MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

8.276 Spring 2007 **April 12, 2007**
Solution to Problem set #7

1. (5 points)

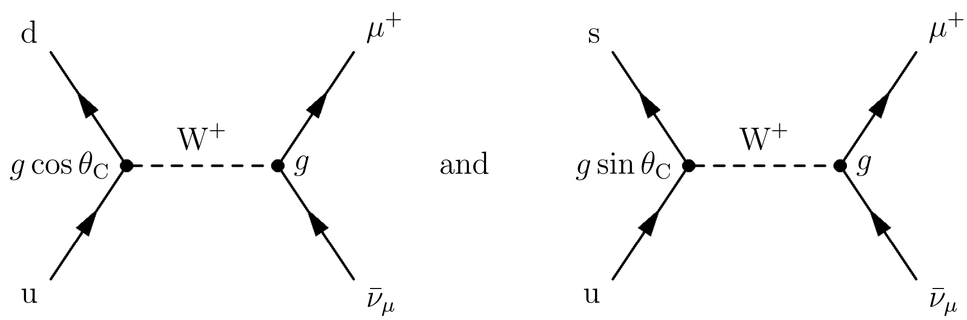
a) $\eta \to 2\pi$ does not conserve parity. (π and η have odd intrinsic parity; (2π) system has even parity.)

b) $\eta \to 3\pi$ does not conserve isospin ($I(\eta) = 0, I(\pi) = 1$), so forbidden as a strong interaction but allowed as electromagnetic decay.

2. (10 points)

$$\pi^+ \to \mu^+ \nu_\mu \qquad [\mathrm{u}\bar{\mathrm{d}} \to \mu^+ \nu_\mu]$$
$$\mathrm{K}^+ \to \mu^+ \nu_\mu \qquad [\mathrm{u}\bar{\mathrm{s}} \to \mu^+ \nu_\mu]$$

The Feynman diagrams are



So the ratio of the decay ratios is

$$\frac{\Gamma(\mathrm{K}^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \frac{f\tau_\pi}{\tau_{\mathrm{K}\mu\nu}} = \frac{F g^4 \sin^2\theta_\mathrm{C}}{g^4\cos^2\theta_\mathrm{C}} = F\tan^2\theta_\mathrm{C}$$

where $f = 0.64$ is the branching fraction for the $\mathrm{K}^+ \to \mu^+\nu_\mu$ decay ($f \approx 1$ for $\pi^+ \to \mu^+\nu_\mu$), and F is the phase space factor due to the different masses of the K and π (and thus the different amounts of energy available).

With $F = 18$, we have

$$\tan^2\theta_\mathrm{C} = \frac{(0.64)(2.6\times10^{-8})}{18(1.2\times10^{-8})} = 0.077$$
$$\Longrightarrow \qquad \theta_\mathrm{C} = 0.27 \approx \sin\theta_\mathrm{C} \qquad \text{(compare (10.18))}$$

3. (15 points)

"Hyperfine splitting" in meson mass spectrum:

$$m(\mathrm{q}_1\bar{\mathrm{q}}_2) = m_1 + m_2 + a\frac{\boldsymbol{s}_1 \cdot \boldsymbol{s}_2}{m_1 m_2}$$

$$\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2 \quad \longrightarrow \quad \boldsymbol{S}^2 = \boldsymbol{s}_1^2 + \boldsymbol{s}_2^2 + 2\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 \quad \longrightarrow \quad \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = \frac{1}{2}\left(\boldsymbol{S}^2 - \boldsymbol{s}_1^2 - \boldsymbol{s}_2^2\right)$$

$$\begin{aligned} \text{Eigenvalues: } \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 &= \frac{1}{2}\left(S(S+1) - s_1(s_1+1) - s_2(s_2+1)\right) \\ &= \frac{1}{2}\left\{S(S+1) - \frac{3}{2}\right\} \qquad \text{for } s_1 = s_2 = \frac{1}{2} \end{aligned}$$

So

$$\begin{aligned} &\text{for} \quad S=0 \quad \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = -\frac{3}{4} \\ &\text{for} \quad S=1 \quad \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = +\frac{1}{4} \end{aligned}$$

