# February 2020 HMMT
# Integration Bee Semifinals

1.
$$\int_0^{2\pi} (\sin x + \cos x)^2 \, dx = 2\pi$$

2.
$$\int (x + e^x)^2 \, dx = \frac{1}{3}x^3 + 2(xe^x - e^x) + \frac{1}{2}e^{2x}$$

3.
$$\int_0^1 \frac{1}{\sqrt{x - x^2}} \, dx = \pi$$

4.
$$\int_0^1 \sqrt{\frac{1-x}{1+x}} \, dx = \frac{\pi}{2} - 1$$

5.
$$\int \frac{xe^x}{(1+x)^2} \, dx = \frac{e^x}{x+1} + C$$

6.
$$\int_0^1 (x(1-x))^2 \lfloor 2x \rfloor \, dx = \frac{1}{60}$$

7.
$$\int_0^{\pi/2} \frac{dx}{1 + (\tan x)^\pi} = \frac{\pi}{4}$$

8.
$$\int_0^{2\pi} \frac{dx}{2 + \cos x} = \frac{2\pi}{\sqrt{3}}$$

9.
$$\int_0^\infty x^5 e^{-x^2} \, dx = 1$$

10.
$$\int_0^1 \sin^{-1}(x) + \cos^{-1}(x) \, dx = \frac{\pi}{2}$$

11.
$$\int \frac{1 - x \log x}{e^x}\, dx = \frac{\log x}{e^x} + C$$

12.
$$\int_0^\infty \frac{\log x}{x^2 + 1}\, dx = 0$$

13.
$$\int_0^{2\pi} \left(\frac{\sin(3x)}{\sin(x)}\right)^3 dx = 14\pi$$

14.
$$\int \sqrt{\sec x}\, \tan x\, dx = 2\sqrt{\sec x} + C$$

15.
$$\int_0^3 \lceil \sqrt{x} \rceil \lfloor \sqrt{x} \rfloor\, dx = 4$$

16.
$$\int \sin^4 x + \cos^4 x\, dx = \frac{3}{4}x + \frac{1}{16}\sin(4x) + C$$

17.
$$\int \frac{x^3}{1 + x^8}\, dx = \frac{1}{4}\tan^{-1}(x^4) + C$$

18.
$$\int \frac{dx}{x^{2/3} + x^{4/3}} = 3\tan^{-1}(\sqrt[3]{x}) + C$$