

# A possible notion of short-term value-based reliability

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*Abstract—*

In this paper we develop a possible notion of short-term value-based reliability for operating an electric power network. As the electric power industry goes through the restructuring process, the operation based on the value-based reliability may be more appropriate than based on the traditional  $(N-1)$  security criteria. The value-based reliability requires a framework where the factors to be considered include not only the cost of energy and the cost of generation reserve (and interruptible load contracts) for anticipated contingencies, but also the cost of generation adjustment, and the cost of demand interruption following each actual contingency. In this framework each contingency needs to be evaluated as a stochastic dynamic process since the effect of a contingency depends on the unpredictable time of its occurrence. It is shown that the new notion developed here can be useful in analyzing the performance of already functioning market or in designing unbundled markets for energy, reserve and adjustment.

*Keywords—*

Value-based reliability, Short-term operation, Uncertainty, Contingency,  $(N-1)$  security criteria

## I. INTRODUCTION

As the electric power industry goes through the restructuring process, there are many technical challenges related to designing new markets for electricity and/or to analyzing the performance of already established markets. This requires a careful consideration of tradeoffs between the overall operating cost and the level of reliability. Included in the overall operating cost are the cost of energy for meeting the expected load demand, and the cost of reserve for preparing for anticipated contingencies and the cost of restoring the system following an occurrence of actual contingency. One should keep in mind that the operation of a system is carried out such that only up to some level of reliability is assured while allowing load interruptions under certain unfavorable system conditions.

To start with understanding a tradeoff between the cost of energy and the cost of reserve already poses a difficult technical challenge for meeting a given level of desired reliability due to the wide range of generator characteristics [1]. Understanding a trade-

off between the overall operating cost and the level of reliability propounds an even more difficult and important technical challenge. In order to address the problem of understanding the latter tradeoff, one must, at the minimum, be able to first, determine the cost of energy and the cost of reserve, under various anticipated uncertainties in load demand, in equipment availability and in the interruption cost, over an appropriate geographical area and a suitable time horizon, and then model the relationship among these costs and uncertainties. Defining the notion of short-term value-based reliability leading to an efficient operating condition is directly related to this latter technical challenge.

In many transitional markets in the United States there are two distinctive features observed that are very important to the study of a tradeoff between the overall operating cost and the level of reliability.

One feature is related to the reliability standards used for determining the level of desired reliability being inherited from the traditional method for operation and planning by a vertically integrated utility. These reliability standards include the loss of load probability (LOLP) for longer term planning purposes and the  $(N-1)$  (security) criteria for shorter term operational purposes.

By measuring the probability of load demand exceeding the available resources, LOLP verifies the adequacy of generation and transmission resources. In case the measured LOLP falls below the desired reliability level, market incentives need to be created in order to encourage generation and transmission expansion while discouraging load growth [3].

By requiring enough resource to be set aside to replace the loss of one equipment in the system, the  $(N-1)$  criteria attempt to assure that there is no interruption in providing electricity service even under a single equipment outage.

The second feature is related to the market rules favoring separate processes for determining the cost of energy and the cost of reserve. In conducting various electricity markets, the supply and the demand in energy markets are cleared first irrespective of the

activities in reserve markets. Then, while the amount of reserve requirement depends on the result of energy market activities, there is very little effect of the previously conducted energy market on the market clearing process of the subsequent reserve markets.

In this paper, the impact of two distinctive features described above on the tradeoff between the overall operating cost and the level of reliability is studied. The main objective is developing a possible mathematical formulation for computing an efficient operating point for managing an electric power system based on a cost-benefit analysis on the effect of contingencies.

The paper is organized as follows:

In Section II we examine the use of  $(N - 1)$  criteria of the regulated industry structure in the new environment of competition and market mechanisms using the cost-benefit analysis. Based on this analysis of  $(N - 1)$  criteria, Section III presents a proposed mathematical formulation for computing an efficient operating point. The proposed mathematical formulation makes clear what the overall operating cost is and what the level of reliability achieved is. In Section IV an approximation method, often referred to as the *ordinal optimization* (OO) method, is proposed to manage a considerable computation complexity required for obtaining value-based reliability decisions. In Section V some concluding remarks are presented.