Using these values, $m_\mathrm{u} = m_\mathrm{d} = 0.310\mathrm{GeV}/c^2$, $m_\mathrm{s} = 0.483\mathrm{GeV}/c^2$, and $a/m_\mathrm{u}^2 = 0.64\mathrm{GeV}/c^2$, we get $a/m_\mathrm{u}m_\mathrm{s} = 0.411\mathrm{GeV}/c^2$, $a/m_\mathrm{s}^2 = 0.264\mathrm{GeV}/c^2$, and the following masses:

	Meson	Calculated mass (GeV/c^2)	Observed mass (GeV/c^2)
0^-	π	0.140	0.135(π^0), 0.140($\pi^\pm$)
	K	0.485	0.494($\mathrm{K}^\pm$), 0.498($\mathrm{K}^0, \bar{\mathrm{K}}^0$)
	$\eta^{(*)}$	0.559	0.549
	$\eta'^{(*)}$	0.349	(0.958) (Whoops!)
1^-	ρ	0.780	0.770
	ω	0.780	0.782
	K^*	0.896	0.892($\mathrm{K}^{*\pm}$), 0.896($\mathrm{K}^{*0}, \bar{\mathrm{K}}^{*0}$)
	ϕ	1.032	1.019

Agreement is good except for the η' so perhaps this mass formula should be applied only to the octet states. However it appears to work for the singlet 1^- state (ϕ).

$^{(*)}$ The $|\eta\rangle$ and $|\eta'\rangle$ contain $\mathrm{u\bar{u}}$, $\mathrm{d\bar{d}}$, and $\mathrm{s\bar{s}}$ quarks:

$$|\eta\rangle = \frac{1}{\sqrt{6}}\left(|\mathrm{u\bar{u}}\rangle + |\mathrm{d\bar{d}}\rangle - 2\,|\mathrm{s\bar{s}}\rangle\right).$$

So $m_\eta = \frac{1}{6}m_{\mathrm{u\bar{u}}} + \frac{1}{6}m_{\mathrm{d\bar{d}}} + \frac{2}{3}m_{\mathrm{s\bar{s}}} = \frac{1}{3}m_\pi + \frac{2}{3}m_{\mathrm{s\bar{s}}}$

$$|\eta'\rangle = \frac{1}{\sqrt{3}}\left(|\mathrm{u\bar{u}}\rangle + |\mathrm{d\bar{d}}\rangle + |\mathrm{s\bar{s}}\rangle\right).$$

So $m_{\eta'} = \frac{1}{3}m_{\mathrm{u\bar{u}}} + \frac{1}{3}m_{\mathrm{d\bar{d}}} + \frac{1}{3}m_{\mathrm{s\bar{s}}} = \frac{2}{3}m_\pi + \frac{1}{3}m_{\mathrm{s\bar{s}}}$

The "η' mass problem" is discussed in the book "Gauge Theories of the Strong, Weak, and Electromagnetic Interactions", C. Quigg (Benjamin, 1983, p.252).

4. (10 points)

a) Use the reactions $\pi^- + \mathrm{p} \to \underset{S=+1}{\mathrm{K}^0} + \underset{S=-1}{\Lambda}$ or $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \Sigma^0$. Let a monoenergetic π^- beam strike a hydrogen target (stationary proton). Two body kinematics ensures that at a given angle to the beam direction, a unique K^0 energy occurs. (Actually, there will be two groups of monoenergetic K^0's, owing to the different masses of the Λ and Σ^0).

