MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

8.276 Spring 2007 **April 24, 2007**
Solution to Problem set #8

1. (5 points)
a)

$$\tau = \frac{\hbar}{\triangle E} = \frac{6.6 \times 10^{-22}\text{MeVs}}{30} = 2.2 \times 10^{-23}\text{s}$$

b) Since the lifetime is characteristic of the strong interaction, strangeness must be conserved in the decay $\Sigma' \to \Lambda + \pi$. Therefore $S(\Sigma') = S(\Lambda) = -1$, since $S(\pi) = 0$.

2. (15 points)

		Mass(initial)	Mass(final)	allowed?
$S = 0$	$\Delta^- \to n + \pi^-$	1232	940+140	yes
	$\Delta^- \to \Sigma^- + \mathrm{K}^0$		1197+498	no
$S = -1$	$\Sigma^{*+} \to \mathrm{p} + \bar{\mathrm{K}}^0$	1385	938+498	no
	$\Sigma^{*+} \to \Sigma^+ + \pi^0$		1189+135	yes
	$\Sigma^{*+} \to \Sigma^+ + \eta$		1189+549	no
	$\Sigma^{*+} \to \Sigma^0 + \pi^+$		1192+140	yes
	$\Sigma^{*+} \to \Lambda + \pi^+$		1116+140	yes
	$\Sigma^{*+} \to \Xi^0 + \mathrm{K}^+$		1315+494	no
$S = -2$	$\Xi^{*-} \to \Sigma^0 + \mathrm{K}^-$	1533	1192+494	no
	$\Xi^{*-} \to \Lambda + \mathrm{K}^-$		1116+494	no
	$\Xi^{*-} \to \Sigma^- + \bar{\mathrm{K}}^0$		1197+498	no
	$\Xi^{*-} \to \Xi^0 + \pi^-$		1315+140	yes
	$\Xi^{*-} \to \Xi^- + \pi^0$		1321+135	yes
	$\Xi^{*-} \to \Xi^- + \eta$		1321+549	no

3. (15 points)
a)

		Mass(initial)	Mass(final)	allowed?
$S=-3$	$\Omega^- \to \Xi^0 + \mathrm{K}^-$	1672	1315+494	no
	$\Omega^- \to \Xi^- + \bar{\mathrm{K}}^0$		1321+498	no

Since no strangeness-conserving decay is allowed by energy conservation, Ω^- cannot decay strongly. So Ω^- lives much longer than the other members of the decuplet.

b) Length of Ω^- track ≈ 5 mm. If $v = 0.1$c,

$$\tau = \frac{0.5\text{cm}}{0.3 \times 10^{10}\text{cm/s}} \approx 1.7 \times 10^{-10}\text{s}.$$

The uncertainties come from the measurement of the length of the truck, the assumption of the speed of the Ω^-, the reproduction of the photograph, and so forth. They could change the result by a factor of up to ten. However, the estimate of the lifetime of the Ω^- is around 10^{-10}s, which is 10^{13} times larger than the lifetime of the other members of the decuplet. Therefore, these uncertainties are not important to the conclusion that the strong interaction is not responsible for the decay of the Ω^-.

c) The weak interaction is responsible for the decay of the Ω^-. Strangeness is not conserved in the decay.

4. (10 points)

The proton wave function in the quark model is

$$p^{\uparrow} = \frac{1}{\sqrt{6}}(2u^{\uparrow}u^{\uparrow}d^{\downarrow} - u^{\uparrow}u^{\downarrow}d^{\uparrow} - u^{\downarrow}u^{\uparrow}d^{\uparrow})$$

The magnetic moment of the proton is

$$\begin{aligned}\mu_p &= \sum_{i=1}^{3} \langle p^{\uparrow} | \mu_i (\sigma_3)_i | p^{\uparrow} \rangle \\ &= \frac{1}{6}\left[4(2\mu_u - \mu_d) + (\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d)\right] \\ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d.\end{aligned}$$

For the neutron, interchange u and d quarks:

$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u.$$

In terms of the nuclear magneton, $\mu_N = \dfrac{e\hbar}{2m_p}$, the quark moments are

