Integration Bee Qualifiers March 2021 HMMT

Instructions: You have 30 minutes to complete 20 integration problems. All logarithms are base e and you may omit the constant of integration for indefinite integrals. Each integral is worth equal points, except for the last integral, which will serve as a tiebreaker. You need not simplify your answers. No outside materials are allowed; don't cheat!

1. $$\int \sin x \cdot \cos x \cdot \tan x \cdot \sec x \cdot \csc x \cdot \cot x \, dx$$

2. $$\int_{\int_0^1 x^2 \, dx}^{\int_0^1 x \, dx} x^3 \, dx$$

3. $$\int \frac{3x+4}{x^2+4x} \, dx$$

4. $$\int_0^1 x \log x \, dx$$

5. $$\int x^x + x^x \log x \, dx$$

6. $$\int_1^\infty \left\lceil \frac{1}{x} - \frac{1}{2021} \right\rceil dx$$

   $\lceil x \rceil$ is the *ceiling function*, which returns the smallest integer greater than x.

7. $$\int_{-2021\pi}^{2021\pi} \sin(\sin(x)) \, dx$$

8. $$\int_0^\infty x^5 e^{-x} \, dx$$

9. $$\int_{-4}^{4} \frac{xe^{|x|} + 16 - x^2}{\sqrt{16-x^2}} \, dx$$

10. $$\int \frac{\sinh x}{e^x + e^{-x}} \, dx$$

11. $$\int \sec^3 x \, dx$$

12. $$\int \frac{dx}{x^{2020} - x}$$

13. $$\frac{\int_0^\pi \sin^{2021}(x) \, dx}{\int_0^\pi \sin^{2019}(x) \, dx}$$

14. $$\int \frac{17 + 23x + 9x^2 + x^3}{e^x(6 + 11x + 6x^2 + x^3)^2} \, dx$$

15. $$\int \sqrt{\tanh x} \, dx$$

16. $$\int_0^\infty \frac{x^3}{e^x + 1} \, dx$$

17. $$\int_0^\infty \frac{dx}{(1 + x^\varphi)^\varphi}$$

    where $\varphi = \frac{1+\sqrt{5}}{2}$.

18. $$\int_0^\infty \frac{\log(1+x)}{x(1+x)} \, dx$$

19. $$\int_0^{2\pi} \tan^{-1}(2^{\cos x}) \, dx$$

20. Estimate, **in scientific notation**, the integral

    $$\int_0^\pi e^{e^x} \, dx$$

    You will be scored by the *ratio* between your answer and the exact answer.

Please submit your answers on the google form, as a PDF or JPG file: `forms.gle/QEPRKWHfcEXLkzM1A`