Massachusetts Institute of Technology
Department of Physics

8.276 Spring 2007 **May 3, 2007**
Solution to Problem set #9

1. (10 points)

$$\pi^- + \mathrm{d} \xrightarrow{?} 2\mathrm{n} + \pi^0$$

Consider angular momentum and parity of the initial and final states:

$$\begin{aligned} \text{For } \pi^- \text{ absorbed at rest}, &\ \ell\ (\pi^-\mathrm{d} \text{ system}) = 0 \\ J = 1 &\ (\text{spin of deuteron} = 1 \\ &\quad \text{spin of } \pi^- = 0) \end{aligned}$$

Pion has negative, or odd, intrinsic parity, so

$$\text{parity of initial state is } \underline{\text{negative}}.$$

In final state, must have $J = 1$, but two neutrons cannot be in the ${}^3\mathrm{S}_1(\ell = 0)$ state because of the Pauli principle. The next lowest energy state is the ${}^3\mathrm{P}_1(\ell = 1)$. This state has negative parity, which when combined with the negative intrinsic parity of the π^0, gives positive parity for the final state. Hence, the reaction does not occur.

If the π^- has appreciable kinetic energy, then the π^-d system could be in a relative $\ell = 1$ state, with parity $(-)(-) = (+)$, so the reaction can occur.

2. (15 points)

$$\begin{array}{lll} u_{\mathrm{I}} = A\sin kr & r < a & \text{with } k^2 = 2m(V+E)/\hbar^2 \\ u_{\mathrm{II}} = Ce^{-\kappa r} & r \geqslant a & \qquad \kappa^2 = -2mE/\hbar^2 \end{array}$$

a)

Continuity: $$u(0) = A\sin ka = Ce^{-\kappa a} \tag{1}$$

$$\left.\frac{\mathrm{d}u}{\mathrm{d}r}\right|_{r=a} = kA\cos ka = -\kappa C e^{-\kappa a}$$

$$\Longrightarrow k\cot ka = -\kappa \tag{2}$$

Normalization: $$\int_0^\infty \left|\frac{u(r)}{r}\right|^2 r^2 \mathrm{d}r = 1$$ (angular part of wavefunction already normalized to 1)

$$A^2 \int_0^a \sin^2 kr \mathrm{d}r + C^2 \int_a^\infty e^{-2\kappa r}\mathrm{d}r = 1$$

$$A^2\left[\frac{a}{2} - \frac{\sin 2ka}{4k}\right] + \frac{C^2}{2\kappa}e^{-2\kappa a} = 1$$

Use (1), (2) and the trigonometric identities $\left(\sin 2x = \dfrac{2\cot x}{1+\cot^2 x},\ \sin^2 x = \dfrac{1}{1+\cot^2 x}\right)$. Skipping few lines of algebra,

$$\frac{A^2}{2}\left[a + \frac{\kappa}{k^2+\kappa^2} + \frac{k^2/\kappa}{k^2+\kappa^2}\right] = 1 \quad \longrightarrow \quad A = \left(\frac{2\kappa}{1+\kappa a}\right)^{1/2}$$

and from (1)

$$\boxed{C = \left(\frac{2\kappa}{1+\kappa a}\right)^{1/2} \sin ka e^{\kappa a}}$$

b)

$$f = C^2 \int_a^\infty e^{-2\kappa r}\mathrm{d}r = \frac{C^2}{2\kappa}e^{-2\kappa a} = \frac{\sin^2 ka}{1+\kappa a}$$

$$\begin{array}{lll} E = -E_B = -2.23\mathrm{MeV} & \longrightarrow & \kappa = 0.232\mathrm{fm}^{-1} \\ V = 35\mathrm{MeV} & \longrightarrow & k = 0.889\mathrm{fm}^{-1} \end{array}$$