## II. COST-BENEFIT ANALYSIS OF $(N - 1)$ CRITERIA

Suppose we emulate implementing the (simplified version of) operational scheme currently being employed in some markets in the Northeast region of US and determine the generation dispatch schedule and capacity reservation schedule. Implied is the poolco type market structure where an Independent System Operator (ISO) conducts an hourly market<sup>1</sup> to meet the forecasted load demand while satisfying contingency criteria. Denote the forecasted load demand for the upcoming hour at node  $d_j$  as  $Q_{d_j}$ .<sup>2</sup>

At the beginning of each hour, the ISO, first, collects the bid price,  $S_{g_i}^e$  from each supplier at node  $g_i$ , participating in the energy market, for its respective energy produced,  $Q_{g_i}^e$ . Then, the generation dispatch schedule for that hour is determined by solving the optimization problem given in Eqs. (1) - (4), often

<sup>1</sup>We ignore so-called unit commitment problem in this paper, which may require an analysis of inter-hourly markets for satisfying optimality conditions [1].

<sup>2</sup>For simplicity we assume that the random deviation of load from its forecasted demand around zero mean is continually compensated by the effort by automated generation control (AGC) devices, and no other random deviation of load around non-zero mean exists.

referred to as *Optimal Power Flow* (OPF) problem or *Location Based Marginal Price* (LBMP) problem so that the system-wide energy cost paid is minimized.

$$Q_{g_i}^{e,*} = \arg \min_{Q_{g_i}^e} \sum_{g_i} Q_{g_i}^e \cdot S_{g_i}^e \quad (1)$$

subject to the load balance constraint

$$\sum_{g_i} Q_{g_i}^e = \sum_{d_j} Q_{d_j} \quad (2)$$

the maximum generation limit

$$Q_{g_i}^e \leq Q_{g_i}^{\max} \quad (3)$$

and the maximum flow constraint

$$\|F_l(Q_{g_i}^e, Q_{d_j})\| \leq F_l^{\max} \quad (4)$$

where  $F_l$  is a real power flow on line  $l$  as a function of generation and load injections to the system. It is assumed that as long as the flow on line  $l$  remains less than or equal to the maximum allowed flow,  $F_l^{\max}$ , no transient stability problem exists in the system.

Still at the beginning of the hour, after solving the OPF problem, the ISO collects the bid price,  $S_{g_i}^c$  from each supplier at node  $g_i$  participating in the reserve market<sup>3</sup> for capacity reserved,  $Q_{g_i}^c$ , and determines capacity reservation by solving the optimization problem given in Eq. (5) so that the system-wide capacity cost paid is minimized.

$$Q_{g_i}^{c,*} = \arg \min_{Q_{g_i}^c} \sum_{g_i} Q_{g_i}^c \cdot S_{g_i}^c \quad (5)$$

subject to the maximum generation limit

$$Q_{g_i}^{e,*} + Q_{g_i}^c \leq Q_{g_i}^{\max} \quad (6)$$

and the  $(N - 1)$  criteria

$$\sum_{g_i} Q_{g_i}^c = Q_{g_i}^{e1,*} + \frac{1}{2} Q_{g_i}^{e2,*} \quad (7)$$

where  $Q_{g_i}^{e1,*}$  and  $Q_{g_i}^{e2,*}$  denote the largest and the second largest dispatched units, respectively, as a result of solving the OPF problem.

During the hour, the ISO may adjust production of generators from the reserved capacity following a contingency so that the interruption of electricity services can be prevented. For example, in the case of a generator outage, the ISO increases the production from reserve capacity to match the loss of generation

<sup>3</sup>We include the case where a supplier in the reserve market may be a load offering an interruptible contract.