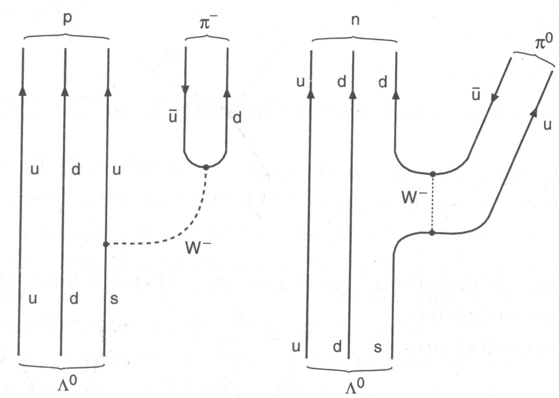
$$\mu_u = \frac{2}{3}\frac{m_p}{m_u}\mu_N, \qquad \mu_d = -\frac{1}{3}\frac{m_p}{m_d}\mu_N,$$

If $m_u = m_d = 340\text{MeV/c}^2$,

$$\begin{aligned}\mu_p &= \left[\frac{4}{3}\left(\frac{2}{3}\right) - \frac{1}{3}\left(-\frac{1}{3}\right)\right] \times 2.76\mu_N = 2.76\mu_N \qquad \text{(observed value 2.79)} \\ \mu_n &= \left[\frac{4}{3}\left(-\frac{1}{3}\right) - \frac{1}{3}\left(\frac{2}{3}\right)\right] \times 2.76\mu_N = -1.84\mu_N \qquad \text{(observed value -1.91)}\end{aligned}$$

5. (5 points)

Although strangeness-changing weak decays (such as $\Lambda \to N\pi$) do not conserve isospin ($\triangle I \neq 0$), they do appear to obey a $\triangle I = 1/2$ rule (see Perkins p.234). At the quark level, this can be understood as a s$\to$u transition (see diagram). The $\triangle I = 1/2$ rule may be applied by postulating that a hypothetical particle with $\triangle I = 1/2$ — called a "spurion" — is added to the left-hand side of the decay process, and thus treating the decay as an isospin-conserving process.



This is particularly easy in the case of $\Lambda \to N\pi$, since the Λ is an $I = 0$ state, and as the "left-hand side" isospin is purely that of the spurion $|\frac{1}{2}, -\frac{1}{2}\rangle$. (We know it's $I_3 = -1/2$ since $I_{3,1} + I_{3,2} = -1/2$, for pπ^- or nπ^0)

$$\Lambda \to \mathrm{p} + \pi^- \qquad |\frac{1}{2}, -\frac{1}{2}\rangle \longrightarrow |\frac{1}{2}, \frac{1}{2}\rangle \cdot |1, 1\rangle \qquad \left(-\sqrt{\frac{2}{3}}\right)^2$$

$$\Lambda \to \mathrm{n} + \pi^0 \qquad |\frac{1}{2}, -\frac{1}{2}\rangle \longrightarrow |\frac{1}{2}, -\frac{1}{2}\rangle \cdot |1, 0\rangle \qquad \left(\sqrt{\frac{1}{3}}\right)^2$$

The Clebsch-Gordan coefficients may be read off from the $j_1 = 1, j_2 = 1/2$. table on p.357.

$$\Longrightarrow \text{Ratio} = 2.$$

6. (10 points)

The isospin states are

$$\begin{aligned} &\mathrm{K}^- = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \\ &\mathrm{d}^- = |0, 0\rangle \\ &\Lambda = |0, 0\rangle \\ &\mathrm{n} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \quad \mathrm{p} = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \\ &\pi^0 = |1, 0\rangle \qquad \pi^- = |1, -1\rangle \end{aligned}$$

The K$^-$d system is pure $I = 1/2$ (initial state). Since the Λ has $I = I_3 = 0$, the final state isospin is determined by the πN isospin, which can be either 3/2 or 1/2. However, isospin conservation allows only $I = 1/2$ in the final state. The Clebsch-Gordan coefficients for $|\mathrm{n}\pi^0\rangle$ and $|\mathrm{p}\pi^-\rangle$ in the $|\frac{1}{2}, -\frac{1}{2}\rangle$ state are $\sqrt{\frac{1}{3}}$ and $-\sqrt{\frac{2}{3}}$ respectively. So

$$\frac{\sigma_a}{\sigma_b} = \left(\frac{\sqrt{1/3}}{-\sqrt{2/3}}\right)^2 = \frac{1}{2}.$$