Organizational Beliefs and Managerial Vision

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Abstract

Can managers have an impact on their firm that goes beyond their direct actions and decisions? This paper shows that a manager with strong beliefs about the right course of action for the firm will attract, through sorting in the labor market, employees with similar beliefs. This alignment of beliefs gives direction to the firm and has important implications for incentives and coordination. The paper then defines vision, in accordance with the management literature, as such strong beliefs about the right course of action and shows that, because of this sorting effect, it is often optimal for a board to hire visionary managers. It also discusses succinctly the link with corporate culture.

1 Introduction

What is the role of a CEO? How do managers influence their companies’ behavior and performance? While economics has done substantial research on, for example, managers’ compensation and incentives, it has usually simply assumed that a CEO’s influence goes solely through her direct

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actions and decisions. The influence and role of a CEO, however, should be positive and normative research questions. The answer to these questions is important not only to the design of managerial incentives, but also to organization design more broadly and to our understanding of the heterogeneity in firm behavior and performance (Mueller 1990, McGahan 1999).

While the management literature has studied extensively the role of the CEO, and in particular the role of managerial vision, the economic literature on these topics (e.g. Rotemberg and Saloner 1993, 2000), discussed in more detail below, is limited. The contribution of this paper is to show that a manager has an important indirect influence on a firm’s behavior and performance through sorting in the labor market. In particular, strong managerial beliefs attract and retain people with similar beliefs, causing an alignment of beliefs within the firm. Existing empirical evidence lends support to this theory. The paper further shows how this alignment of beliefs has important implications for incentives, coordination, and direction setting. It then defines vision, in accordance with the original managerial and psychology literature on the topic, as a strong belief of the manager about the right course of action and shows that it might often be optimal to hire a manager with such vision.

The model and results. The paper studies a simple model which focuses on the interaction between a manager’s beliefs and those of his employees. In this model, employees can spend effort on developing new initiatives. If an employee comes up with a project, his manager has to decide on implementation. If the project gets implemented and is a success, the employee gets part of the revenue through ex-post bargaining. However, at the time of the project generation and implementation there is uncertainty about what kind of projects (A vs. B) will be successful. The key to the analysis is that the manager and the employee may openly differ in their beliefs about the right course of action. This means that I do not impose the common prior assumption, an approach that will be justified in more detail.

As mentioned before, the focus of this paper is on the sorting effect induced by strong managerial beliefs. Before we can tackle this issue, however, we need to understand the impact of managerial beliefs on the employees’ effort (motivation) and utility (satisfaction). The paper shows that a stronger belief of the manager motivates those employees who agree with him. The reason is simply that these employees get easier approval for the projects they undertake, and thus that
their efforts have a higher expected payoff. Furthermore, these employees will also have higher utility or satisfaction. Analogously, employees who disagree with the manager will get demotivated and get lower satisfaction from stronger managerial beliefs. These differential effects on utility then give rise to sorting: a firm attracts employees with beliefs that are similar to those of its manager. Existing evidence in economics and psychology supports such sorting. The paper then shows how the resulting alignment of beliefs affects some key outcomes. For one thing, the alignment of the employees’ beliefs with those of the manager give direction to the firm. It also indirectly aligns the beliefs of the employees with each other, which improves coordination. Finally, the sorting also eliminates precisely those employees who got most demotivated and the lowest satisfaction from these strong managerial beliefs. This suggests that strong managerial beliefs might be profitable.

In line with the original management and psychology literature on the topic, discussed later, the paper then defines ‘vision’ as a strong belief by the manager about the future and about the right course of action for the firm. To evaluate the profitability of hiring a manager with such vision, the paper takes the perspective of an outsider, such as the board, with a ‘reference’ belief. It shows that under weak conditions it is optimal for such outsider to hire a manager who has strictly stronger beliefs than his own. The paper finally connects the sorting mechanism to the formation of corporate culture.

The appendices explore the robustness of the conclusions and some related results of independent interest. One issue is the impact of vision on wages and stock options. The most interesting conclusions here are that sorting attracts exactly people who value stock options the highest and that there may be a spurious relationship between stock options and effort. Another result is that vision and strong beliefs increase the variance of the performance: the best (and worst) companies will be those with visionary managers even when vision has no real performance effects. This may induce an important selection bias in studies of vision.

The literature. The idea that a manager’s influence goes beyond her direct actions and decisions is implicitly well accepted in the management literature and business press. The literature on culture, for example, has stressed the role of a leader’s values and beliefs in the formation of corporate culture (Donaldson and Lorsch 1983, Schein 1985, Kotter and Heskett 1992). The literature on vision, which started with Bennis and Nanus (1985) and Tichy and Devanna (1986) and built on
the theories of charismatic and transformational leadership (House 1977, Burns 1978), also focuses on the implications of characteristics and beliefs of managers and leaders for employees’ actions and decisions to explain the performance of organizations.

Economics has largely neglected these issues. An important exception are Rotemberg and Saloner (2000) who present a formal model of vision and why it may matter. Extending their work on leadership styles and strategy (Rotemberg and Saloner 1993, 1994), they consider a firm with two employees, or product divisions, working on different projects. Vision in their model is a bias of the manager that makes him favor one division over the other. Such vision improves the incentives of one employee at the cost of reducing the incentives of the other. While this paper thus shows that managerial preferences may matter to firm performance, it does not allow to address the question of firm heterogeneity: in their world, all firms would hire equally biased managers and perform exactly the same. The focus on within-firm bias also limits the model to incentive considerations and does not allow to study such issues as decision making, sorting or coordination. In more recent work, Hart and Holmström (2002) consider how managerial vision may be a determinant of firm boundaries. Their argument is essentially that in a world with incomplete contracts, a manager’s bias matters and that different activities may therefore need different managers. More distantly related is the work by Goel and Thakor (2000), who argue that people who underestimate project risks have a higher probability to win in tournaments and thus get elected as leader, and that such overoptimism may compensate for risk aversion. Hermalin (1998) assumes that leaders, for some exogenous reason, have better information and then studies how they can communicate that information to others. The literature on culture or delegation will be discussed as these topics come up.

On the empirical side, Bertrand and Schoar (2001) show that the policies of a firm depend on the identity of the CEO. This, and especially their finding that a manager’s policies are correlated with the MBA program he or she attended, fits the idea that CEO’s may differ in their beliefs about what is right and that that fact has real implications for firm behavior and performance. Section 6 will review more evidence that throws light on the theory presented in this paper, particularly the sorting mechanism. Baum, Locke, and Kirkpatrick (1998), finally, find evidence of a positive influence of vision on venture growth and survey earlier empirical studies on managerial vision.
The key contribution of this paper is clearly the sorting effect: it considers the formation of a firm’s work force from a perspective other than productive ability and derives real economic consequences from this sorting, in terms of incentives and coordination. It also relates the concepts of vision and culture in ways that are consistent with the management literature (Schein 1985).

The next section explains the model setup and discusses how the definition of vision in this paper fits that of the original management literature on the topic. Section 3 considers the impact of managerial beliefs on employees’ decisions, motivation and satisfaction. On that foundation, section 4 derives the sorting effect and its implications for incentives and coordination, while section 5 considers when vision would be profitable. Section 6 discusses existing evidence that is relevant to the theory presented, while section 7 considers concisely the implications for organizational culture. Section 8 concludes and suggests further research topics. Appendix A discusses the impact of changes in the assumptions or set-up and some potential objection to the theory. Appendices B through D were discussed earlier. Appendix E treats differing priors, while appendix F contains the proofs.

2 The Model

A sketch of the model  Remember the basic model, as sketched in the introduction. Employees try to develop initiatives. The probability that an employee ‘comes up with something’ equals his effort e. If an employee comes up with something, the manager decides whether to implement it. In making that implementation decision, the manager considers not only the project’s expected revenue but also its organization-wide implementation cost, which is a random variable I. If the project gets implemented and is a success, the employee gets part of the revenue through ex post bargaining.

The key element of the model is the presence of uncertainty about which projects will be successful and generate revenue. In particular, employees have to choose which projects to spend effort on: A- or B-type projects. These project types are mutually exclusive. The success and revenue of a project depends on its fit with the (unknown) state of the world x ∈ {a, b}. In particular, X-type projects are successful if and only if the state is x = 1. Successful projects generate a revenue of 1 while failures generate no revenue. Note that the state may include any factor that
has a bearing on what the optimal action is, including evolution of the industry, core competences of the firm, or ‘the right way of doing things’.

All agents in the model have their own subjective belief about the likelihood of each state. These beliefs may differ but are common knowledge. This implies, by Aumann (1976), that the agents start from differing priors. It also implies that agents will not update their beliefs merely because they are confronted with a different opinion. Appendix E shows that this approach is consistent with the economic paradigm. Harsanyi (1967), for example, observed that ‘by the very nature of subjective probabilities, even if two individuals have exactly the same information and are at exactly the same high level of intelligence, they may very well assign different subjective probabilities to the very same events’. The reason for following this approach is simply that common knowledge of differing priors is the most transparent and parsimonious way to study the issues in this paper.

Working with differing priors raises the issue how to measure expected profits and thus how to determine the optimality of vision. To that purpose, this paper uses the perspective of an outsider with a ‘reference’ belief. This outsider can be interpreted as the researcher doing the analysis, the board, the financial markets, or even a truly objective observer, though the latter interpretation sometimes causes confusion.

I will use the notation $\mu_{i,Y}$ for the probability that agent $i$ assigns to the event that the state is $Y$. Employee $E$, for example, believes that with probability $\mu_{E,A}$ the state is $A$. The strength of an agent’s beliefs will turn out to play an important role in the analysis. I will denote this by $\nu_i = \max(\mu_{i,A}, \mu_{i,B}) \in [1/2, 1]$, i.e. $\nu_i$ is the strength of $i$’s belief in the state he considers most likely. An agent has a ‘stronger belief’ or ‘stronger conviction’ if $\nu_i$ is larger. Finally, $p$ will denote the reference belief that the true state is $a$.

What follows gives a more detailed description of some elements in the model.

**Agents, utilities, and beliefs** The model has 3 types of agents: firms, managers, and employees. In the analysis of optimal vision, imagine the firm to be represented by the board (or the owner) who chooses the manager. This board maximizes expected profits using the reference belief $p$. Each firm also has a manager who hires its employees and decides on implementation. Managers maximize expected firm profit based on their own subjective belief $\mu_{M,Y}$. Employees, finally, choose
Hiring process

1 Employee chooses firm (if there is more than one firm).
2 Firm makes initial wage offer $\tilde{w}$.
3 Employee accepts or rejects. Upon rejection, employee gets outside wage $w = 0$.

Project

1 Employee chooses (one and only one) type of project $X \in \{A, B\}$ and invests effort $e \in [0, 1]$ (cost $c(e)$ sunk).
2 Employee generates project with probability $e$.
3 Manager observes project type and implementation cost, which is a random variable $I \sim U[0, 1]$.
4 Manager decides whether to implement (cost $I$ sunk).
5 Employee and manager observe whether the project will be a success (i.e., they observe the state).

Renegotiation

1 Employee can ask for a raise (i.e., decides on wage renegotiation).
2 Manager and employee renegotiate wage (see below). Upon breakdown, employee decides to stay (and get $w = \tilde{w}$) or leave (and get $w = 0$) but the project will be a failure either way.

Payoff

1 Successful projects generate 1, failures generate 0.
2 Wages paid (according to renegotiation).

Figure 1: Timeline of game

projects and spend effort on developing them. They maximize their expected revenue net of any cost of effort. In doing so they also use their own beliefs $\mu_{E,Y}$.

Actions and timing

The precise timing is indicated in figure 1. Stages 1, 2, and 4 are straightforward. The renegotiation in stage 3 is according to Nash bargaining with relative bargaining power $\gamma_E, \gamma_M > 0$, with $\gamma_E + \gamma_M = 1$. This means that a dollar gain from agreement will be split $(\gamma_E, \gamma_M)$. If bargaining breaks down, the project fails and generates 0 revenue while the employee can choose either to stay with wage $w = \tilde{w}$ or to leave the firm and take his outside wage $w = 0$. This renegotiation is just a way to assure that the employee cares about the outcome. We would obtain the same results if, for example, the employee cares about the outcome because it affects his outside options or future promotions. The results would also not change if the firm could also ask for renegotiation. Appendix A considers further modifications to the setup of the model.

The assumption that the employee can quit in stage 3 when the negotiations break down will make the equilibrium wage independent of the employee’s beliefs. While this makes the analysis more transparent, it also eliminates potentially interesting wage effects. Appendix C therefore considers an alternative setup that allows to study such wage effects of beliefs.

Contractibility

Implicit in this timeline are a number of assumptions as to what variables are contractible. In particular, the timeline implicitly assumes that the agent’s effort $e$ and the project type are economically too complex to contract on. It also assumes that employees, without spending
any effort, can come up with bad (zero-revenue) projects that an outsider cannot distinguish from
good ones so that ‘coming up with a project’ is not contractible either. Furthermore, future
revenue becomes economically contractible only after the project has been implemented. All these
assumptions can be justified by the difficulty of describing a project that does not yet exist. It
then follows that the only possible contract at the start of the game is a fixed-wage contract (which
may in principle depend on the employee’s belief) as described in the timeline. The description
of the game also implicitly assumes that the employee’s support is needed until the end for the
project to become a success, and that he can withdraw that support at will to force a renegotiation.
Appendix A discusses how these contractibility and renegotiation assumptions affect the results.
That appendix also considers other variations on the basic model.