from the outage so that the supply and the demand continue to balance. When portion of reserved capacity is used for adjusting generation, the respective generators need to be compensated according to some prior agreement,  $S_{g_i}^a$ . Thus, the ISO solves the optimization problem in Eq. (8) following  $i$ th contingency so that the system-wide adjustment cost paid is minimized,

$$Q_{g_i}^{a,*}(t_i) = \arg \min_{Q_{g_i}^a(t_i)} \sum_{g_i} Q_{g_i}^a(t_i) \cdot S_{g_i}^a \cdot \left(1 - \frac{t_i}{3600}\right) \quad (8)$$

where  $t_i$  is the time in seconds at which the  $i$ th contingency occurs, subject to the load balance constraint

$$\sum_{g_i} Q_{g_i}^{e,*} + \sum_{t_j=t_1}^{t_{i-1}} Q_{g_i}^{a,*}(t_j) + Q_{g_i}^a(t_i) = \sum_{d_j} Q_{d_j} \quad (9)$$

the reserved capacity constraint<sup>4</sup>

$$\sum_{t_j=t_1}^{t_{i-1}} Q_{g_i}^{a,*}(t_j) + Q_{g_i}^a(t_i) \leq Q_{g_i}^{c,*} \quad (11)$$

and the maximum flow constraint

$$\left\| F_l \left( Q_{g_i}^e + \sum_{t_j=t_1}^{t_{i-1}} Q_{g_i}^{a,*}(t_j) + Q_{g_i}^a(t_i), Q_{d_j} \right) \right\| \leq F_l^{\max} \quad (12)$$

If no solution is found, then the ISO starts to shed load demand until there is solution to the optimization problem in Eq. (8).

This process of solving three separate linear programming (LP) optimization problems repeats itself at each hour.

Under this operational scheme it is possible that even when the availability of a single generator deteriorates while the availabilities of all other generators improve, the system-wide reliability level may actually deteriorate as well. For example, if the entire reserved capacity comes from the generator of degraded availability with strict transmission capacity limit in case of contingencies, which is possible under the scheme described above, generator outages lead to service interruptions due to transmission capacity

<sup>4</sup>The reserve capacity constraint for interruptible load may be expressed as

$$\sum_{t_j=t_1}^{t_{i-1}} Q_{g_i}^{a,*}(t_j) + Q_{g_i}^a(t_i) = \begin{cases} 0 \\ Q_{g_i}^{c,*} \end{cases} \quad (10)$$

since no arbitrary partial interruption is possible at the time of writing.

limits and the increased unavailability of reserved capacity from that generator. If the cost to the service interruptions is accounted for as well as the prices paid for acquiring energy, capacity and generation adjustment, then the overall cost may fluctuate arbitrarily since only an arbitrary level of reliability in terms of service interruptions may be assured. This is due to a lack of understanding of the tradeoff between the overall operating cost and the level of reliability. It is particularly disturbing since the cost to the service interruptions usually tend to be much higher than the overall operating cost.

A further examination of sequential approach to optimization problems in Eqs. (1) and (8) reveals that the optimality of overall operating cost cannot be assured. Eq. (8) neglects the effect of generation adjustment as a result of  $i$ th contingency on the necessary generation adjustment following  $(i+1)$ th,  $(i+2)$ th,  $\dots$  contingencies. For example, suppose the optimal adjustment for generators  $g_i$  and  $g_j$  following  $i$ th contingency alone may be  $Q_{g_i}^{a,*}(t_i) < 0$  and  $Q_{g_j}^{a,*}(t_i) = -Q_{g_i}^{a,*}(t_i) + C^{st}$ , where  $C^{st}$  denotes some constant. Further suppose the optimal adjustment for generators  $g_i$  and  $g_j$  following  $(i+1)$ th contingency alone may be  $Q_{g_i}^{a,*}(t_{i+1}) = -Q_{g_i}^{a,*}(t_i)$  and  $Q_{g_j}^{a,*}(t_{i+1}) = C^{st}$ . If generation adjustment costs are fairly high and  $(i+1)$ th contingency takes place immediately following  $i$ th contingency, then it is possible that the optimal adjustment for generators  $g_i$  and  $g_j$  following  $i$ th contingency while considering  $(i+1)$ th contingency may be  $Q_{g_i}^{a,*}(t_i) = 0$  and  $Q_{g_j}^{a,*}(t_i) = C^{st}$ .

