

# Integration Bee Qualifiers February 2020 HMMT

Time Limit: 20 minutes

*All logarithms are base  $e$*

*You may omit the constant of integration*

*The integrals are ordered in terms of approximate difficulty*

*All integrals are worth equal points*

*Ties will be broken by the highest-numbered integral solved*

1.

$$\int_0^{2\pi} \min(\sin x, 0) dx$$

2.

$$\int_0^e \log(2^{\cos^2 x}) + \log(4^{\frac{1}{2} \sin^2 x}) dx$$

3.

$$\int_0^{\pi/2} (\sin x)^3 dx$$

4.

$$\int \frac{\sec^2 x}{\tan x(\tan x + 1)} dx$$

5.

$$\int e^{\sin x} (e^x \cos(e^x) + \sin(e^x) \cos(x)) dx$$

6.

$$\int x e^x \sin x dx$$

7.

$$\int_0^{10} [x][x] dx$$

$[x]$  rounds  $x$  down to the nearest integer, while  $\lceil x \rceil$  rounds  $x$  up to the nearest integer.

8.

$$\int \frac{\sin x}{\sin x + \cos x} dx$$

9.

$$\int_0^{\pi/2} \sin^2(\sin x) + \cos^2(\cos x) dx$$

10.

$$\int_0^{\pi/2} \sin^7 x \cos^7 x dx$$

11.

$$\int_0^{\infty} \frac{x}{1 + e^x} dx$$