I also make a number of explicit assumptions.

**Assumption 1** Employees’ beliefs are independent draws from a distribution of beliefs $F$ on $[0,1]$, with continuous density $f$.

When indifferent about which firm to apply to, employees randomize between the firms with
equal probability. When indifferent about what action to undertake, employees do as their manager
prefers. When indifferent about implementation, managers do as their employee prefers.

**Assumption 2**
- The implementation cost $I$ is distributed uniformly on $[0,1]$.
- The cost of effort $c(e)$ is twice continuously differentiable.
- $c''(e) > 0; c(0) = c'(0) = 0; c(1) > \gamma_E \gamma_M$.

**Assumption 3** $p \geq 1/2$.

This last assumption is without any loss of generality since we can always rename the states and
projects.

One further remark on differing beliefs facilitates the interpretation of the model. The distrib-
ution of beliefs is implicitly assumed to be generated by the following information process. All
agents start with a common prior on the state $x \in \{a, b\}$ that puts equal probability on both states.
All agents subsequently get a common signal that, for example, the true state is $a$. The agents,
however, have their own subjective opinion about the correctness of that signal and these beliefs
are common knowledge. In particular, it is commonly known that agent $i$ thinks that the signal is correct with probability $\mu_{i,A}$. Note here that differing beliefs about the correctness of the common signal is just a special case of differing priors$^2$. Bayesian updating then leads the agent to believe that the probability of state $a$ is $\mu_{i,a}$. The ‘reference’ belief $p$ is the board’s belief about the signal. Note that a ‘visionary’ manager is overconfident relative to the reference belief.

A practical example To fix ideas, think back a few years to the time that the Internet was close to taking off and consider a software product manager who is preparing the next version of his product. His key issue is whether to add and improve traditional features or to focus instead on adding Internet capabilities. The future success of his product may depend crucially on this choice. Complicating matters is the fact that the CEO has the final say on any new release. Consider now the case that the product manager believes the Internet is no more than a fad and developing Internet capabilities a complete waste of resources which might put him fatally behind his competitors. His CEO, however, is a true believer and has made clear to her product managers that they should focus on making their products Internet-ready.

In this case, contracting on direct output is problematic since it is difficult to define what Internet-ready means, what good implementation means, or what the relative importance of different features is. Software development efforts are also difficult to measure objectively. Finally, his product’s success is obviously a key factor in the product manager’s future wage negotiations (or promotions), but it is difficult to contract on long in advance given the fundamental uncertainties in the industry$^3$.

At this point it is useful to make a remark regarding the level of analysis in this paper. The model represents essentially a CEO and the management team directly below him, or a team leader and his immediate team members. It should be noted, however, that the effects of a CEO’s vision might influence employees $n$ levels down the hierarchy, via a cascade of the mechanism discussed here.

Operational definition of vision Since this paper started out as an inquiry into the precise workings of managerial vision, the definition of vision is directly inspired by the original management literature on the topic. The term ‘vision’ was introduced in the management literature via research
on leadership (Bennis 1982, Bennis and Nanus 1985) that built on the charismatic (House 1977) and transformational (Burns 1978) theories of leadership. Vision was defined as ‘a mental image of a possible and desirable future state of the organization’ (Bennis and Nanus 1985) and the ability to formulate a vision was found to be a key discriminating characteristic of leaders. While most of the subsequent research-based literature stuck quite closely to this original meaning, the popular press and some more recent work have used the concept much more loosely.

This paper operationalizes that ‘mental image of a desirable state of the organization’ as a more prosaic ‘very strong belief about the optimal future state of the organization’. While this captures the core elements of ‘vision’ as the term is used in the original research-based literature, some characteristics that are mentioned in that literature are absent from the model but should be explored in further research. Most important is probably the claim that a vision should be attractive (Bennis and Nanus 1985). Temporarily abstracting from these aspects is useful for two reasons. It results, first of all, in a very transparent analysis. Second, the results in this paper replicate very well the benefits that the managerial literature attributes to vision (motivation, direction setting, consensus, coordination). This suggests that this belief aspect of vision is indeed one of its core elements.

Vision is thus defined here operationally as a strong belief by the manager about the optimal course of action for the firm. A manager who says that ‘anything is possible’ has no vision, while one who claims that ‘in five years handhelds will have completely displaced PC’s’ conveys a strong sense of vision.

In principle, a manager would thus be visionary (from the board’s perspective) if he has a stronger belief than the board, i.e. if $\mu_M > \max(p, (1 - p))$. Given that we assumed $p \geq 1/2$, however, the interesting case is when the manager has a stronger belief than the reference belief. The operational definition will thus be that the manager is visionary if $\mu_{M,A} > p$.

### 3 Preliminary Analysis: Decisions, Motivation, and Satisfaction

To get to the sorting effect, we first need an understanding how individual employees react to their manager’s beliefs. This is the subject of this section. Along the way, I will also develop and discuss some results that are of independent interest.
To analyze the effect of managerial beliefs on an individual employee, consider the model of section 2 with the firm having only one employee. Let $\mu_{E,Y}$ and $\mu_{M,Y}$ denote the beliefs of the employee and the manager that the correct course of action is $Y$. Throughout this and the following sections, hats will indicate optimized choice variables and value functions. So $\hat{e}$ is the employee’s optimal choice of effort while $\hat{u}$ is his optimized utility. To make the notation more transparent, the dependence of the maximizers on other variables will be suppressed.

I reason now by backwards induction. The renegotiation process will give the firm a gross revenue $\gamma_M$ if the project is implemented and turns out to be a success, and zero otherwise. Given that the manager can observe the project type, he will thus allow a project $Y$ to be implemented if and only if $\gamma_M \mu_{M,Y} \geq I$. Prior to the revelation of $I$, the project will thus be implemented with probability $\gamma_M \mu_{M,Y}$, which gives the employee an expected payoff from proposing a project of $\gamma_E \gamma_M \mu_{E,Y} \mu_{M,Y}$. In choosing the type of initiative and $e$, the employee thus solves:

$$\max_{e \in [0,1], Y \in \{A,B\}} e \gamma_M \gamma_E \mu_{E,Y} \mu_{M,Y} - c(e)$$

The next proposition now says that whoever has the stronger beliefs or conviction about what should be done, will determine what the employee will do. Remember that $\nu_i = \max(\mu_{i,A}, \mu_{i,B})$ denotes the strength of belief of agent $i$.

**Proposition 1** If the manager has the stronger conviction, i.e. $\nu_M \geq \nu_E$, then the employee undertakes the action that his manager prefers, i.e. $X = \arg\max_{Y \in \{A,B\}} \mu_{M,Y}$. Otherwise he follows his own opinion, i.e. $X = \arg\max_{Y \in \{A,B\}} \mu_{E,Y}$.

The intuition is simple. If the manager and the employee agree on the optimal action, then $E$ chooses of course that action. If they have different opinions, the employee will have to ‘disappoint’ one of the two. Since the roles of their beliefs are symmetric in the employee’s utility function, it is optimal to ‘disappoint’ the one who holds the weaker belief (i.e. the belief closer to 1/2).

Given this symmetry, one might wonder what the difference between the employee and the manager really is: why do we say that managers have a vision while employees ‘only’ have beliefs? The difference is, first, that the manager influences the decision of the employee but not the other way around and, second, that the manager also influences other employees.
A different way to look at proposition 1 is to say that the manager keeps a strong influence over the project type, even though the choice is formally delegated to the employee. In many non-routine jobs, such indirect authority might be a more effective way to influence the course of action than direct authority, since, among other things, the manager has to get involved only after the project has been successfully developed. For this kind of decision processes, the earlier results then imply the following.

Corollary 1 (‘Visionary managers have more influence.’) The prior probability that the project choice is according to the manager’s belief increases in \( \nu_M \), the manager’s conviction in his view of the world.

While the manager’s opinion has an important influence on the decisions of the employee, it is also a key determinant for the employee’s effort and utility, i.e. his motivation and satisfaction. The following proposition essentially says that a stronger belief of the manager will motivate the employee and increase his utility if the employee acts according to the manager’s beliefs. Such stronger beliefs, however, will demotivate and reduce the utility of an employee who goes against the manager’s opinion. To state this formally, let \( N \) be an open neighborhood of \( \mu_E \) and \( \mu_M \) on which the chosen project type \( X \) remains identical and let \( 0 < \mu_v,A < 1 \) for both agents.

Proposition 2 Employee effort \( \hat{e} \) and utility \( \hat{u} \) strictly increase in the conviction of the manager \( \nu_M = \max(\mu_{M,A}, \mu_{M,B}) \) (resp. in the employee’s own conviction \( \nu_E \)) on \( N \) if the employee undertakes the action that the manager strictly prefers \( X = \arg\max_{Y \in \{A,B\}} \mu_{M,Y} \) (resp. that he himself strictly prefers).

Analogously, employee effort \( \hat{e} \) and utility \( \hat{u} \) strictly decrease in his manager’s conviction (resp. his own conviction) on \( N \) if he undertakes the opposite action of what his manager strictly prefers: \( X = \arg\min_{Y \in \{A,B\}} \mu_{M,Y} \) (resp. of what he himself strictly prefers).

The intuition is simple. Suppose that the employee undertakes a project that is the right course of action according to his manager. As the manager is more convinced of that action, the probability that he will implement the project increases. This will increase the expected payoff to the employee from trying to develop the project, which indeed motivates him and gives him higher utility.
This result can be loosely interpreted as follows:

- Employees with no specific opinion on the correct action ($\mu_E$ close to 1/2) get more motivated by managers who know precisely what they want, no matter what they want. The same is true for employees whose utility depends only on implementation or approval, and not on the final success (since this case is formally equivalent to setting $\mu_{E,X} = 1$ for the likelihood of whichever action is chosen).

- Employees with a strong opinion about the correct path of action will be very motivated under managers who agree with them (and more so as the manager is more convinced of that opinion). But they will be very demotivated under managers with a different opinion.

These statements fit casual empiricism.

4 The Sorting Effect

4.1 Basic analysis

The effects of beliefs on effort and utility cause sorting in the labor market. The basic argument runs as follows.

- Employees get higher utility working for firms that espouse a vision they agree with. Firms get higher profits from employees who agree with their vision, since the latter are more motivated. An efficient labor market should therefore match employees and firms with similar beliefs. Since sorting determines the type of employees a firm attracts, which then influences its profit, this might on itself constitute a sufficient reason for deviating from the reference belief.

- Once sorting has taken place, the beliefs of the employees and the manager are more aligned. This will decrease or even eliminate the demotivating effect that vision had on some employees, so that vision becomes more effective.

Empirical support for such sorting will be discussed in section 6.

To study these effects formally, consider again the model of section 2 but let the employee, with belief $\mu_{E,A}$, have the choice between two firms, $F_1$ and $F_2$, with managers $M_1$ and $M_2$ who have beliefs $\mu_{M_1,A}$ and $\mu_{M_2,A}$, where we assume wlog. $\mu_{M_1,A} \geq \mu_{M_2,A}$.
There is again a unique subgame perfect equilibrium outcome, which gives sorting as indicated in figure 2.

**Proposition 3** Let $\bar{\mu} = \frac{1 - \mu_{M_2,A}}{\mu_{M_1,A} + 1 - \mu_{M_2,A}}$. In any subgame perfect equilibrium, all employees with $\mu_{E,A} > \bar{\mu}$ end up being hired by $M_1$, while any employee with $\mu_{E,A} < \bar{\mu}$ will be hired by $M_2$. $\bar{\mu}$ decreases in both $\mu_{M_1,A}$ and $\mu_{M_2,A}$.

![Diagram](image_url)

Figure 2: The choice of firm and action (indicated by the horizontal brackets) by the employees in function of the beliefs of both the two managers ($\mu_{M_1,A}$ and $\mu_{M_2,A}$) and the employees (whose beliefs are distributed over $[0, 1]$ according to $F$) for two constellations of the managers’ beliefs (in top and bottom part of the figure).

It can be shown that this allocation of employees is also the unique stable matching and the unique element in the core (defined by weak domination) of the corresponding cooperative matching game.

To see intuitively what is happening consider first the upper graph of figure 2. There are two managers who have approximately opposite beliefs. Consider the situation of an employee with belief $\mu_E = 1/2$. Personally this employee doesn’t see any difference between the two alternatives. All he thus cares about is the probability of implementation. So he will go with the manager with the strongest conviction, which is $M_1$. Given that his preference is strict and utilities are continuous, the cutoff $\bar{\mu}$ must be strictly to the left of 1/2.

Note two things:

1. The employee with $\mu_E = 1/2$ is closer to $M_2$ in terms of beliefs, but goes to firm $F_1$, since $M_1$ ‘knows better what he wants’.

2. As $M_1$ gets more convinced, he becomes more attractive to work for. In particular, an
employee that before was indifferent will now go to work for \( M_1 \). So \( \mu \) shifts to the left as \( \mu_{M_1} \) shifts to the right. The same is true for \( M_2 \). This gives the lower graph.

The result is also striking in the sense that the firm with the stronger vision attracts precisely these employees who take action according to its manager’s beliefs.\(^8\)

**Corollary 2** If manager \( M_1 \) has the stronger belief, i.e. \( \nu_{M_1} > \nu_{M_2} \), then any employee hired by \( F_1 \) will choose the action preferred by its manager, i.e. \( X = \arg \max_{Y \in \{A,B\}} \mu_{F_1,Y} \) for \( F_1 \), while any employee hired by \( F_2 \) will choose the other action (which then might or might not be preferred by \( M_2 \)).