Plus, even though there is a clear relationship in costs from solving Eqs. (5) and (8) via constraints in Ineq. (11) the optimization problem in Eq. (5) does not take, either implicitly or explicitly, the cost from solving Eq. (11) into account. Intuitively, the ISO may reduce the overall operating cost by allocating reserve capacity to a generator with relatively high capacity cost but extremely low adjustment cost rather than allocating to a generator with relatively low capacity cost but very high adjustment cost. However, the market rules according to Eqs. (5) and (8) do not allow such flexibility.

Finally, the constraints in Ineqs. (3), (6) and (11) link the cost from solving optimization problems in Eqs. (1), (5) and (8), yet this linkage is not taken into consideration either implicitly or explicitly in the optimization problems. For example, there may be a case where a cheaper generator is not fully dispatched in order to reduce the prices paid for acquiring reserve capacity. The operational scheme according to the market rules ignoring interrelationship is due to a lack of understanding of the tradeoff between the cost

of energy and the cost of reserve.

It is clear from the discussion presented here that there exist a need for a new mathematical formulation for computing an efficient operating point, which considers, at the minimum, the tradeoffs between the overall operating cost and the level of reliability and between the cost of energy and the cost of reserve.

### III. MATHEMATICAL FORMULATION FOR COMPUTING EFFICIENT OPERATING POINT

We begin by choosing a time horizon and a model suitable for representing the dynamics of contingencies. In order to simplify the formulation it is required that no simultaneous contingencies can take place within the shortest interval for the time scale chosen. For simplicity without loss of generality a single second interval is assumed to fit this requirement. This assumption allows to model contingencies using the discrete time (in seconds) equivalent of Poisson process of random event, i.e. binomial process, where  $\lambda(\cdot)$  and  $\mu(\cdot)$  refer to the failure rate and the repair rate of a given equipment,  $(\cdot)$ , respectively.

Under the presence of contingencies, the overall cost at each time step,  $k = 1, 2, \dots, 3600$  consists of adjustment cost,  $c_{g_i}^a[k]$  as well as the interruption cost,  $c_{d_j}^a[k]$  where we have used a slightly simplified notation of  $c_{g_i}^a[k]$  to represent the adjustment cost in product form in Eq. (8), i.e.,

$$c_{g_i}^a[k] = Q_{g_i}^a \cdot S_{g_i}^a \cdot \left(1 - \frac{k}{3600}\right) \quad (13)$$

In addition to the overall cost at each time step, there are the initial cost associated with acquiring energy,  $c_{g_i}^e$ , and the initial cost for purchasing reserve capacity,  $c_{g_i}^c$  for providing the electricity services. Similarly, for  $(\cdot) = e$  or  $c$

$$c_{g_i}^{(\cdot)} = Q_{g_i}^{(\cdot)} \cdot S_{g_i}^{(\cdot)} \quad (14)$$

Using the notation developed so far the problem of finding an efficient operating condition can be formulated as an optimization problem given in Eq. (15).

$$[Q_{g_i}^{e,*}, Q_{g_i}^{c,*}, Q_{d_j}^{a,*}, Q_{d_j}^{a,*}[k]] = \quad (15)$$

$$\mathbf{E} \left\{ \sum_{g_i} c_{g_i}^e + \sum_{g_i} c_{g_i}^c + \sum_{k=1}^{3600} \left( \sum_{g_i} c_{g_i}^a + \sum_{d_j} c_{d_j}^a \right) \right\}$$

subject to the load balance constraints

$$\sum_{g_i} Q_{g_i}^e = \sum_{d_j} Q_{d_j} \quad (16)$$

$$\sum_{g_i} y_{g_i,k} + \sum_{d_j} \sum_{t=1}^k Q_{d_j}^a = \sum_{d_j} Q_{d_j} \quad (17)$$

the maximum generation limits

$$Q_{g_i}^e \leq Q_{g_i}^{\max} \quad (18)$$

$$Q_{g_i}^e + Q_{g_i}^c \leq Q_{g_i}^{\max} \quad (19)$$

$$-Q_{g_i}^e \leq \sum_{t=1}^k Q_{g_i,t}^a \leq Q_{g_i}^c \quad (20)$$

and the maximum flow constraints

$$\|F_l(Q_{g_i}^e, Q_{d_j})\| \leq x_{l,0} \cdot F_l^{\max} \quad (21)$$

$$\left\| F_l \left( Q_{g_i}^e + \sum_{t=1}^k Q_{g_i,t}^a, Q_{d_j} - \sum_{t=1}^k Q_{d_j,t}^a \right) \right\| \leq x_{l,k} \cdot F_l^{\max} \quad (22)$$

