWLEDGMENT

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