The intuition is simply that an agent who goes to \( F_2 \) and undertakes action \( A \) would have been better off going to \( F_1 \) while still undertaking \( A \), and vice versa.

The result also says that firm 2 gets ‘pushed’ into taking the other action, even if its manager thinks it should take in fact the same action as firm 1. It thus follows that firm 2 might be better off hiring a manager with the opposite vision of firm 1, or one whose vision is still stronger. This raises the broader issue how firms will compete on vision, which is partially analyzed in appendix D.

Note, finally, that there is an implicit assumption in this model that firms are not limited in size: they hire any employee who comes their way. In the presence of many candidate-employees, this leads to the rather surprising result that the more visionary firm tends to be larger and have employees with more diverse beliefs. In reality, however, firms are not so flexible in terms of their size. Taking into account such limitations would eliminate these results. They also tend to disappear as the number of project types increases.

The alignment of beliefs, both between a manager and his employees and among the employees, has broad implications. The next two subsections simply consider the implications for incentives and coordination that follow directly from the current model. It is easy, however, to conjecture additional implications, not only for incentives and coordination, but also for experimentation, learning, communication, conflict, and delegation. This awaits further research.

### 4.2 The impact of sorting on incentives

In section 3 we concluded that stronger managerial beliefs could demotivate and reduce employees’ utility, that is, if they favored the other action so strongly as to go against the manager’s
opinion. Sorting, however, will reduce this negative effect on incentives since potential employees who strongly disagree with the manager will go elsewhere. In particular, in the current setting the negative effect on incentives will be completely eliminated for the more visionary of the two firms since all its employees end up choosing according to the manager’s beliefs. Vision will thus always motivate.

**Corollary 3** If $M_1$ has the stronger belief ($\nu_{M_1} > \nu_{M_2}$) then the effort and utility of $F_1$’s employees increase in $\mu_{M_1, A}$ and thus in $\nu_{M_1}$.

### 4.3 The impact of sorting on coordination

The literature on vision has stressed its coordinating effects (Bennis and Nanus 1985). While a full study of coordination goes beyond this paper, it is probably useful to illustrate how strong managerial beliefs can indeed improve coordination. The key conclusion is that, although vision has coordination benefits independent of sorting, it is again the sorting-induced alignment of beliefs that has most impact. In particular, there are 3 ways how vision can align employees’ actions:

1. In choosing their actions, employees are influenced by their manager’s beliefs. Since all employees face the same managerial beliefs, their actions will become more aligned.

2. Employees who take actions that fit the manager’s beliefs exert more effort than others.

3. The sorting effect directly aligns the beliefs of the employees with those of the manager and thus with each other. This also aligns their actions.

To study these effects more formally consider a slight modification of the original model. Let $i$ be an employee of firm $j$. As a measure of coordination, let $\tau_i$ denote the percentage of employees of $i$’s firm ($i$ included) that take the same action as $i$, and let $\bar{\tau}_j$ denote the average $\tau_i$ over all employees $i$ that are part of firm $j$. Consider then the following generalization of the implementation cost. Let $c_j^i(I)$ denote the cost to firm $j$ of implementing employee $i$’s action, with $I$ still a random variable, $I \sim U[0, 1]$. Assume that the $I$-draw is the same for all employees of firm $j$. While in the original setup $c_j^i(I) = I$, the cost will now depend on the level of coordination in firm $j$. In particular, the analysis considers two specifications.
with \(0 < \alpha < 1\). The first specification corresponds with a situation where there are scale effects in implementing actions of identical type. The second captures the notion that a less focused firm has more trouble implementing initiatives of any kind. In both cases, analytical considerations played an important role in the choice of specification. Note that the original model corresponds to \(\alpha = 1\).

It can be shown that this formulation is equivalent to letting the probability of coming up with a project be \((\alpha + (1 - \alpha)\tau)e\) (instead of simply \(e\)), using the appropriate \(\tau\).

**Coordination absent sorting** Absent sorting, there will typically be some employees of the firm that undertake \(A\) while others undertake \(B\). An employee’s optimal action will then depend on the actions of all other employees in his firm. This creates the possibility of multiple equilibria, as is often the case with coordination games.

This case becomes analytically more tractable when we consider the limit as the number of potential employees goes to infinity and let the agents choose their actions simultaneously. Make furthermore the following assumption.

**Assumption 4** The manager’s belief \(\frac{1}{2} < \mu_{M,A} < 1\). The distribution of beliefs \(F\) First Order Stochastically Dominate some symmetric distribution.

This assumption essentially says that the distribution of beliefs weakly favors the side that the manager believes in (or, alternatively, that the manager’s beliefs do not go against those of the population).

**Proposition 4**

- Assume \(A^4\) and \(S2\). In the limit as the number of employees \(n \rightarrow \infty\), the measure of coordination \(\tau\) increases in the strength of belief of the manager.

- Assume \(A^4\) and \(S1\). In the limit as the number of employees \(n \rightarrow \infty\), \(\tau\) of the equilibrium with the highest number of employees choosing \(A\), increases in the strength of belief of the manager.
• Assume $A_4$ and $S1$. In the limit as the number of employees $n \to \infty$, there exists $\hat{\alpha} < 1$ such that $\forall \alpha \geq \hat{\alpha}$, $\tau$ increases in the strength of belief of the manager.

Note that the last part of the proposition says that vision improves coordination, ‘as long as coordination is not too important’. The reason for this limitation is that when coordination is very important, agents will usually coordinate in equilibrium. They may, however, coordinate on $B$ even though most of them, including the manager, believe in $A$. In such case where the manager’s beliefs go against the equilibrium outcome, stronger managerial beliefs may actually cause coordination problems.

**Coordination with sorting** The situation with sorting is much more simple. Consider a setting with 2 firms. Assume that there are a finite number $n$ of potential employees and that in stages 1 and 2 they take their firm and action choices sequentially in some random order.

**Proposition 5** In each firm, the measure of coordination $\tau_j = 1$ a.s., both under $S1$ and under $S2$.

Note that in this case we did not take the limit $n \to \infty$ and did not assume $A_4$.

While this result is not very surprising, it is very strong. It suggests that sorting in the labor market might be a very effective coordinating device. If coordination is really important, then this might be a reason to have different firms.

### 4.4 Visionary organizations

The analysis thus suggests the following characteristics of a ‘visionary organization’:

- Even when employees choose their initiatives without intervention from the top, they choose what management would want them to choose. This strengthens the case for delegation.

- The employees’ initiatives are well coordinated without any explicit coordination effort or mechanism.

- Visionary firms sometimes attract employees who do not really agree with the vision, but who are attracted by its conviction.
• Vision motivates all employees, including those who actually think the other project would be better.

Note also that there is typically a group of employees who have even stronger beliefs than the CEO. Since the ‘employees’ in this model are typically executives, they have on their turn important decision power over the projects of their subordinates. From the CEO’s (subjective) perspective it may therefore make sense to impose artificially inflated minimal rates of return for approval of the projects of these subordinates. Note that a CEO himself is typically not subject to such approval criteria. While this is a potential explanation for the inflated investment hurdles that are often observed (Kaplan 1986, Poterba and Summers 1995), a more likely conjecture is that these high hurdle rates are a response to employees’ purposely inflating their revenue estimates of projects they like. This is indeed what casual empiricism suggests.

5 Profitability of vision

A key question remains: is vision good or bad? In other words, can we say anything about when a company gains from hiring a CEO with vision? In line with the discussion in section 2 on the outsider’s perspective, I will consider here the question ‘Given some reference \( p \), where we assume \( 1 > p > 1/2 \), is the optimal belief of the manager \( \mu_{M,A} > p \)?’.

The essential conclusions of this section are as follows. Absent sorting, vision is optimal for a restricted but important class of belief distributions. Sorting makes again a big difference: with sorting vision is optimal under much more general conditions. Furthermore, the impact of vision decreases as the uncertainty goes to zero.

Before going into the formal analysis, it is useful to summarize the forces that determine the optimal CEO belief.

• The motivation/demotivation effect.

• The influence on the project choice.

• The coordination effect.

• The cost of wrong implementation decisions.
All these are modified by the presence of sorting.

The analysis that follows considers the optimality of vision with and without sorting. It does not, however, consider the coordination effect since the latter just strengthens the results while complicating the analysis significantly.

5.1 Profitability of vision absent sorting

Consider first the case without sorting. With employees’ beliefs drawn from a distribution $F$, the firm’s reference expected profits can be written\(^{10}\):

$$E[\pi] = \int_0^{\mu_{M,B}} \hat{e}_M^2 \mu_{M,B} \left((1-p) - \frac{\mu_{M,B}}{2}\right) f(u)du + \int_{\mu_{M,B}}^{1} \hat{e}_M^2 \mu_{M,A} \left(p - \frac{\mu_{M,A}}{2}\right) f(u)du$$

Since the balance of forces depends on the distribution of beliefs, we cannot say anything in full generality. There exists, however, an interesting class of distributions that does allow clear conclusions. Consider in particular the following restriction.

**Assumption 5** All agents think $A$ is the optimal project, i.e. supp $F \subset (1/2, 1]$.

This assumption will, for example, be satisfied when all employees approximately hold the reference belief. It eliminates all employees who get demotivated or switch actions as the manager gets more convinced. The remaining trade-off is then between the motivation effect and the cost of wrong implementation.

**Proposition 6** Let $A$ hold. If, in deviation from the general setup, the agent always comes up with a project, independent of his effort, then the unique optimal belief is the reference belief. If the probability of coming up with a project is $e$, then vision is strictly optimal.

The intuition is simple. As long as there is some effect of effort, the motivation effect dominates, since the effect of wrong implementations is second order at $\mu_{M,A} = p$. When the motivation effect is completely absent, then the cost of making wrong decisions will make it optimal to hold the reference belief.

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For a practical example, let $\gamma_E = \gamma_M = 1/2$ and $c(e) = \frac{e^2}{8}$. In that case, the optimal belief is $\mu_{M,A} = \min(\frac{4p}{3}, 1)$ for any distribution of beliefs that satisfies A5.

5.2 Profitability of vision with sorting

When sorting occurs, an important cost of vision gets eliminated for the most visionary firm: no employee will get demotivated by the manager’s vision. Moreover, at small levels of overconfidence the cost of wrong implementations is second order since it concerns only projects that go marginally the other way. This suggests that ‘vision is always good in moderate amounts’. There is still one caveat, however: it is theoretically possible that all potential employees hold beliefs opposite to the reference belief $p$. A visionary firm ($\mu_M > p$) could then end up with nearly no employees and thus nearly no profits.

For the formal analysis, let the focal firm face one competitor whose manager holds the reference belief $p$. Consider any of the following two assumptions.

Assumption 6 The support of $F$ is contained in $[(1-p), 1]$.

Assumption 7 The distribution of beliefs $F$ First Order Stochastically Dominates some symmetric distribution\(^{11}\) and $1/2 < p < 1$.

This second assumption says that the distribution of beliefs weakly favors the side of the reference belief, in the sense that it can be generated from some symmetric distribution by moving some probability mass up. This holds for example when $F(x) \leq 1 - F(1 - x)$ or when $F$ is the Beta-distribution $F(x; a, b) = \frac{\int_0^x u^{a-1}(1-u)^{b-1}du}{\int_0^1 u^{a-1}(1-u)^{b-1}du}$ with $0 < b \leq a < \infty$.

The following results confirm that vision is optimal under very weak conditions.

Proposition 7 Under A6 or A7, vision is optimal (against a firm whose manager holds the reference belief).

Figure 3 shows the optimal belief in function of $p$ when $\gamma_E = \gamma_M = 1/2$ and $c(e) = \frac{e^2}{8}$ and the employee beliefs are distributed $U[0, 1]$.

Appendix D considers, for a specific parameterization and a slightly modified setup, the equilibrium in a more general game in which both firms can choose their manager. The key result is that the optimal managerial beliefs become still more extreme.
The optimal belief (against a firm with ‘reference’ beliefs) in function of the reference belief for $\gamma_E = \gamma_M = 1/2$ and $c(e) = \frac{e^2}{N}$.

The effect of uncertainty

One would conjecture that the impact of vision decreases as the uncertainty about the true state goes to zero. This would indeed explain why vision seems more important in high tech industries than, say, in the steel industry. This is indeed what happens in the model. The reason is that with less uncertainty in the reference belief, there is automatically less room for a manager to have stronger beliefs than the reference and thus for vision to make a difference.

With sorting, however, we have to be careful in stating the result. In this case, the gain from vision has two components. The first is the gain from inducing sorting with a minimum (limit) deviation from the reference belief, which I call the pure sorting effect. The second is the extra gain from holding a belief that is strictly greater than $p$, which I call the gain beyond the pure sorting effect. The result applies only to the latter.

For the formal statement of the result, note that the correct measure for ‘uncertainty’ is $p(1-p)$, the variance of the binomial distribution generated by the reference probability. The condition in the following proposition that $p \to 1$ thus captures the fact that the uncertainty decreases (given that we assumed $p \geq 1/2$).

**Proposition 8**  

*Absent sorting, the profit gain from vision, if any, converges to zero as $p \to 1$.  

Formally $\max_{\mu_M,A \geq p} E[\pi] - E[\pi | \mu_M,A = p] \to 0$ as $p \to 1$.  