where the generator output,  $y_{g_i,k}$  is kept track of at each time step according to Eq. (23)

$$y_{g_i,k} = x_{g_i,k} \cdot \left( Q_{g_i}^e + \sum_{t=1}^k Q_{g_i,t}^a \right) \quad (23)$$

and, the generator availability,  $x_{g_i,k}$ , and the transmission line availability,  $x_{l,k}$  are kept track of at each time step according to Eq. (24)

$$x_{(\cdot),k+1} = \begin{cases} \begin{cases} 0 & \text{with the probability of } \lambda_{(\cdot)} \\ & \text{if } x_{(\cdot),k} = 1 \end{cases} \\ \begin{cases} 1 & \text{with the probability of } (1 - \lambda_{(\cdot)}) \\ & \text{if } x_{(\cdot),k} = 0 \end{cases} \\ \begin{cases} 0 & \text{with the probability of } (1 - \mu_{(\cdot)}) \\ & \text{if } x_{(\cdot),k} = 0 \end{cases} \\ \begin{cases} 1 & \text{with the probability of } \mu_{(\cdot)} \\ & \text{if } x_{(\cdot),k} = 1 \end{cases} \end{cases} \quad (24)$$

where  $(\cdot) = g_i$  or  $l$ . It should be noted that the maximum flow constraint given in Ineq. (22) is not complete as  $F_l^{\max}$  itself is a function of  $y_{g_i,k}$ . However, the current technology requires that  $F_l^{\max}$  is defined ahead of time over longer time scale of typically season to year because  $F_l^{\max}$  circumscribing sub-transient problem requires numerous off-line studies, which cannot be done in near real time operation.

Graphically, the decision process described above can be represented using the tree structure in Figure 1. In this figure each node represents a particular system status of generator output, the generator availability, and the transmission line availability, and each branch represents the transition to new system status based on adjustment decision and a

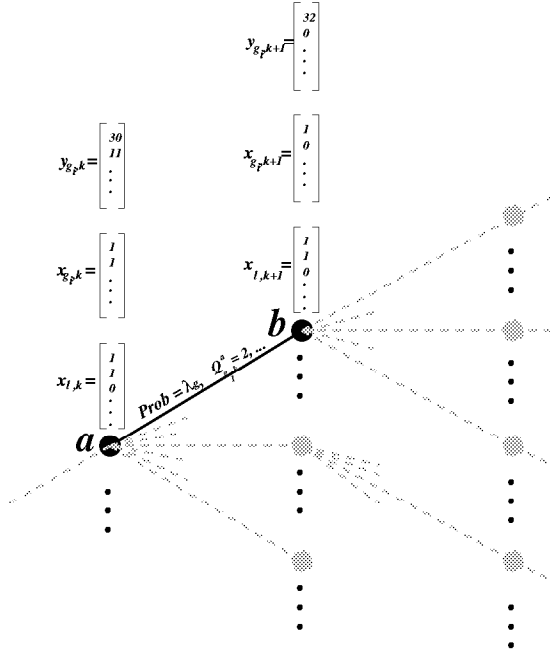


Fig. 1. Decision tree representation of the proposed formulation

possible single contingency with associated probability. For example, node  $a$  represents the system where generators,  $g_1$ , and  $g_2$  among many generators are in service, each producing 30MW and 11MW, respectively, and transmission lines 1 and 2 are in service while transmission line 3 is out among many transmission lines at time step  $k$ . Node  $b$  represents the system where generator  $g_1$  is still in service while the generator  $g_2$  is out among many generators, each producing 32MW following an adjustment and 0MW following an outage, respectively, and the system-wide transmission line status remains same as at node  $a$ . The associated probability for the branch between nodes  $a$  and  $b$  is, thus,  $\lambda_{g_2}$ , and the adjustment of  $Q_{g_1,k}^a = 2$  among many generators. It is recognized that the proposed optimization problem in Eq. (15) is a dynamic programming (DP) optimization problem.