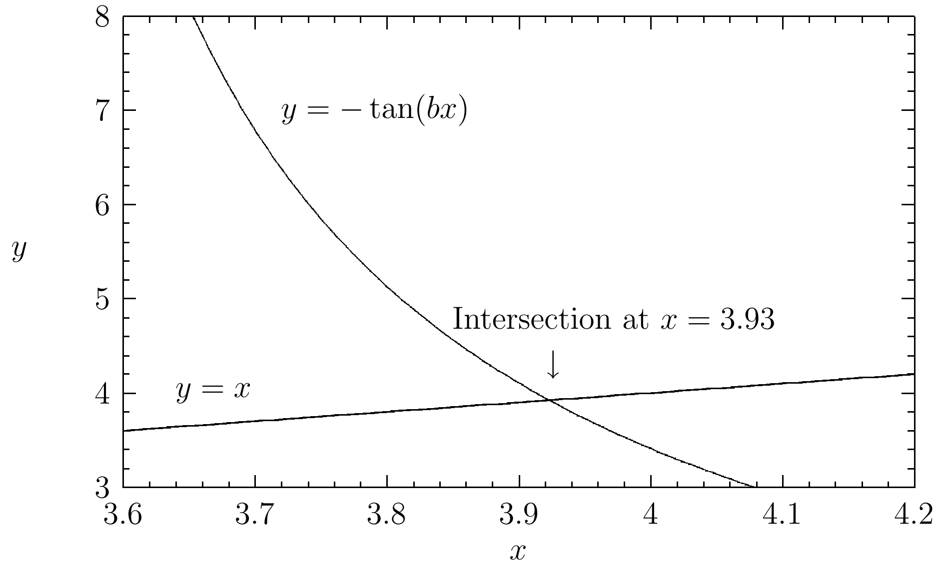
$$f = \frac{\sin^2(0.889\times 1.7)}{1+(0.232)(1.7)} = \boxed{0.71} \qquad \text{(large!)}$$

3. (10 points)

a) let $x = \dfrac{k}{\kappa} \quad \longrightarrow \quad \dfrac{k}{\kappa} = -\tan\left(\dfrac{k}{\kappa}\kappa a\right) \quad \longrightarrow \quad x = -\tan bx \quad \text{with} \quad b = \kappa a$

b) $b = (0.232)2 = 0.464$ (see Problem 2)

Graphical solution: we know $bx > \dfrac{\pi}{2}$, so don't have to plot starting at $x = 0$



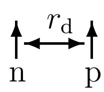
$$x = 3.93 = \frac{k}{\kappa} \quad \longrightarrow \quad k = 0.912\text{fm}^{-1}$$

$$\frac{\hbar^2 k^2}{M_N} + 2.23 = V_0 \quad \longrightarrow \quad V_0 = 36.7\text{MeV}$$

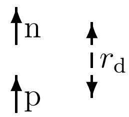
4. (10 points)

The tensor force term is

$$V_T = f(r)\left[3(\boldsymbol{s}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{s}_2 \cdot \hat{\boldsymbol{r}}) - \boldsymbol{s}_1 \cdot \boldsymbol{s}_2\right] \equiv f(r) S_{12}$$

In orientation 1 

$\boldsymbol{s}_1 \cdot \hat{\boldsymbol{r}} = \boldsymbol{s}_2 \cdot \hat{\boldsymbol{r}} = 0 \qquad S_{12} = -\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = -\frac{1}{4}$

In orientation 2 

$S_{12} = 3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - \frac{1}{4} = \frac{1}{2}$

a) So V2-V1 $= 3V_T(r_\mathrm{d}) = \frac{3}{4} \cdot 20 = 15\mathrm{MeV}$

b) In orientation 2, the force is <u>attractive</u> (think of the analogy with two magnetic dipole); in 1, it is repulsive. Therefore 2 is the real deuteron ground state orientation.

c) This is consistent with the prolate ("cigar-shaped") charge distribution deduced from the positive quadrupole momentum.