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• When vision is optimal under sorting, the profit gain from vision beyond pure sorting converges to zero as \( p \to 1 \). Formally 
\[
\max_{\mu_M, \mu_A} p E[\pi] - \lim_{p \to 1} E[\pi] \to 0 \quad \text{as} \quad p \to 1.
\]

6 Evidence

While this theory has not been tested explicitly, there exists empirical evidence that speaks to many of the underlying issues.

**Sorting** The key prediction of this paper is that the beliefs of the CEO will attract employees with similar beliefs. There exists evidence on a closely related issue: how the fit in terms of values influences hiring, satisfaction, and turnover. Meglino, Ravlin, and Adkins (1989), for example, show that more value congruence between a manager and his subordinate is associated with higher satisfaction and commitment on the part of the latter. Chatman (1991) shows that a higher person-organization fit with respect to organizational values leads to higher satisfaction and a lower intent to leave the firm. Judge and Bretz (1992) showed that a fit in work values significantly improves the probability of accepting a job offer. Chatman, Carroll, and Harrison (2001) show that the probability of a high-level employee exiting the firm increases as his beliefs about corporate values differ more from those of his business unit manager. Since all these studies relate to values instead of beliefs, it is important to note that there is a very close relationship between the two. The management literature, for example, uses the concepts almost interchangeably. Savage derived beliefs from preferences. The current model could even be reformulated in terms of values instead of beliefs. For all these reasons, I do believe that this research on values is relevant to the issues considered in this paper.

More indirect evidence comes from CEO and non-CEO turnover. In particular, the theory implies that when the CEO gets replaced by someone with different beliefs, then this will cause turnover in the levels below him. Hayes, Oyer, and Schaefer (2002) show indeed a positive association between turnover of the CEO and non-CEO top executives. The association is stronger when the new CEO is an outsider. This is consistent with the theory if we believe that outsiders are more likely to have different beliefs. Moreover, even the year after CEO turnover, non-CEO top
managers are more likely to leave the firm’s top management group. This suggests that the effect is not just due to group layoffs. More casual evidence for the theory comes from the business press discussions on the consequences of mergers and acquisitions.

**Impact of the CEO**  Implicit in the paper is also the idea that the beliefs of the CEO can make a difference to firm behavior and performance. Casual empiricism suggests clearly that the person of the CEO matters to a firm’s behavior and performance. Managers disagree as much about the right actions as academics disagree about the right theories. There is also more systematic evidence on this issue. Bertrand and Schoar (2001) show that the identity of CEO’s and CFO’s covaries with important policy variables such as internal versus external growth or aggressiveness of financial policies. While they refrain from identifying the origin of their manager-specific effects, the results are obviously consistent with the current theory. In particular, the fact that the MBA programs these managers attended are correlated with their policies suggests that beliefs about the right course of action might be an important factor.

More indirect evidence comes from the fact that much of the performance differences among firms cannot be accounted for by industry or market share (Schmalensee 1985, Rumelt 1991, McGahan and Porter 1997). Along the same lines, the study by Bertrand and Schoar (2001) was partially motivated by the fact that much of the heterogeneity in financial policies remains unaccounted for.

### 7 Organizational Culture

The dominant definition of organizational culture in the sociological and management literature is that of culture as shared beliefs or assumptions (Schein 1984, Schein 1985, Kotter and Heskett 1992). Note now that the sorting mechanism in this paper causes the set of beliefs within a firm to be relatively homogenous, but different from the set of beliefs in other firms. This corresponds thus quite well with the idea of corporate culture as shared beliefs. As such the paper shows how a manager’s vision may play a key role in the formation of culture. This is consistent with the management literature, which has often argued that CEO’s and founders have a profound influence on culture. Schein (1985), for example, focuses on the role of leaders in the formation of culture, as the title *Organizational Culture and Leadership* suggests. In discussing how culture may be
changed, Kotter and Heskett (1992) stress the importance of bringing in a manager with strong beliefs. They write that ‘[a]lthough it is difficult to validate, it would appear that in many (possibly all) cases [of successful cultural change] these [leaders] were asked to run their organizations because they had those beliefs and visions.’ This perspective on culture as shared beliefs and its implications for the formation and development of culture is developed in two parallel papers (Van den Steen 2002b, Van den Steen 2002a).

In giving a perspective on the formation of culture, these papers are complementary to existing economic perspectives on corporate culture (Kreps 1990, Cremer 1993, Lazear 1995, Hermelin 1999). In particular, the current work is most closely related to Cremer (1993), whose definition of culture as shared knowledge is similar to a definition of culture as shared beliefs, and Lazear (1995), who studies an evolutionary process in which elements of culture are transmitted like genes.

8 Conclusion

This paper showed how vision, in the sense of a strong belief by the manager, causes sorting in the labor market that aligns the beliefs of employees with those of the manager, and indirectly with those of each other. This alignment had important consequences for incentives and coordination. The paper concluded that vision is profitable under weak conditions. In the appendices, it also derives some results that are of independent interest. Most important is probably the insight that vision increases the variance of the performance. Overall, the paper suggests clearly that a manager’s indirect influence on his firm’s behavior and performance might be as important as his direct actions and decisions.

There are many interesting extensions to this work:

- The current analysis is essentially static. How strong beliefs interact with learning or how a CEO will choose his successor are interesting dynamic issues that are left to explore. The issues of communication, influence, and conviction are also completely absent from this model. Lazear (1995) presented some results in this sense.

- The firms in this model focus all their energy single-mindedly on one course of action. This raises the question under which circumstances the firm would do better to hedge its bets by
spending part of its resources on the other option. This is obviously related to the issues of diversity (Athey, Avery, and Zemsky 2000) and autonomous strategic action by middle management (Burgelman 1983, 1991, 1994, Rotemberg and Saloner 2000).

- Homogeneity of beliefs is likely to have implications far beyond the ones discussed here.

While I cited already some empirical and ‘casual’ evidence that supports the theory, real testing remains to be done. The most effective method would probably be experimental. This is facilitated by the fact that only the employees’ perceptions matter, so that individual experiments suffice. Alternatively, one could test how employee motivation and satisfaction, and firm hiring and firing are affected by the fit in beliefs between the employee and the organization or manager, as has been done in the psychology and sociology literature for values. A direct empirical test of the vision-performance relationship is complicated by its non-monotone form. Testing the second moment prediction (that more visionary firms have more variation in their results) might be more powerful.

From a broader perspective, I believe that economic theory, and in particular organizational economics, has much to gain from studying the consequences of differences in beliefs.
Notes

1 This deterministic connection between action-state fit and success simplifies the theory but is not necessary. In particular, in a multi-period model, it might be better to specify a probabilistic connection so that the state does not get completely revealed by the outcome of the first period.

2 In particular, agents not only have (prior) beliefs about the state \( x \in \{a, b\} \), but also about what game they are playing, how correct their information is, etc. In this particular case, agent \( i \) puts probability one on the signal being correct with probability \( \mu_{i,A} \), but agent \( j \) puts probability one on \( \mu_{j,A} \), which might be different.

3 In this context, it is interesting to note that just before Microsoft announced what later became known as the .NET initiative, Bill Gates handed the title of CEO to Steve Ballmer. The latter had a reputation of believing much more in the Internet than Gates himself.

4 Kouzes and Posner (1987) define it as ‘an ideal and unique image of the future’; Kotter (1990) defined vision as ‘a description of something (...) in the future, often the distant future, in terms of the essence of what it should be.’ The Cambridge Dictionaries Online defines vision as ‘the ability to imagine how a country, society, industry, etc. will develop in the future and to plan in a suitable way’. A clear example of a widely expanded notion of vision is the work of Collins and Porras (1994) who essentially use the term ‘visionary’ as a more catchy synonym for ‘admired’. Such use of the term has given it a nearly magical meaning. People who are used to this more liberal use of the term, will consider the current definition very narrow.

5 The proof of lemma 1 shows that all SP equilibria have the same outcomes and are equivalent to one in which the firm offers a wage \( \tilde{w} = 0 \) and the employee accepts. Appendix C considers a variation of the model that allows to study wage effects.

6 An extension of the model might also complement the theory of delegation (Vickers 1985, Prendergast 1993, Aghion and Tirole 1997, Baker, Gibbons, and Murphy 1999, Zabojnik 2001). The main conjecture would be that, (all else equal) with effort complementary to the probability of success, the project type decision should be taken by the person with the more important non-contractible effort.

7 Note that effects similar to the ones described here can occur in other types of markets. In particular, investors (in financial markets) will be willing to pay more for equity in firms whose managers have beliefs that are similar to their own.

8 It should be noted that this extreme outcome is partially due to the limited state-space. Nevertheless, even with richer state-spaces, I conjecture that the essence of the result carries over.

9 I let the agents choose sequentially in random order since that procedure excludes some ‘unreasonable’ equilibria that appear when they choose simultaneously. Note that this differs from the setup without sorting. In that case, the issue of unreasonable equilibria doesn’t arise, while sequential choice does complicate things enormously there.

10 Remember that hats indicate optimal choice variables and value functions, and that the dependence of maximizers on parameters has been suppressed. In particular, \( \hat{e} \) is function of the type of action taken, \( \gamma_E, \gamma_M, \mu_{M,A} \) and \( u \).

11 A distribution \( F \) first-order stochastically dominates a distribution \( G \) when \( F \) is generated from \( G \) by adding to
every outcome some non-negative random variable. An alternative definition is that $F \leq G$, i.e. some probability mass of $G$ is shifted upwards to obtain $F$.

12This is not necessarily the same as ‘exploitation’ in the sense of March (1991) or the absence of innovation. In particular, the manager’s vision can focus the firm’s actions on innovation and exploration, at the cost of exploitation. It is plausible, however, that vision often leads to exploitation at the cost of exploration.
A Robustness of the Results and Potential Objections

This appendix considers how the results are affected by changes in assumptions or in setup.

A.1 Contractibility and renegotiation assumptions

Consider first what happens when effort $e$ would become contractible. Assume in particular that, after the employee has accepted the wage offer $\tilde{w}$, the firm can offer an extra effort-based compensation $b(e)$. If the employee rejects, the game just proceeds as before. If the employee accepts, this effort compensation becomes non-renegotiable (while the wage $\tilde{w}$ remains renegotiable)\textsuperscript{13}. The following informal argument suggests that all qualitative results are preserved in this case. Let $\tilde{e}$ and $\hat{e}$ denote the effort that the employee would choose with and without the extra compensation scheme. Any compensation scheme $b(e)$ can be replicated by one that also induces $\tilde{e}$ and that simply consists of a bonus $\tilde{b} = b(\tilde{e})$ if and only if the employee chooses $e = \tilde{e}$. This bonus must be non-negative (since the employee can always reject $b(e)$ and choose $\tilde{e}$ anyways).

It is also straightforward that we must have $\tilde{e} \geq e$ (since the firm will never pay anything extra for a lower effort) and $\tilde{b} = [\tilde{e} - \hat{e}]\mu_{\mu,\mu,\mu,\gamma,\gamma,\gamma} - [c(e) - c(\tilde{e})]$ (since this is the minimum that the firm has to offer to make the employee willing to choose $\tilde{e}$). It now follows already that the employee’s project choice and utility are the same as in the original game. The satisfaction and sorting effects are thus preserved. Moreover, the effort will be larger than before and moves with the manager’s $\nu_i$ as in the original game.

If instead of effort, I made the project type contractible in the way I just described, then the qualitative effects would again be preserved. The choice of project type will still be influenced by both beliefs although the manager’s belief will get more weight. The employee’s motivation and satisfaction will still depend on his own and his manager’s belief in the action undertaken. So we also get sorting. The case where both project type and effort are contractible in the way described is essentially a combination of both cases, so that we would expect the qualitative results to be again preserved.

A second issue is the non-contractibility of the agent’s participation, which leads to the ex-post renegotiation. We noted already that the results extend to the case where the employee gets instead some exogenously determined benefit, such as improved outside options or satisfaction from a successful project. A very different case, however, is that where the firm can make an up-front offer of wage plus bonus, which are then non-renegotiable. The choice of the optimal bonus introduces a second optimization problem in the game, which complicates the analysis. While the original results seem to hold under appropriate restrictions on the third derivatives, a full analysis of this case awaits further research. Alternatively, this game could be simplified by assuming that the size of the bonus is exogenously given, but this brings us back to the above model with exogenously determined benefits.
A.2 Other modifications

Consider now some more structural changes to the model. A first important modification is the timing of the renegotiation. We could for example imagine that the firm and the employee renegotiate at the time of implementation (i.e. that the employee’s support is critical for implementation). It can be shown that the employee will then undertake the project that the manager considers best, that he spends more effort as the manager has stronger belief, and that he gets the higher expected utility from working for the manager with the stronger beliefs. It thus also follows that vision is optimal. The key change, however, is that the sorting is not based any more on the employee’s beliefs (since his wage gets fixed before the project gets realized).

A different set of modifications pertains to the role of employee effort $e$. In particular, in the model employee utility was strictly increasing and supermodular in $e, \mu_{M,Y}$ and $\mu_{E,Y}$. While this appears to be the more natural case, these properties do not necessarily always hold in modified games$^{14}$. The property that the employees’ utility is increasing in the manager's belief in the project he undertakes, tends to hold in most situations. In that case, vision still causes sorting and an increase in satisfaction. The complementarity between $e$ and $\mu_{M,Y}$ however, is more fragile. In some situations, the motivation effect may get lost or even reversed. If so, the optimality of vision depends on the exact strength and interaction of the different effects.