In the proposed optimization problem the trade-off between the cost of energy and the cost of reserve is incorporated by the DP formulation while the tradeoff between the overall operating cost and the level of reliability is made explicit through the inclusion of cost of service interruption. Some interesting formulations given in [2], in comparison, does not consider this tradeoff by relying on probabilistic criteria of LOLP instead and may be limited in implementation for cases where the activities in the energy market is completely separate and the nec-

essary compensation for generator outage is already contracted without the probability of losing load.

#### IV. APPLICATION OF ORDINAL OPTIMIZATION (OO) TECHNIQUE TO THE PROPOSED FORMULATION

Although the proposed formulation is conceptually relevant, it may not be very practical due to its computational complexity. Given the lack of structures as the gradient of cost as a function of inputs, finding the optimal solution to the proposed formulation in Eq. (15) requires searching through  $2^{3600}$  nodes even when accounting only for a single equipment failure.

When dealing with the optimization problems requiring computationally intensive search, the so-called ordinal optimization (OO) method has been proven very effective.[4] The strength of the OO method lies in its considerable savings in computational time based on the idea of the goal softening where the objective is settling for any solution belonging to the “good enough” subset instead of finding the optimal solution.

Consider the operation of the 2-bus electric power network shown in Figure 2 for the duration of one hour. The network is composed of 2 generation sub-

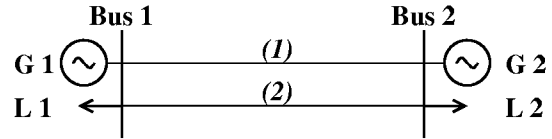


Fig. 2. One-line diagram of 2-bus electric power network

stations, 2 load centers and two identical parallel lines connecting two buses. The generation substation at bus 1 consists of 8 identical hydro units while the generation substation at bus 2 consists of 8 identical thermal units.

The generation units and transmission lines in the network are subject to random outages (and repairs following the outages) according to pre-specified binomial processes. The failure rates of generators at bus 1 and at bus 2 are assumed to be given as  $6.2 \times 10^{-7}$  and  $1.4 \times 10^{-7}$ , respectively. The failure rate and the repair rates of the transmission lines are given as  $1.3 \times 10^{-8}$  and  $2.5 \times 10^{-5}$ .

Under the presence of these random events the cost of operating the network consists of initial generation cost paid for meeting the inelastic demand of 15MW at bus 1, and of 45MW at bus 2, reserve capacity cost paid to generators and curtailment contract cost paid to loads for allowing system operator to adjust generation and curtail load as different events take place

within the hour, and finally adjustment cost paid to generators and load in case the generation re-dispatch and/or demand curtailment take place following an event.

As before, at the beginning of the hour the system operator determines initial generation and load assignment including the amount of initial generation dispatch, the amount of generation capacity reservation and the amount of load curtailment contract, necessary for balancing the supply and demand. During the hour, the system operator then uses these generation capacities and load curtailments for continuously balancing the supply and demand following the various events. The optimization problem of interests is computing the initial generation and load assignment so that the expected value of the overall operating cost is minimized.

In addition, there exist a hard limit on the maximum generation of 40MW at each bus and a strict constraint of maximum transfer of 12.5MW on each line.

The adjustment cost of generation is determined by

$$c_{g_1}^a(Q_{g_1}^a[k]) = 2.5(Q_{g_1}^a[k])^2 \quad (25)$$

and

$$c_{g_2}^a(Q_{g_2}^a[k]) = 3.0(Q_{g_2}^a[k])^2 \quad (26)$$

and the load adjustment (curtailment) cost is given by

$$c_{d_1}^a(Q_{d_1}^a[k]) = \begin{cases} 0 & \text{if } Q_{d_1}^a[k] = 0 \\ \text{sgn}(Q_{d_1}^a[k]) \cdot 10^5 & \text{if } |Q_{d_1}^a[k]| = 7 \\ \text{sgn}(Q_{d_1}^a[k]) \cdot 2.9^6 & \text{if } |Q_{d_1}^a[k]| = 8 \\ \text{sgn}(Q_{d_1}^a[k]) \cdot 3^6 & \text{otherwise} \end{cases} \quad (27)$$

