Auction Algorithms for Path Planning, Network Transport, and Reinforcement Learning

by

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This monograph represents "work in progress," and will be periodically updated. It more than likely contains errors (hopefully not serious ones). Furthermore, the references to the literature are incomplete. Your comments and suggestions to the author at dbertsek@asu.edu are welcome.

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ABOUT THE AUTHOR

Dimitri Bertsekas studied Mechanical and Electrical Engineering at the National Technical University of Athens, Greece, and obtained his Ph.D. in system science from the Massachusetts Institute of Technology. He has held faculty positions with the Engineering-Economic Systems Department, Stanford University, and the Electrical Engineering Department of the University of Illinois, Urbana. Since 1979 he has been teaching at the Electrical Engineering and Computer Science Department of the Massachusetts Institute of Technology (M.I.T.), where he is McAfee Professor of Engineering. In 2019, he joined the School of Computing and Augmented Intelligence at the Arizona State University, Tempe, AZ, as Fulton Professor of Computational Decision Making.

Professor Bertsekas' teaching and research have spanned several fields, including deterministic optimization, dynamic programming and stochastic control, large-scale and distributed computation, artificial intelligence, and data communication networks. He has authored or coauthored numerous research papers and nineteen books, several of which are currently used as textbooks in MIT classes, including "Dynamic Programming and Optimal Control," "Data Networks," "Introduction to Probability," and "Nonlinear Programming." At ASU, he has been focusing in teaching and research in reinforcement learning, and he has developed several textbooks and research monographs in this field since 2019.

Professor Bertsekas was awarded the INFORMS 1997 Prize for Research Excellence in the Interface Between Operations Research and Computer Science for his book "Neuro-Dynamic Programming" (co-authored with John Tsitsiklis), the 2001 AACC John R. Ragazzini Education Award, the 2009 INFORMS Expository Writing Award, the 2014 AACC Richard Bellman Heritage Award, the 2014 INFORMS Khachiyan Prize for Life-Time Accomplishments in Optimization, the 2015 MOS/SIAM George B. Dantzig Prize, and the 2022 IEEE Control Systems Award. In 2018 he shared with his coauthor, John Tsitsiklis, the 2018 INFORMS John von Neumann Theory Prize for the contributions of the research monographs "Parallel and Distributed Computation" and "Neuro-Dynamic Programming." Professor Bertsekas was elected in 2001 to the United States National Academy of Engineering for "pioneering contributions to fundamental research, practice and education of optimization/control theory, and especially its application to data communication networks."

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Contents

Preface

Invisible hand metaphor:

> **The unseen constant interplay of self-interested individual pressures on market supply and demand, which cause the natural movement of prices and goods, while fulfilling the best interest of society as a whole.**
>
> **Adam Smith, "The Wealth of Nations," 1776**

The purpose of this monograph is to present new research and an up-to-date account relating to the auction algorithm for the classical assignment problem, introduced by the author in the 1979 paper [Ber79] and studied together with its variations over a period of more than 40 years, including a detailed treatment in the book [Ber98]. The auction algorithm is highly intuitive, and involves an invisible hand-like market mechanism that resembles a real-life auction. Other features of the algorithm are that it has excellent computational complexity, and it is amenable to distributed asynchronous implementation. We will describe the auction algorithm and its properties for the assignment problem, and we will focus on adaptations of the algorithm to solve other network optimization problems, including new methods for path planning and network transport problems.

The network flow problems that we will discuss are some of the most important and frequently encountered classes of optimization problems. They arise naturally in the analysis and design of large engineering systems, such as communication, transportation, and manufacturing networks. They can also be used to model important classes of combinatorial problems, such as assignment, shortest path, and traveling salesman problems, which in turn arise in a broad variety of applications. Moreover, they are an integral part of several types of artificial intelligence software, such as those involving knowledge graphs and path planning.

We pay special attention to three major mathematical network optimization problems:

(a) The *assignment* problem, which involves matching the elements of two finite sets, on a one-to-one basis, at minimum cost.

(a) The *shortest path* problem, which involves finding a minimum cost path between designated origin(s) and destination(s).

(c) The *transhipment problem*, also known as the *network transport problem*, which involves supply and demand points, but also arc capacity constraints.

The transhipment problem, contains the other two as special cases, but it can also be reformulated as an assignment problem. Moreover the shortest path problem can also be reformulated as an assignment problem. Because of these relations, algorithms that are used to solve one type of problem, can be adapted to solve any other problem. This is a fundamental conceptual point, which will be important for our development.

Much of the new research in this book relates to adaptations of the auction algorithm for the assignment problem, which apply to path construction problems, including the classical shortest path problem. Auction algorithms for path construction have an intuitive form, and will be used as the basis for solution of other problems, including max-flow, transportation, and transhipment problems. Auction/path construction algorithms will also be applied, in both exact and approximate form, in an important application area: reinforcement learning and sequential decision making, particularly in contexts where the popular Monte Carlo tree search and real-time dynamic programming methods have been used.

The first chapter serves as an introduction to our principal network problem formulations and their interrelations, as well as auction algorithms, and their relation to primal and dual optimization. We focus primarily on our two principal paradigms, assignment and shortest path. Moreover, in Section 1.4.4, we provide a preview of ways that auction algorithms can be fruitfully incorporated within the broad framework of the reinforcement learning methodology.

Detailed discussions of auction algorithms for assignment and shortest path problems are given in Chapters 2 and 3, respectively. Extensions of the algorithmic ideas of Chapters 2 and 3 to transhipment and other network flow problems are given in Chapter 4. The application and adaptation of auction algorithms to reinforcement learning will be discussed in Chapter 5.

The author's 1998 network optimization book [Ber98] (freely available on-line) serves to provide support for some of the more mathematical aspects of the present book, including some proofs and computational complexity analyses. It uses similar notation and terminology, includes a large number of theoretical exercises, and discusses a broader range of network algorithms and applications.

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