Finally, one might wonder about the impact of the allocation of authority. We consider two cases of interest. First, if the manager were to choose the type of project (while the employee still chooses his own effort level), his criterion would put strictly more weight on his own beliefs. Second, the case in which the employee makes the implementation decision is identical to a situation where the manager happens to have the same belief as the employee. In particular, the analysis implies that the firm would want to hire overconfident employees.

A.3 Potential objections

An important potential objection to this theory is the observation that most decisions, especially those on lower levels, are taken without the CEO’s explicit approval. There are a number of reasons why this poses less of a problem than it might seem. First of all, sorting happens even if a CEO intervenes only sporadically. Second, CEO’s do get involved in the key decisions, and that are the ones employees care most about. Third, there is a cascading effect. The CEO’s beliefs will cause a sorting on the level just below him. The firm will thus attract and retain top managers whose beliefs are aligned with those of the CEO. This sorting then leads to the same effect on the next level. Moreover, according to the results on optimal selection of managers, the CEO will want to hire subordinates who have stronger beliefs than his own. The effect on the next level might thus even be stronger.
Another potential objection is that there might be more effective mechanisms to obtain the effects that vision has in this paper. In particular, wouldn’t it be possible to simply adapt the budget procedures to accept projects more easily? There are a number of reasons why such solution does not work, except when combined with the right managerial beliefs. First and most important, the estimates of future revenues and costs are subjective. Whether top management considers them reasonable and thus accepts the project proposal as reliable depends of course on their beliefs. What matters to employees is the final decision by the relevant manager and thus his beliefs. Budget procedures can only provide tweaks to these fundamental determinants. Consider, for example, the fact that AT&T’s top management originally believed that mobile phones would never surpass the status of gadget. Even very easy budget procedures would never have made them pursue this innovation seriously. On the other hand, Motorola thought that there was a huge market for globally functioning mobile phones, even if the phone weighed a few pounds. More restrictive budget procedures probably wouldn’t have convinced them not to sink $6 billion dollars in the Iridium project. A second problem is that of commitment. Budget approval procedures tend to be under the authority of the CEO, who can overrule them at any time. So, in the end, the manager’s beliefs will matter anyways. Finally, such laxer budgeting procedures cannot replicate the sorting effect since the project types are non-contractible.

An objection of a very different kind is the question why it isn’t sufficient for the CEO to ‘proclaim’ the vision: why can’t the CEO simply ‘decide’ that the firm will now follow a more quality-oriented strategy? The issue here is of course again one of commitment. If the lower level managers don’t expect the CEO to follow up on his claims, then they will also not change their actions. The only real commitment, apart from contracting, is the fact that the manager really believes in the strategy. This issue is well recognized by management consultants, who will typically spend more time convincing top management that the conclusions are right than deriving the conclusions themselves. It is also interesting to observe that Ballmer, who was known as an Internet-believer, took over from Gates as CEO of Microsoft less than a month before Microsoft officially announced its .NET strategy.

Probably a more fundamental question is how management can convincingly communicate its beliefs to employees or build a reputation for holding certain beliefs. This paper sidesteps that important question by assuming common knowledge of beliefs, in order to keep the analysis transparent. It is an important and interesting issue for further research.
Why the Best and Worst Firms Are Always Visionary

Vision and strong beliefs increase the variance of performance. If you act as if you knew the future and you turn out to be right, then your actions will be ex-post optimal, even if they were ex-ante suboptimal given the objective odds. So we would expect that even when vision is not optimal, ex-post the best (but also worst) firms in the market will be those with visionary managers. This is especially important for empirical analyses of the effects of vision on firm performance, since it may induce a strong selection bias.

To confirm this argument formally, consider an economy with $N$ firms with $K$ employees each. Each employee of firm $n$ faces a choice of action $X_n \in \{A_n, B_n\}$. The state of the world relevant to firm $n$ is an independent draw $x_n$ from $\{a_n, b_n\}$ with probabilities $p$ and $(1-p)$ respectively, where we assume $1 > p > 1/2$. All employees hold the reference belief $\mu_e = p$, which implies $A_5$. Let the agent always come up with a project, independent of his effort, so that the reference belief (‘no vision’) is optimal by proposition 6. The managers’ beliefs are independent draws from a distribution of beliefs $G$ with support $[p, 1]$ and with an atom of size $0 < P[p] < 1$ at the endpoint $p$. Any such draw thus results with probability $P[p]$ in a manager who holds the reference belief. With probability $1 - P[p]$, the draw will be a ‘visionary’ manager with a belief $\mu_v > p$. Assume that the firms face equivalent opportunities: the implementation cost $I_k$ of the $k$th employee’s project is identical for all firms. The following proposition confirms that visionary firms will have extreme results:

**Proposition B.1** As the number of firms $N \to \infty$ and the number of employees per firm $K \to \infty$, the probability that the best (and worst) firms have visionary managers (as indicated in figure 4) and that the profit difference with any firm with an objective manager is strict, converges to one. The probability of being ex-post the best (or worst) performing firm increases in the firm’s rank in terms of strength of its manager’s belief.

**Proof:** The probability that the manager of a randomly selected firm has belief $\mu \geq x$ for some $x$ such that $1 > x > p$, is $1 - G(x) > 0$. That fact combined with the fact that $1 > p > 0$ implies that both the event that ‘there exists some firm with belief $\mu \geq x$ which turns out to be correct about the true state of the world’ and the event that ‘there exists some firm with belief $\mu \geq x$ which turns out to be wrong about the true state of the world’ are almost surely true in the limit as $N \to \infty$.

The difference in profit between a visionary firm with belief $\mu \geq x > p$ and an ‘objective’ firm with belief $p$ that turn out to be correct equals $\sum_{k=1}^K I(\gamma_{k,M} < I_k \leq \gamma_{k,M}) (\gamma_{k,M} - I_k)$ which is almost surely strictly positive for $K \to \infty$. Analogous arguments show that there are strict differences in profitability between an objective firm that is right and one that is wrong and between an objective firm that is wrong and a (strictly) visionary firm that is wrong. Combined with the earlier conclusion, this proves the first part of the proposition.
For the last part of the proposition, consider a firm that has the $m$'th rank in terms of strength of belief. The probability that the firm turns out to be the weakly best performing firm equals the probability that this focal firm is correct while the $m-1$ firms with stronger beliefs turn out to be wrong, and is thus $(1-p)^{m-1}p$. The probability that it turns out to be the worst performing is analogously $p^{m-1}(1-p)$. Both decrease in $m$, so that they increase as the firm is ranked higher in terms of belief strength.

The intuition is exactly the one set forth at the start of this appendix. The result confirms the initial conjecture.

**Figure 4:** The extreme performance of visionary companies

**Corollary B.1 (In Search of Excellence)** For a large enough number of firms and employees per firm, the very best firms in the market have (almost always) visionary managers.

**Proof:** This follows from the proof of proposition B.1.

This might also explain the observation that many famous ‘visionary’ managers were actually founders or co-founders of their firm (e.g. Steve Jobs, Sam Walton, Bill Gates, Larry Ellison, Scott McNealy). The theory suggests that these people might actually have had too strong beliefs (from an ex ante perspective) but happened to be right. Note also that such extreme believers will spend extreme effort on their ideas.

This spurious effect will be stronger as there is more underlying uncertainty (which might explain why 4 out of the 5 names above come from the software sector):

**Proposition B.2** The difference in ex-post profitability between the firm of the most visionary manager ($\mu_v = 1$) and that of the closest objective manager increases in the reference uncertainty $p(1-p)$.

**Proof:** Note that, given that we assumed $p > 1/2$, it is sufficient to prove that that difference increases as $p$ decreases to $\frac{1}{2}$.

Consider first a visionary $\mu_v = 1$ firm that turns out to be right. The objective firm that is closest in terms of profit is just one that is right. The difference in profitability is $\sum_{k=1}^{K} I_{\gamma_{M}p < I_k \leq \gamma_{M}} (\gamma_{M} - I_k)$ which increases as $p$ decreases since each $\gamma_{M} - I_k$ term is positive and the number of terms increases as $p$ decreases. The proof for a maximally visionary firm that turns out to be wrong is analogous.

The intuition is that objective managers are very cautious in markets with high uncertainty. There is thus much more room for overconfidence to make a difference.

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C The Effect on Wages and Compensation Forms

The original game had no wage effects. While this makes the model more transparent, it also eliminates some potentially interesting elements. To study the effect of beliefs and vision on wages, modify the original setup as follows:

• Instead of the firm making a take-it-or-leave-it wage offer to the employee in stage 1, the firm and the employee negotiate the wage. Assume here Nash bargaining with bargaining power $0 \leq \delta \leq 1$ for the employee and $(1 - \delta)$ for the firm (i.e. a dollar gain from agreement gets split $(\delta, (1 - \delta))$).

• When the firm and the employee renegotiate, the employee’s only outside option is continuing with the firm at his original wage. He does not have the option any more to take the outside job at wage $w = 0$.

The effect on wages is actually quite complex. Consider first intuitively what happens within a firm. Assume that employees have already sorted and that the wage is fixed. The expected utility of an employee now increases in the belief of his manager in the firm’s current course of action\(^{18}\). It follows that, at the earlier negotiation, an employee is willing to accept a lower wage. But a manager with stronger belief also expects a higher gain from having the employee and is thus willing to offer a higher wage. The balance between those two will depend, among other things, on the relative bargaining power of both parties. But that is not all. The sorting effect also plays a role and again with two effects. On the one hand, the manager with the stronger belief is assured to get the employees that believe in what he believes. They are willing to work for relatively low wages, while the manager is willing to offer them relatively high wages. On the other hand, that firm also gets employees who do not believe in what the manager believes. They require higher wages, while the manager doesn’t want to pay them as much as those other employees. Overall, the balance is again not clear. Finally, the compensation package complicates things even further. Many of the employees believe stronger in the course of action of the firm than the manager does. Paying them with call options is thus very cheap (since they value these higher than the manager does)\(^{19}\). In that case, the compensation will also look lower from an outsider’s perspective than it looks from the employees’ perspective.

While a full analysis would lead us too far, the following proposition formalizes the first part of the above argument for a situation with 2 firms with $c(e) = \frac{x^2}{2}$. Let both managers have beliefs strictly between zero and one, and let $X_j$ denote the actions that employees of firm $j$ undertake.

Proposition C.1 For any set of managerial beliefs $\mu_{M_1, X_1}$ and $\mu_{M_2, X_2}$,

• there exists $\delta_1 > 0$ such that for all $\delta < \delta_1$, the wage of each employee of firm $j$ decreases in $\mu_{M_j, X_j}$, the belief of its manager in the course of action of the firm, $X_j$. 

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• there exists $\hat{\delta}_2 < 1$ such that for all $\delta > \hat{\delta}_2$, the wage of each employee of firm $j$ increases in in $\mu_{M_j, X_j}$, the belief of its manager in the course of action of the firm, $X_j$.

The proof is below.

As mentioned before, the compensation form can also depend on the differences in beliefs. In particular, the manager and the employee will typically differ in their beliefs and thus in their valuation of stock and stock options. To study this more formally, consider a situation with two firms and assume that the firm can pay part of the compensation in stock options, subject to some restriction on the wage part $\tilde{w} \geq A$.

The up-front bargaining now also determines the form of compensation\textsuperscript{20}. Assume further that there is no volatility. To exclude any incentive effects, consider finally the limit as the number of employees goes to infinity. Let $X$ denote the action that the employees of the focal firm undertake.

**Observation 1** For all employees with $\mu_{E, X} > \mu_{M, X}$, the equilibrium compensation package maximizes the share of the wage that is paid in stock options. Within one firm, employees with stock options spend more effort than employees without stock options (even though stock options have no incentive effect).

**Proof:** Consider wlog. the firm that undertakes $A$. Let the expected value of a share in this firm conditional on $A$ being correct be $V_A$, and analogous for $V_B$. Since these are the only possible outcomes, any reasonable stock option has its strike price between the two. Let this strike price be denoted $p$. From the perspective of the manager, the value of such option is $\mu_{M, X}(V_A - p)$ while it is worth $\mu_{E, X}(V_A - p)$ to the employee. Whenever, $\mu_{E, X} > \mu_{M, X}$, paying in stock options thus creates value. Since the bargaining will select a Pareto-optimal outcome, the wage will be paid maximally in stock options for these employees.

Note that employees with stock options are those with the stronger beliefs, and thus also those that spend most effort. (Remember that we considered $n \to \infty$ to exclude any incentive effects.) This implies the second part of the observation. $\blacksquare$

Note how this gives a spurious correlation between stock options and effort levels.

These results are consistent with the empirical analysis of Oyer and Schaefer (2002), who conclude that broad-based options are, among others, consistent with the idea that they are used as a cheap way to pay employees with overoptimistic beliefs about the prospects of the firm.

