

Integration Bee Qualifiers November 2020 HMMT

Instructions: You have 30 minutes to compute 20 integration problems. All logarithms are base e and you may omit the constant of integration for indefinite integrals. Each integral is worth equal points, except for the last integral, which will serve as a tiebreaker. You need not simplify your answers. No outside materials are allowed; don't cheat!

1.

$$\int_{20}^{2020} |x - 20| + |x - 2020| dx = 4000000$$

2.

$$\int_0^1 x\sqrt{1-x^2} dx = \frac{1}{3}$$

3.

$$\int e^{e^{e^x} + e^x + x} dx = e^{e^{e^x}}$$

4.

$$\int_0^{20} \max(5, \min(15, x)) dx = 200$$

5.

$$\int (\log x)^3 dx = x(\log^3 x - 3\log^2 x + 6\log x - 6)$$

6.

$$\int (\sin x)^{e^x} \cdot e^x \cdot (\cot x + \log \sin x) dx = (\sin x)^{e^x}$$

7.

$$\int_0^1 \frac{1}{1 + \frac{1}{1+\frac{1}{x}}} dx = \frac{1}{2} + \frac{1}{4} \log(5/3)$$

8.

$$\int \frac{e^{\frac{1}{\log x}}}{x \log^2 x} dx = -e^{1/\log x}$$

9.

$$\int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \frac{\pi}{4}$$

10.

$$\int \frac{1}{\sqrt{(1-x^2)(1+\sin^{-1}(x))}} dx = 2\sqrt{1+\sin^{-1}(x)}$$

11.

$$\int_0^1 \int_0^1 \frac{dx dy}{1-xy} = \frac{\pi^2}{6}$$

12.

$$\int_0^\pi \sin x \sin 2x \sin 4x \sin 8x dx = 0$$

13.

$$\int_0^\infty \int_0^{10} ye^{-xy} dy dx = 10$$

14.

$$\int_0^\pi \frac{1}{4 - \cos x} dx = \frac{\pi}{\sqrt{15}}$$

15.

$$\int_0^1 \frac{1}{1+x^6} dx = \frac{\pi}{6} + \frac{\sqrt{3}}{6} \log(2 + \sqrt{3})$$

16.

$$\int_0^1 \frac{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{1+x^2} dx = \frac{\pi^2}{16}$$

17.

$$\int_0^\infty \log(1 - e^{-x}) dx = -\frac{\pi^2}{6}$$

18.

$$\int_0^\infty \frac{\sin(x^3)}{x} dx = \frac{\pi}{6}$$

19.

$$\int_0^1 \log(x) \log(1-x) dx = 2 - \frac{\pi^2}{6}$$

20. Estimate, **as a decimal**, the integral

$$\int_0^{\pi/2} \tan \cos x dx \approx 1.332141$$

You will be scored by the absolute difference between your answer and the exact answer.