and

$$c_{d_2}^a(Q_{d_2}^a[k]) = \begin{cases} 0 & \text{if } Q_{d_2}^a[k] = 0 \\ \text{sgn}(Q_{d_2}^a[k]) \cdot 3.5 \times 10^5 & \text{if } |Q_{d_2}^a[k]| = 23 \\ \text{sgn}(Q_{d_2}^a[k]) \cdot 8.7^6 & \text{if } |Q_{d_2}^a[k]| = 22 \\ \text{sgn}(Q_{d_2}^a[k]) \cdot 9^6 & \text{otherwise} \end{cases} \quad (28)$$

for bus 1 and bus 2, respectively. The initial generation dispatch cost function is given by

$$c_{g_1}(Q_{g_1}[0]) = Q_{g_1}^2[0] \quad (29)$$

and

$$c_{g_2}(Q_{g_2}[0]) = 2.5Q_{g_2}^2[0] \quad (30)$$

the reserve capacity allocation cost function is given by

$$c_{g_1}^c(Q_{g_1}^c[0]) = 0.25(Q_{g_1}^c[0])^2 \quad (31)$$

and

$$c_{g_2}^c(Q_{g_2}^c[0]) = 0.1(Q_{g_2}^c[0])^2 \quad (32)$$

and the load curtailment contract cost function is given by

$$c_{d_1}^c(Q_{d_1}^c[0]) = 2.5(Q_{d_1}^c[0])^2 \quad (33)$$

and

$$c_{d_2}^c(Q_{d_2}^c[0]) = 3.0(Q_{d_2}^c[0])^2 \quad (34)$$

for bus 1 and bus 2, respectively. The generation amount at each time step  $k$  is determined by taking the outages and the adjustment into consideration as given by

$$Q_{g_i}[k+1] = \left( \frac{n_{g_i}[k] + e_i \cdot w_{k,i}}{n_{g_i}[k]} \right) Q_{g_i}[k] + Q_{g_i}^a[k] \quad (35)$$

The outages also affect the available reserve capacity, load demand and equipment status in Section III.

The constraints for the problem can be represented by

$$0 \leq Q_{g_i}[0] \leq 40 \quad (36)$$

and

$$0 \leq Q_{g_i}^c[0] \leq 40 - Q_{g_i}[0] \quad (37)$$

$$\begin{aligned} -Q_{g_i}[k] \left( \frac{n_{g_i}[k]}{n_{g_i}[0]} \right) &\leq n_{g_i}[k] \sum_{k'=1}^k \left( \frac{1}{n_{g_i}[k']} \right) Q_{g_i}[k']^a \\ &\leq \left( \frac{n_{g_i}[k]}{n_{g_i}[0]} \right) Q_{g_i}^c[k] \end{aligned} \quad (38)$$

and the discrete curtailment contract structure constraint of

$$Q_{d_1}^c[0] = \begin{cases} 7 \\ 0 \end{cases} \quad (39)$$

$$Q_{d_1}^c[0] = \begin{cases} 23 \\ 0 \end{cases} \quad (40)$$

and

$$Q_{d_1}^a[k] = \begin{cases} \begin{cases} 0 \\ -15 \\ 0 \\ -8 \\ -15 \\ 8 \\ 0 \\ -7 \end{cases} & \text{if } Q_{d_1}[k] = 0, \text{ and } Q_{d_1}^c[0] = 0 \\ \begin{cases} -8 \\ -15 \\ 8 \\ 0 \\ -7 \end{cases} & \text{if } Q_{d_1}[k] = 0, \text{ and } Q_{d_1}^c[0] = 7 \\ \begin{cases} 8 \\ 0 \\ -7 \end{cases} & \text{if } Q_{d_1}[k] = 8 \\ \begin{cases} 15 \\ 0 \\ 0 \\ 15 \\ 8 \\ 0 \end{cases} & \text{if } Q_{d_1}[k] = 15, \text{ and } Q_{d_1}^c[0] = 0 \\ \begin{cases} 15 \\ 8 \\ 0 \end{cases} & \text{otherwise} \end{cases} \quad (41)$$