**Proof of Proposition [C.1]:** Let $X$ denote the project undertaken by the employees of the focal firm. Let the initial wage be $\tilde{w}$. Absent any project, the payoffs to the employee and the firm are respectively $\tilde{w}$ and $-\tilde{w}$. If the project turns out to be a success if the employee continues, the joint gain is 1. It follows that the payoff to the employee in case of a successful project is $\tilde{w} + \gamma_E$ and to the firm $-\tilde{w} + \gamma_M$ (before the investment cost). The firm’s expected gain from implementing a project $X$ is thus $\gamma_M \mu_{M, X}$. Since the firm implements only if that is larger than $I$, the firm will implement with probability $\gamma_M \mu_{M, X}$. So the net
expected return to the firm of an employee coming up with a project is

\[
\int_{\gamma_{M,\mu,M,X} \geq I} [\gamma_M \mu_{M,X} - I] dI = \int_0^{\gamma_M \mu_{M,X}} [\gamma_M \mu_{M,X} - I] dI = (\gamma_M \mu_{M,X})^2 - \frac{(\gamma_M \mu_{M,X})^2}{2} = \frac{(\gamma_M \mu_{M,X})^2}{2}
\]

while the expected net return to the employee is

\[
\int_{\gamma_M \mu_{M,X} \geq I} \gamma_E \mu_{E,X} dI = \gamma_M \gamma_E \mu_{M,X} \mu_{E,X}
\]

The optimal effort of the employee is then \( \hat{e} = \gamma_M \gamma_E \mu_{M,X} \mu_{E,X} \). So at the time of the first wage negotiations, the expected net payoffs for the employee and the firm conditional on wage \( \hat{w} \) are \( \hat{w} + \frac{(\gamma_M \gamma_E \mu_{M,X} \mu_{E,X})^2}{2} \) and \(-\hat{w} + \frac{(\gamma_M \gamma_E \mu_{M,X} \mu_{E,X})(\gamma_M \mu_{M,X}^2)}{2} \). Since the outside options for both are zero, the joint gain is \( \frac{(\gamma_M \gamma_E \mu_{M,X} \mu_{E,X})^2 + (\gamma_M \gamma_E \mu_{M,X} \mu_{E,X})(\gamma_M \mu_{M,X}^2)}{2} \) so that according to Nash bargaining with bargaining power \( \delta \) for the employee, the employee’s expected utility is \( \delta \frac{(\gamma_M \gamma_E \mu_{M,X} \mu_{E,X})^2 + (\gamma_M \gamma_E \mu_{M,X} \mu_{E,X})(\gamma_M \mu_{M,X}^2)}{2} \) so that the wage is

\[
\hat{w} = \frac{\gamma_M^2 \gamma_E^2 \mu_{E,X}^2}{2} (\delta \gamma_M \mu_{M,X} - (1 - \delta) \gamma_E \mu_{E,X})
\]

Finally,

\[
\frac{d\hat{w}}{d\mu_{M,X}} = \frac{\gamma_M^2 \gamma_E \mu_{E,X} \mu_{M,X}}{2} \left[ 3\delta \gamma_M \mu_{M,X} - 2(1 - \delta) \gamma_E \mu_{E,X} \right]
\]

This derivative is negative when \( 3\delta \gamma_M \mu_{M,X} < 2(1 - \delta) \gamma_E \mu_{E,X} \). This will be the case for all employees if \( 3\delta \gamma_M \mu_{M,X} < 2(1 - \delta) \gamma_E (1 - \mu_{M,X}) \) since with sorting, the belief of the employee with the weakest belief is at least \( 1 - \mu_{M,X} \). With

\[
\hat{\delta}_1 = \frac{2 \gamma_E (1 - \mu_{M,X})}{3 \gamma_M \mu_{M,X} + 2 \gamma_E (1 - \mu_{M,X})}
\]

the first part of the proposition thus holds.

The derivative is positive when \( 3\delta \gamma_M \mu_{M,X} > 2(1 - \delta) \gamma_E \mu_{E,X} \). This will be the case for all employees if \( 3\delta \gamma_M \mu_{M,X} > 2(1 - \delta) \gamma_E \) since the belief of an employee is at most 1. With \( \hat{\delta}_2 = \frac{2 \gamma_E}{3 \gamma_M \mu_{M,X} + 2 \gamma_E} \) the second part of the proposition holds.
D Competing on Vision

The main paper derived conditions under which vision is optimal against a firm with reference beliefs. When both firms are allowed to choose their manager (and his beliefs), we would expect them to ‘overbid’ each other in trying to attract the most favorable group of employees. This appendix explores that conjecture for a specific parametrization.

In the original setup, there were a discrete number of potential employees. Each employee’s belief was an independent draw from \( F \). Firms were not limited in size. To make the analysis tractable I modify this setup as follows. There is a \([0, 1]\) continuum of potential employees. Employee \( x \in [0, 1] \) has belief \( y = F^{-1}(x) \) so that there is again a distribution of beliefs \( F \). If there are two firms, then each can hire at most a group of employees of Lebesgue measure \( 1/2 \). Assume further that at the very start of the game both firms simultaneously choose the beliefs of their manager. If they choose the same belief, then each is considered the highest with probability \( 1/2 \). Furthermore, let again \( \gamma_R = \gamma_M = 1/2 \) and \( c(e) = \frac{1}{8} e^2 \). The employee’s effort is thus again \( \hat{e} = \mu_{M,A} \mu_{E,A} \).

Consider now a situation in which \( F \) has support \([1/2, 1]\), continuous density \( f \), and median \( \hat{\mu}_F \). Both a single firm and a firm that faces a firm with the reference belief would set \( \hat{\mu}_{M,A} = \min(4\sqrt{3}, 1) \). With two firms simultaneously choosing their manager’s belief, the unique symmetric equilibrium is determined as follows. Let \( \Pi_H(\mu) \) denote the profit of the focal firm when its belief \( \mu \) is the stronger, and analogous for \( \Pi_L(\mu) \). If \( \Pi_L(\hat{\mu}_{M,A}) < \frac{\Pi_H(1)+\Pi_L(1)}{2} \) then the equilibrium is for both to play \( \mu = 1 \) for sure. If that is not the case, then the equilibrium is as follows. Each player sets \( \check{\mu} = 1 \) with probability \( P \) and randomizes with support \([\hat{\mu}_{M,A}, \check{\mu}]\) according to

\[
G(x) = \frac{\Pi_L(\hat{\mu}_{M,A}) - \Pi_L(x)}{\Pi_H(x) - \Pi_L(x)}
\]

where \( P \) is defined by \( P = 2\frac{\Pi_H(1)+\Pi_L(\hat{\mu}_{M,A})}{\Pi_H(1)-\Pi_L(1)} \) and \( \check{\mu} \) is defined by \( 1 - \max(P, 0) = \frac{\Pi_L(\hat{\mu}_{M,A})-\Pi_L(\hat{\mu})}{\Pi_H(\hat{\mu})-\Pi_L(\hat{\mu})} \).

Note that both firms almost surely choose a manager with belief that is stronger than \( \hat{\mu}_{M,A} \), the optimal belief for a single firm and for a firm that faces a firm with the reference belief. It follows that in this case, competition unambiguously increases the optimal level of vision or overconfidence. Moreover, the average strength of belief increases in a \emph{median}-preserving spread of the distribution. With different parametrizations and distributions, I expect the same qualitative results to hold, but that analysis is beyond the scope of this paper.
E A Note on ‘Differing Beliefs’ in Economic Modeling

The model in this paper differs in one respect from most economic models: the agents knowingly entertain differing beliefs (without having private information). The reason for this assumption is pragmatic: differences in beliefs are at the heart of the issues studied here, and assuming common knowledge of differing beliefs is the most transparent and parsimonious way to study this question. Differing beliefs do not contradict the economic paradigm: while rational agents should use Bayes’ rule to update their prior with new information, nothing is said about those priors themselves, which are primitives of the model. In particular, absent any relevant information agents have no rational basis to agree on a prior. Harsanyi (1967), for example, observed that ‘by the very nature of subjective probabilities, even if two individuals have exactly the same information and are at exactly the same high level of intelligence, they may very well assign different subjective probabilities to the very same events’. The best argument for the traditional use of common priors is Aumann’s (1987) argument that they allow us to ‘zero in on purely informational issues’. Conversely, differing priors allow us to zero in on the implications of open disagreement and differing beliefs.

Van den Steen (2001) considers this issue in more detail. Among other things, it argues against the idea that differing priors might allow us to explain anything, it discounts the theoretical possibility that agents will make very large or infinite bets, and shows that the epistemic foundations for Nash equilibria in the sense of Aumann and Brandenburger (1995) extend to this context with differing priors on the payoff-space.

Utility versus beliefs An alternative way of modelling would be to incorporate beliefs in the utility function. Rotemberg and Saloner (2000) and Hart and Holmström (2002) can be interpreted in this way.

While this approach may seem more general, there are a number of issues. First of all, there are real differences between utility and beliefs. Utilities can be influenced by incentive contracts and beliefs by new data, but not the other way around. One implication is that beliefs provide a stronger commitment: financial incentives based on firm performance can undo biased preferences, but will not change beliefs about what the firm has to do to maximize that performance. Second, the benefits of stock-based compensation also differ. The use of stock options as a cheap way to compensate ‘believers’ does not work when there are only utility differences. A third implication relates to the dynamics of the situation. As more information becomes available, beliefs will change but utilities won’t. The two models thus have different implications for change and resistance to change. Overall, modelling beliefs via utilities risks missing out on important phenomena.

Apart from these analytical differences, there are obvious differences in interpretation. Since the phenomena in this paper are more naturally described in terms of beliefs, it makes sense to model beliefs explicitly.
F Proofs of the propositions

Lemma 1 All subgame perfect equilibria of this game have the same project types, effort levels and payoffs, and are equivalent to one in which the firm offers \( \hat{w} = 0 \) and the employee accepts.

Proof: Let us first determine the full equilibrium by backwards induction. Assume that the firm has made a wage-offer \( \hat{w} \) and the employee has accepted. Let \( \hat{w} = \max(\hat{w}, 0) \). The outside options in the bargaining are \( \hat{w} \) for the employee and \(-\hat{w}\) for the firm. These are also the final payoffs in case there is no successful project at the start of the renegotiation stage. Furthermore, Nash bargaining when there is a successful project will give the employee \( w = \gamma_E + \hat{w} \). The employee will thus always ask for renegotiation and end up with this wage while the firm gets \( \gamma_M - \hat{w} \).

Consider now the firm’s decision when it gets a proposal for a project of type \( X \). The firm implements iff \( \mu_{M,X}(\gamma_M - \hat{w}) + (1 - \mu_{M,X})(-\hat{w}) - I(= \mu_{M,X}\gamma_M - \hat{w} - I) \geq -\hat{w} \) or iff \( \mu_{M,X}\gamma_M \geq I \) i.e. with probability \( \mu_{M,X} \gamma_M \).

The employee’s payoff upon generating a project is \( \mu_{E,X}\gamma_M \mu_{M,X}(\gamma_E + \hat{w}) + (1 - (\mu_{E,X}\gamma_M \mu_{M,X}))\hat{w} = \hat{w} + \gamma_E \gamma_M \mu_{M,X} \mu_{E,X} \) while it is just \( \hat{w} \) without any project. So the employee solves \( \max_{\gamma \in [0,1]} \gamma_E \gamma_M \mu_{M,X} \mu_{E,X} - c(\epsilon) \). It follows that the employee chooses the project, say \( X \), with the highest \( \mu_{E,Y} \mu_{M,Y} \), and then chooses \( \epsilon \) to solve: \( \max_{\epsilon \in [0,1]} \epsilon \gamma_E \gamma_M \mu_{M,X} \mu_{E,X} - c(\epsilon) + \hat{w} \). This is non-negative by \( A[2] \) and the fact that the employee can set \( \epsilon = 0 \). Since \( \bar{w} = 0 \), the employee accepts any \( \hat{w} \).

The firm’s payoff from offering a wage \( \hat{w} \) is

\[
\hat{e} \int_0^{\gamma_M \mu_{M,X}} (\gamma_M \mu_{M,X} - I)dI - \max(\hat{w}, 0) = \hat{e} \int_0^{\gamma_M \mu_{M,X} \mu_{E,X}} (\gamma_M \mu_{M,X} - I)dI - \max(\hat{w}, 0)
\]

so the firm offers \( \hat{w} \leq 0 \), so that \( \hat{w} = 0 \). Any such wage gives the same payoff.

Proof of Proposition 1: Lemma 1 says that the employee will choose the action \( Y \in \{A, B\} \) with the highest \( \mu_{E,Y} \mu_{M,Y} \). Let \( \nu_i > \nu_j \) and let wlog. \( A = \arg\max_Y \mu_{i,Y} \). If also \( \arg\max_Y \mu_{i,Y} = A \) then \( \mu_{E,A} \mu_{M,A} = \nu_i \nu_j > 1/4 > (1 - \nu_i)(1 - \nu_j) = \mu_{E,B} \mu_{M,B} \), else \( \mu_{E,A} \mu_{M,A} = \nu_i (1 - \nu_j) > (1 - \nu_i) \nu_j = \mu_{E,B} \mu_{M,B} \).

In any case, the employee chooses indeed the action preferred by \( i \). If \( \nu_M = \nu_E \) and \( \nu_M \neq \mu_E \), then, by \( A[1] \) the employee does as his manager prefers.

Proof of Corollary 1: Let, essentially wlog, \( \mu_{M,A} > 1/2 \), so that \( \nu_M = \mu_{M,A} \). The probability that the decision follows the manager’s belief is \( \int_{1 - \mu_{M,A}}^{1} dF \) which increases in \( \mu_{M,A} \) and thus in \( \nu_M \).