$$Q_{d_2}^a[k] = \begin{cases} \text{if } Q_{d_2}[k] = 0, \text{ and } Q_{d_2}^c[0] = 0 & \begin{cases} 0 \\ -45 \\ 0 \end{cases} \\ \text{if } Q_{d_2}[k] = 0, \text{ and } Q_{d_2}^c[0] = 23 & \begin{cases} -22 \\ -45 \\ 22 \end{cases} \\ \text{if } Q_{d_2}[k] = 22 & \begin{cases} 0 \\ -23 \end{cases} \\ \text{if } Q_{d_2}[k] = 45, \text{ and } Q_{d_2}^c[0] = 0 & \begin{cases} 45 \\ 0 \\ 45 \end{cases} \\ \text{otherwise} & \begin{cases} 23 \\ 0 \end{cases} \end{cases} \quad (42)$$

the constraint of continuously balancing of supply and demand expressed as

$$\sum_{g_i} Q_{g_i}[k] = \sum_{d_j} Q_{d_j}[k] \quad (43)$$

and finally the flow limit on transmission lines given by

$$|Q_{g_1}[k] - Q_{d_1}[k]| \leq 12.5n_t[k] \quad (44)$$

In applying the OO method, first we choose a set of 1000 possible generation dispatch and capacity reservation schedule satisfying the load demand. Then, a large number of hourly random contingencies are generated for simulations. Finally, each schedule is evaluated against randomly generated contingencies via Monte Carlo simulations and is ranked from 1st through 1000th. The simulation results are given in Figure 3. Given that the estimated overall operating

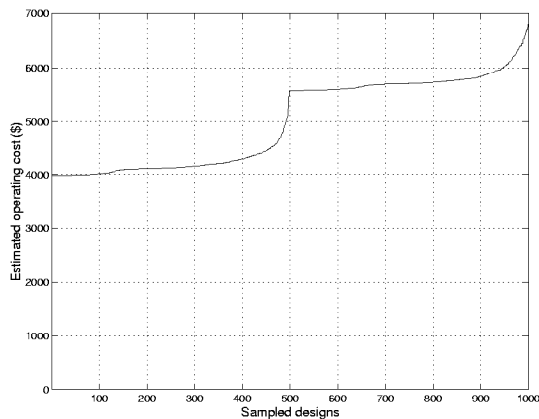


Fig. 3. Estimated overall operating cost to 1000 sampled designs

cost is around \$4,900 for the generation dispatch and capacity reservation schedule determined through sequentially solving optimization problems in Eqs. (1) and (5), a case can be made for importance of explicitly incorporating the tradeoffs between the overall

operating cost and the level of reliability and between the cost of energy and the cost of reserve.

## V. CONCLUSION

In this paper we develop a possible notion of short-term value-based reliability and a mathematical formulation for computing an efficient operating point for managing an electric power system based on a cost-benefit analysis on the effect of contingency. As the electric power industry goes through the restructuring process, a reliable operation of the system based on such analysis may be more appropriate than traditional operation of the system based on simple compliance to  $(N - 1)$  security criteria. The solution to proposed formulation represents the optimal operating condition measured in terms of overall operating cost seen from the perspective of entire system. Thus, the solution becomes relevant for analyzing the performance of already functioning market or for designing unbundled markets for energy, reserve and adjustment.

It is shown that computing an efficient operating point requires an explicit incorporation of tradeoffs between the overall operating cost and the level of reliability and between the cost of energy and the cost of reserve in the relevant optimization problems. Based on empirical results a strong argument can be made for re-design of market which moves away from the operational scheme represented by Eqs. (1), (5) and (8) and move towards new paradigm of Eq. (15).

Much more work is needed to include other aspects of reliability including reactive power, frequency regulation and transient stability problems into market design.

## ACKNOWLEDGMENT

The authors greatly appreciate the financial support provided for this research by the National Grid Company - USA.

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