Proof of Proposition 2: I first show that, with \( X \) denoting the project undertaken by the employee, \( \hat{e} \) and \( \hat{u} \) strictly increase in \( \mu_{i,X} \) on \( N' \). With \( l(\epsilon) = c'(\epsilon) \), we have that \( \hat{e} = l^{-1}(\gamma_E \gamma_M \mu_{E,X} \mu_{M,X}) \) so that
\[ \frac{d\hat{e}}{d\mu_{i,X}} = [l^{-1}(\cdot)]^\gamma \gamma_{M\mu_{i,X}} \] which is strictly positive. This implies the first part of the statement. The second part follows from applying an envelope theorem on the employee’s problem \[ \max_{e \in [0,1]} e \gamma_{E \mu_{E,X}} \mu_{M,X} - c(e) \].

Assume now that the manager strictly prefers project A, i.e. \( \mu_{M,A} > 1/2 \), so that \( \nu_M = \mu_{M,A} \). If now \( X = A \) then \( \frac{d\hat{e}}{d\mu_M} = \frac{d\hat{e}}{d\mu_{M,A}} = \frac{d\hat{e}}{d\mu_{M,X}} > 0 \). If \( X = B \), then \( \mu_{M,X} = \mu_{M,B} = 1 - \mu_{M,A} = 1 - \nu_M \), so that \( \frac{d\hat{e}}{d\mu_M} = \frac{d\hat{e}}{d\mu_{M,A}} = -\frac{d\hat{e}}{d\mu_{M,X}} < 0 \).

The arguments for increases and decreases in utility, and for the analogous relationships with respect to the employee’s conviction and project preference, are completely analogous.

**Proof of Proposition 3:** Remember that an employee of firm \( F_1 \) who undertakes \( Y \) gets a payoff \( \hat{e}\gamma_{E \mu_{E,Y}} \mu_{M,Y} - c(\hat{e}) \).

I claim first of all that in any SPE, all employees (with \( \mu_{E} \neq \check{\mu} \)) hired by \( F_1 \) choose A and all those hired by \( F_2 \) choose B. This follows by contradiction: Consider any employee who applies to \( F_1 \) but chooses action B. He would be strictly better off applying to \( F_2 \) and still undertaking B.

Next, given that \( F_1 \) (resp. \( F_2 \))-employees choose A (resp. B), an employee strictly prefers \( F_1 \) if \( \max_{e \in [0,1]} e \gamma_{E \mu_{E,A}} \mu_{M,A} - c(e) > \max_{e \in [0,1]} e \gamma_{E \mu_{E,B}} \mu_{M,B} - c(e) \) or if (by an envelope theorem argument) \( \mu_{E,A} \mu_{M_1,A} > \mu_{E,B} \mu_{M_2,B} \) or if \( \mu_{E,A} > \check{\mu} \).

An analogous argument shows that if \( \mu_{E} < \check{\mu} \) then the employee will definitely choose firm \( F_2 \). The fact that \( \check{\mu} \) decreases in \( \mu_{M_1,A} \) and \( \mu_{M_2,A} \) follows from its definition.

**Proof of Corollary 2:** This follows directly from the proof of proposition 3.

**Proof of Corollary 3:** By the earlier results and assumptions, all employees of \( F_1 \) choose A. The corollary then follows from monotone comparative statics and an envelope theorem on the employee’s problem.

**Proof of Proposition 5:** Note first that the outcome (in terms of sorting and actions) specified in proposition 3 a.s. Pareto dominates all other outcomes by the following argument. For each agent the very best outcome is that he can go to the firm as specified in proposition 3, while all other agents who go to this firm choose exactly the same action as he does. This is indeed what happens in the outcome of proposition 3. Moreover, as long as none of the agents has \( \mu_{E,A} = \check{\mu} \), there is for each other outcome at least one agent who strictly prefers the outcome in proposition 3. It follows that the outcomes specified in proposition 3 almost surely Pareto dominates all other outcomes.

Furthermore, (conditional on no employee having belief \( \check{\mu} \)) any employee always strictly prefers the proposition 3 outcome to any outcome in which he goes to a firm or takes an action that is different from the one
This proves the first part of the proposition. But then it follows by a backwards induction argument that the outcome of the proposition is the unique subgame-perfect equilibrium outcome.

Proof of Proposition [4] Consider the first part of the proposition.

Fix a number of agents $n$. In any equilibrium, there is a (possibly empty) subset of the employees who undertake $A$ and a (possibly empty) subset who undertake $B$. Furthermore, in any equilibrium, there exist two agents with consecutive beliefs $\bar{\mu}^-$ and $\bar{\mu}^+$ such that all agents with belief $\mu_{E,A} \leq \bar{\mu}^-$ undertake $B$ and all agents with belief $\mu_{E,A} \geq \bar{\mu}^+$ undertake $A$. For further reference, denote the belief of the agent with belief just smaller than $\bar{\mu}^-$ as $\bar{\mu}^-$ and that of the agent with belief just larger than $\bar{\mu}^+$ as $\bar{\mu}^+$. 

Let $F_n^+(x) = \lim_{\epsilon \to 0} F_n(x + \epsilon)$ and $F_n^-(x) = \lim_{\epsilon \to 0} F_n(x - \epsilon)$. Note that when employee $i$ undertakes $A$ then, in equilibrium, $\tau_i = F_n^+(\bar{\mu}^-)$, while if he undertakes $A$ then $\tau_i = 1 - F_n^-(\bar{\mu}^+)$. Furthermore, in equilibrium, $\tau = F_n^+(\bar{\mu}^-)^2 + (1 - F_n^-(\bar{\mu}^+))^2$. Denote $\tau^- = F_n^+(\bar{\mu}^-)(1 - F_n^-(\bar{\mu}^-))^2$ and $\tau^+ = F_n^+(\bar{\mu}^+)(1 - F_n^-(\bar{\mu}^+))^2$.

The condition for an equilibrium is twofold. The agent with belief $\bar{\mu}^-$ must find it optimal to choose $B$

$$(\alpha + (1 - \alpha)\tau^-)\gamma_E \gamma_M \mu_{M,B}(1 - \bar{\mu}^-) \geq (\alpha + (1 - \alpha)\tau^-)\gamma_E \gamma_M \mu_{M,A} \bar{\mu}^-,$$

and the agent with belief $\bar{\mu}^+$ must find it optimal to choose $A$

$$(\alpha + (1 - \alpha)\tau^+)\gamma_E \gamma_M \mu_{M,B}(1 - \bar{\mu}^+) \leq (\alpha + (1 - \alpha)\tau^+)\gamma_E \gamma_M \mu_{M,A} \bar{\mu}^+.$$

As the number of employees $n \to \infty$, $\bar{\mu}^+ - \bar{\mu}^- \xrightarrow{a.s.} 0$, $\bar{\mu}^- - \bar{\mu}^- \xrightarrow{a.s.} 0$, $\bar{\mu}^+ - \bar{\mu}^+ \xrightarrow{a.s.} 0$ (uniformly over all equilibria). Moreover, by the Glivenko-Cantelli lemma, $F_n$ converges to $F$ almost surely (in the supremum norm). It thus follows that $\tau^+ - \tau \xrightarrow{a.s.} 0$ and $\tau - \tau^- \xrightarrow{a.s.} 0$. It follows that in the limit the above conditions for an equilibrium combine to 

$$(\alpha + (1 - \alpha)\tau^-)\gamma_E \gamma_M \mu_{M,B}(1 - \bar{\mu}^-) = (\alpha + (1 - \alpha)\tau^-)\gamma_E \gamma_M \mu_{M,A} \bar{\mu}^-$$

(where we could have written the equation equally well in terms of $\bar{\mu}^+$). This simplifies to $\mu_{M,B}(1 - \bar{\mu}^-) = \mu_{M,A} \bar{\mu}^-$. It follows that $\tau = F(\mu_{M,B})^2 + (1 - F(\mu_{M,B}))^2$. Given $\alpha$ this increases in $\mu_{M,A}$.

This proves the first part of the proposition.

The logic for the second part is the same. Only now, the two equilibrium conditions are

$$(\alpha + (1 - \alpha)F_n^+(\bar{\mu}^-))\gamma_E \gamma_M \mu_{M,B}(1 - \bar{\mu}^-) \geq (\alpha + (1 - \alpha)F_n^-(\bar{\mu}^-))\gamma_E \gamma_M \mu_{M,A} \bar{\mu}^-,$$
and
\[(\alpha + (1 - \alpha)F_n^+ (\mu^+))\gamma_E^E \gamma_M \mu_{M,B} (1 - \mu^+) \leq (\alpha + (1 - \alpha)(1 - F_n^- (\mu^-)))\gamma_E^E \gamma_M \mu_{M,A} \mu^+.
\]

As the number of employees \(n \to \infty\), these now combine to
\[(\alpha + (1 - \alpha)F_n^+ (\mu^-))\gamma_E^E \gamma_M \mu_{M,B} (1 - \mu^-) = (\alpha + (1 - \alpha)(1 - F_n^- (\mu^-)))\gamma_E^E \gamma_M \mu_{M,A} \mu^-,
\]
or, since \(F_n^+ (\mu) \xrightarrow{a.s.} F (\mu)\) for any \(\mu\),
\[(\alpha + (1 - \alpha)F (\mu^-))\mu_{M,B} (1 - \mu^-) = (\alpha + (1 - \alpha)(1 - F (\mu^-)))\mu_{M,A} \mu^-,\]

This gives
\[
\mu^- = \frac{(\alpha + (1 - \alpha)F (\mu^-))\mu_{M,B}}{(\alpha + (1 - \alpha)(1 - F (\mu^-)))\mu_{M,A} + (\alpha + (1 - \alpha)F (\mu^-))\mu_{M,B}}
\tag{1}
\]

Since the right-hand side is a monotone function from \([0,1]\) to \([0,1]\), the equation has a fixed point by Tarski’s fixed point theorem. Moreover the right hand side decreases in \(\mu_{M,A}\) so that, by Milgrom and Roberts (1994) the smallest (and largest) fixed point also decreases in \(\mu_{M,A}\). Again, combined with A4 it follows that \(\tau\) increases in \(\mu_{M,A}\). This proves the second part of the proposition.

Consider now the last part of the proposition. The derivative of the right-hand side of equation 1 for \(\mu^-\) equals
\[
(1 - \alpha) f (\mu^-) \mu_{M,B} \mu_{M,A} + \alpha \mu_{M,A} \left\{ (\alpha + (1 - \alpha)(1 - F (\mu^-)))\mu_{M,A} + (\alpha + (1 - \alpha)F (\mu^-))\mu_{M,B} \right\}^2
\]

Since \(f\) is bounded (since it is continuous), this derivative is continuous in \(\alpha\) and equals zero at \(\alpha = 1\). There thus exists \(\alpha^* < 1\) such that the derivative is strictly smaller than one for \(\alpha \leq \alpha^* \leq 1\). Combined with the fact that the right hand side of equation 1 is strictly larger than 0 at \(\mu^- = 0\) and strictly smaller than 1 at \(\mu^- = 1\), it implies that there is a unique fixed point for \(\alpha \geq \alpha^*\). Using again Milgrom and Roberts (1994), that fixed point decreases in \(\mu_{M,A}\), so that \(\tau\) increases in \(\mu_{M,A}\).

\begin{lemma}
Absent sorting, the optimal \(\mu_{M,A}\) increases in \(p\).
\end{lemma}
**Proof** : It is sufficient to show that $E[\hat{\pi}_O]$ is supermodular in $p$ and $\mu_{M,A}$. The profit equation is:

$$
E[\hat{\pi}_O] = \int_{0}^{\mu_{M,B}} \hat{e}_M^2 (1 - p) \mu_{M,B} - \frac{\mu_{M,B}^2}{2} f(u) du \\
+ \int_{\mu_{M,B}}^{1} \hat{e}_M^2 (p \mu_{M,A} - \frac{\mu_{M,A}^2}{2}) f(u) du
$$

where we suppressed notation that indicates that $\hat{e}$ depends on both agents’ beliefs and on the action taken. The cross partial of this function in $(p, \mu_{M,A})$ is positive.

**Proof of Proposition 6** : I first want to show that $\mu_{M,A} \geq 1/2$. By lemma 2 above, it is sufficient to show this for $p = 1/2$. By contradiction, assume that $\mu_{M,A} < 1/2$ while $p = 1/2$, then firm profits are:

$$
E[\hat{\pi}_O] = \int_{1/2}^{\mu_{M,B}} \hat{e}_M^2 \mu_{M,B} \frac{\mu_{M,A}}{2} f(u) du + \int_{\mu_{M,B}}^{1} \hat{e}_M^2 \mu_{M,A} \left( \frac{\mu_{M,B}}{2} \right) f(u) du
$$

Consider now what happens if we select instead a manager with belief $\hat{\mu}_{M,A} = 1 - \mu_{M,A} > 1/2$.

- Employees who before chose $A$ will still choose $A$, but their effort strictly increases. This implies that the second term strictly increases.
- Employees who before chose $B$ will now choose $A$. By the relation between $\mu_{M,A}$ and $\hat{\mu}_{M,A}$, the $\mu_{M,X}$ (the manager’s belief in the action chosen by the employee) remains the same. $\mu_{E,X}$ on the contrary increases (since by A5 all employees believe more in $A$ than in $B$), so that again employee effort increases. This implies that the first term increases.

This implies that overall the firm profits increase, so that $\mu_{M,A} < 1/2$ is not optimal.

Consider now the case that the employee always comes up with a project, independent of his effort. The employee sets $\hat{e} = 0$ and undertakes the action that maximizes $\mu_{E,Y} \mu_{M,Y}$. Since $\hat{\mu}_{M,A} \geq 1/2$, profit equals $E[\hat{\pi}_O] = \int_{1/2}^{1} \hat{e}_M^2 \mu_{M,A} \left( p - \frac{\mu_{M,A}}{2} \right) f(u) du$ which is maximized at $\hat{\mu}_{M,A} = p$. This proves the first part of the proposition. For the second part, the firm profit when $\mu_{M,A} \geq 1/2$ is $E[\hat{\pi}_O] = \max_{\mu_{M,A}} \int_{1/2}^{1} \hat{e}_M^2 \mu_{M,A} \left( p - \frac{\mu_{M,A}}{2} \right) f(u) du$ where the maximum is well defined since the profit function is continuous in $\mu_{M,A}$ on $[1/2, 1]$. The derivative of the integrand (for $\mu_{M,A}$) is strictly positive for $1/2 \leq \mu_{M,A} \leq p$ and continuous in $\mu_{M,A}$. It thus follows that the optimal $\mu_{M,A}$ is strictly larger than $p$ and thus that vision is optimal.

For the profitability with sorting, remember that we assume that $1 > p > 1/2$ and that the focal firm
$F$ faces one competitor with belief $\mu = p$. We first introduce some notation. Let $\hat{\pi}_H = \max_{\mu \geq p} E[\pi]$ when $F$ attracts all employees with $\mu_{E,A} \geq \hat{\mu}$, and let $\hat{\mu}_{FH}$ be the corresponding maximizer. Let analogously $\hat{\pi}_L = \max_{\mu \leq p} E[\pi]$ when $F$ attracts all employees with $\mu_{E,A} \leq \hat{\mu}$, and let $\hat{\mu}_{FL}$ be the maximizer. Note that this implies that $0 \leq \hat{\mu}_{FL} \leq p \leq \hat{\mu}_{FH} \leq 1$.

Let $\hat{\pi}_L$ be the profit of $F$ when $\mu_{F,A} = p$ but $F$ attracts all employees with $\mu_{E,A} < (1-p)$; $\hat{\pi}_H$ be the profit of $F$ when $\mu_{F,A} = p$ but $F$ attracts all employees with $\mu_{E,A} \geq (1-p)$; $\hat{\pi}_M$ be the profit of $F$ when $\mu_{F,A} = p$ and employees are allocated randomly between the two firms with equal probability. Note that we always have that $\hat{\pi}_H \geq \hat{\pi}_M$ and $\hat{\pi}_L \geq \hat{\pi}_L$.

Finally, let $F^-(x) = \lim_{x \downarrow x} F(u)$ and $F^+(x) = \lim_{x \uparrow x} F(u)$.

**Lemma 3** If $F^-(1-p) < 1$ then $\hat{\mu}_{FH} > p$. If $F^+(1-p) > 0$ then $\hat{\mu}_{FL} < p$. Finally, if $F^-(1-p) < 1$ or $F^+(1-p) > 0$ then either $\hat{\pi}_L > \hat{\pi}_M$ or $\hat{\pi}_H > \hat{\pi}_M$ or both. If both conditions are satisfied (which is the case when $F$ has full support), then the optimal belief is strictly different from the reference belief.

**Proof:** Consider the first part of the lemma, so assume $1 - F^-(1-p) > 0$. Conditional on $\mu_{F,A} \geq p$ and $F$ attracting all the employees with $\mu_{E,A} \geq \hat{\mu}$, its optimal profits are: $\hat{\pi}_H = \max_{\mu_{F,A} \geq \hat{\mu}} \int_{1-p}^1 \gamma_p^2 \left( \frac{2\hat{\mu}}{\mu_{F,A}} - \frac{\hat{\mu}^2}{2} \right) d\mu f(u) du$ with $\hat{\mu} = \frac{1-p}{\mu_{F,A} + 1-p}$. This profit function is (right)continuously differentiable in $\mu_{F,A}$ on $[p, 1)$. Its right derivative in $\mu_{F,A}$ at $\mu_{F,A} = p$ is:

$$\left[ \frac{d \hat{\pi}_H}{d \mu_{F,A}} \right]_{\mu_{F,A} = p}^+ = \int_{1-p}^1 \gamma_p^2 \frac{2\hat{\mu}}{\mu_{F,A}} \frac{d \hat{\mu}}{d \mu_{F,A}} f(u) du - \hat{\mu} \gamma_p^2 \frac{2\hat{\mu}}{\mu_{F,A}} \frac{d \hat{\mu}}{d \mu_{F,A}} f(1-p)$$

The second term is non-negative since $\frac{d \hat{\mu}}{d \mu_{F,A}} < 0$. The first term is strictly positive since $F(1-p) > 1$ and $\frac{d \hat{\mu}}{d \mu_{F,A}} > 0$. This implies that the optimal $\hat{\mu}_F > p$. Note that this also implies that $\hat{\pi}_H > \hat{\pi}_M$.

The argument for the second part is analogous and implies $\hat{\pi}_L > \hat{\pi}_L$.

I now show that if $F^-(1-p) < 1$ or $F^+(1-p) > 0$ then either $\hat{\pi}_L > \hat{\pi}_M$ or $\hat{\pi}_H > \hat{\pi}_M$ or both. Just checking definitions of $\hat{\pi}_L$, $\hat{\pi}_H$, and $\hat{\pi}_M$ shows that $\hat{\pi}_L + \hat{\pi}_H = 2\hat{\pi}_M$. But, we always have that $\hat{\pi}_H > \hat{\pi}_M$ and $\hat{\pi}_L > \hat{\pi}_L$ with one of these strict when $F^-(1-p) < 1$ or $F^+(1-p) > 0$. This implies that under that condition $\hat{\pi}_L + \hat{\pi}_H > \hat{\pi}_L + \hat{\pi}_H = 2\hat{\pi}_M$ which implies that $\max(\hat{\pi}_L, \hat{\pi}_H) > \hat{\pi}_M$.

The very last part follows from the fact that when $F^-(1-p) < 1$ and $F^+(1-p) > 0$ then $\hat{\mu}_{FH} > p$ and $\hat{\pi}_{FL} < p$.

**Proof of Proposition 7:** For $A[7]$, this follows immediately from the lemmas that follow. For $A[6]$, it is immediate that the optimal belief must be $\mu \geq p$ since a firm with $\mu < p$ has no employees. Next, there exist
some $\mu > p$ that gives the focal firm higher profits than $\mu = p$ (since with $\mu > p$ all the employees prefer the focal firm, while they randomize between the two when $\mu = p$). Finally, the right-derivative (in the manager’s belief) of firm profit at $\mu = p$ is strictly positive, so that the optimal belief subject to $\mu \in (p, 1]$ is well-defined.

\textbf{Lemma 4} Vision is optimal (against a firm with reference beliefs) for any symmetric distribution of beliefs.

\textbf{Proof:} Fix a symmetric distribution of beliefs $F$. Note that we always have that $F^{-1}(1 - p) < 1$, so that $\hat{\mu}_F > p$.

Consider first the case that $p = 1 - p = 1/2$. By symmetry we have $\hat{\pi}_H = \hat{\pi}_L$ so that vision ($\hat{\mu}_F > p$) is (weakly) optimal.

As $p$ increases, $\hat{\pi}_H$ strictly increases since $\frac{d\hat{\pi}_H}{dp} = \frac{\partial \hat{\pi}_H}{\partial p} = \int_0^1 e^{\gamma_2} \mu_{F_H} f(u)du > 0$, while $\hat{\pi}_L$ (weakly) decreases since $\frac{d\hat{\pi}_L}{dp} = \frac{\partial \hat{\pi}_L}{\partial p} = - \int_0^1 e^{\gamma_2} \mu_{F_L} f(u)du \leq 0$. This implies that for all $p > 1/2$, $\hat{\pi}_H > \hat{\pi}_L$. ■

\textbf{Lemma 5} Let $G$ and $H$ be distribution functions on $[a, b]$, with $H$ FOSD $G$. Let $k(\theta, x) = E_{u \sim \theta H + (1 - \theta) G} [f_3(x, u) \mid f_1(x) \leq u]$ with $\theta \in [0, 1]$, $a \leq f_1 \leq f_2 \leq b$ and $f_3$ $u$-measurable. Let finally $K(\theta) = \max_{x \in X} k(\theta, x)$ be well-defined for $\theta \in \{0, 1\}$.

If $f_3(x, u)$ increases in $u$ (for fixed $x$), then $K(1) \geq K(0)$.

\textbf{Proof:} Let $f_3(x, u)$ be increasing in $u$. Since $H$ FOSD $G$, the basic theorem on FOSD says that for any fixed $x \in X$, $k(1, x) = E_{u \sim H} [f_3(x, u) \mid f_1(x) \leq u \leq f_2(x)] \geq E_{u \sim G} [f_3(x, u) \mid f_1(x) \leq u \leq f_2(x)] = k(0, x)$.

Let $\hat{x}_H \in \argmax_{x \in X} k(1, x)$ and $\hat{x}_G \in \argmax_{x \in X} k(0, x)$ which exist by assumption. We then have: $K(1) = k(1, \hat{x}_H) \geq k(1, \hat{x}_G) \geq k(0, \hat{x}_G) = K(0)$ which proves the lemma. ■

\textbf{Lemma 6} If vision is optimal for some belief-distribution $G$, then it is optimal for any belief-distribution $H$ that FOSD $G$.

\textbf{Proof:} The fact that vision is optimal for some belief-distribution $G$ implies that $\hat{\pi}_{H,G} \geq \hat{\pi}_{L,G}$ where \[ \hat{\pi}_{H,G} = \max_{\mu_{F_H} \geq p} \int_{\hat{\mu}_H}^1 e^{2\gamma_2} \left( p\mu_{F_H} - \frac{\mu_{F_H}^2}{2} \right) g(u)du \] with $\hat{\mu}_H = \frac{1-p}{\mu_{F_H} + 1-p}$ and $\hat{\pi}_{L,G} = \max_{\mu_{F_L} \geq 1-p} \int_0^{\hat{\mu}_L} e^{2\gamma_2} \left( (1-p)\mu_{F_L,B} - \frac{\mu_{F_L,B}^2}{2} \right) du$ with $\hat{\mu}_L = \frac{1-\mu_{F_L}}{p+1-\mu_{F_L}}$.

Define now $\tau_H(\mu_{F_H}, p, \hat{\mu}_H, u) = \hat{\gamma}_2 e^{\gamma_2} \left( p\mu_{F_H} - \frac{\mu_{F_H}^2}{2} \right)$ if $u \geq \hat{\mu}_H$ and zero otherwise. Define analogously $\tau_L(\mu_{F_L}, p, \hat{\mu}_L, u) = \hat{\gamma}_2 e^{\gamma_2} \left( (1-p)\mu_{F_L,B} - \frac{\mu_{F_L,B}^2}{2} \right)$ if $u \leq \hat{\mu}_L$ and zero otherwise. Then we can write $\hat{\pi}_{H,G} = \frac{1}{p+1-\mu_{F_L}}$.
\[ \max_{\mu_{FH}} \int_0^1 \tau_H(\mu_{FH}, p, \bar{\mu}_H, u) g(u) du \] and \[ \hat{\pi}_{L,G} = \max_{\mu_{FL}} \int_0^1 \tau_L(\mu_{FL}, p, \bar{\mu}_L, u) g(u) du. \] By lemma [5] it suffices to show that \( \tau_H \) increases and \( \tau_L \) decreases in \( u \), to conclude that \( \hat{\pi}_{H,H} \geq \hat{\pi}_{H,G} \geq \hat{\pi}_{L,G} \geq \hat{\pi}_{L,H} \) which would imply the proposition. The rest of this proof shows that that is indeed the case.

Note, first, that the optimal \( \mu_{FH} \) and \( \mu_{FL,b} \) must be such that
\[
\left( p \mu_{FH} - \frac{\mu_{FH}^2}{2} \right) > 0 \quad \text{and} \quad \left( (1 - p) \mu_{FL,b} - \frac{\mu_{FL,b}^2}{2} \right) > 0
\]
since otherwise profits are non-positive while, in each case, it is always possible to set \( \mu = p \), which gives strictly positive profits. But then the inequalities follow immediately: For \( \tau_H \) (using the fact that \( \bar{\mu}_H \) is no function of \( u \)) the derivative is zero for \( u < \bar{\mu}_H \), the function makes a jump upwards at \( \bar{\mu}_H \), and the derivative for \( u > \bar{\mu}_H \) is
\[
\gamma_p^2 \left( p \mu_{FH} - \frac{\mu_{FH}^2}{2} \right) \frac{d\hat{\pi}}{du}
\]
which is positive (since \( \frac{d\hat{\pi}}{du} \) is positive for employees who undertake \( A \)). An analogous argument for \( \tau_L \) shows that it is decreasing. ■

**Proof of Proposition 8:** For the first part of the proposition, note that with \( \hat{\mu}_M = \arg\max_{\mu_{M,A} \geq p} (E[\pi]) \), we have \( p \leq \hat{\mu}_M \leq 1 \). Clearly, as \( p \uparrow 1 \), \( \hat{\mu}_M \uparrow 1 \). This combined with the continuity of the expected profit \( E[\hat{\pi}_O] \), implies the proposition.

For the second part of the proposition, note that vision is optimal so that \( p < \hat{\mu}_M \leq 1 \). Clearly, as \( p \uparrow 1 \), \( \hat{\mu}_M \to 1 \). But this, combined with continuity of the profit function \( E[\hat{\pi}_O] \), implies the proposition. ■

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References