How do we know this beam is pure K^0 (no $\bar{\mathrm{K}}^0$)? Conservation of Strangeness. [One must make sure the π^- beam energy is below the threshold for $\pi^- + \mathrm{p} \to \underset{S=+1}{\mathrm{K}^0} + \underset{S=-1}{\bar{\mathrm{K}}^0} + n$.]

b) "Strangeness oscillations" can occur in the neutral kaon system: We start out with a pure $S = +1$ state, which is a superposition of $CP = \pm 1$ states. Since the $CP = +1$ eigenstate decays much more rapidly than the $CP = -1$ state, after some time we will have predominantly $CP = -1$, which is a mixture of $S = \pm 1$. As a result, $\bar{\mathrm{K}}^0$s will appear ($S = -1$), which can interact to produce hyperons:

$$\begin{aligned}\bar{\mathrm{K}}^0 + \mathrm{p} &\to \Sigma^+ + \pi^0 \\ &\to \Lambda + \pi^+ \quad \text{etc.}\end{aligned}$$

5. (10 points)

a) $\underset{S=+1}{\mathrm{K}^0}$, $\underset{S=-1}{\bar{\mathrm{K}}^0}$ have different interaction probabilities:

$$\begin{aligned} &\mathrm{K}^0 + \mathrm{p} \to \mathrm{K}^0 + \mathrm{p}, \mathrm{K}^0 + \mathrm{n} \\ &\bar{\mathrm{K}}^0 + \mathrm{p} \to \bar{\mathrm{K}}^0 + \mathrm{p}, \\ \text{but also} \quad &\to \Sigma^+ + \pi^0, \Sigma^0 + \pi^+, \Lambda^0 + \pi^+ \end{aligned}$$

(Similarly for reactions on neutrons) So $\bar{\mathrm{K}}^0$, with more reaction channels available, will be attenuated more strongly.

b) $|\mathrm{K}_2\rangle = \frac{1}{\sqrt{2}}(|\mathrm{K}^0\rangle + |\bar{\mathrm{K}}^0\rangle)$ (equal components of $|\mathrm{K}^0\rangle$ and $|\bar{\mathrm{K}}^0\rangle$). After passing through matter, we have

$$|\mathrm{K}_2'\rangle = \frac{1}{\sqrt{2}}(f|\mathrm{K}^0\rangle + \bar{f}|\bar{\mathrm{K}}^0\rangle) \quad \text{with} \quad f \neq \bar{f},$$

since $|\mathrm{K}^0\rangle$ and $|\bar{\mathrm{K}}^0\rangle$ interact differently. Write

$$\begin{aligned} |\mathrm{K}_2'\rangle &= \frac{f+\bar{f}}{2\sqrt{2}}(f|\mathrm{K}^0\rangle + \bar{f}|\bar{\mathrm{K}}^0\rangle) + \frac{f-\bar{f}}{2\sqrt{2}}(f|\mathrm{K}^0\rangle - \bar{f}|\bar{\mathrm{K}}^0\rangle) \\ &= \frac{1}{2}(f+\bar{f})|\mathrm{K}_2\rangle + \frac{1}{2}(f-\bar{f})|\mathrm{K}_1\rangle \end{aligned}$$

Since $f - \bar{f} \neq 0$, K_1s are "regenerated." (see diagram below)

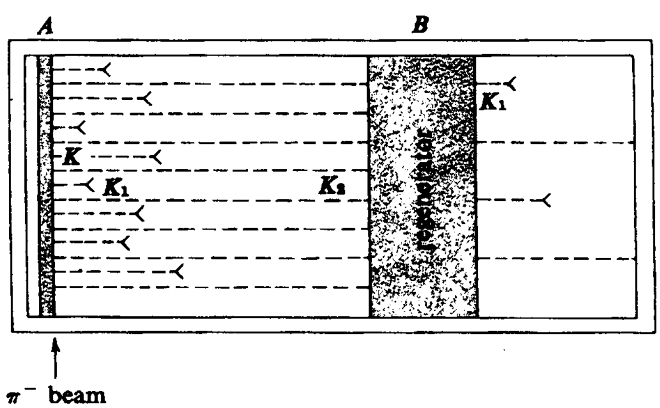


Figure 19-6 Schematic diagram showing the regeneration of K_1 events in a multiplate cloud chamber. The symbol ⟨ indicates the decay

$$K_1 \to \pi^+ + \pi^-$$

In target A, pions generate K^0. The K_1 component decays fast and only K_2 reach the regenerator B. In crossing it some of them undergo strong interactions and emerge as K^0 having a regenerated K_1 component. [From A. Pais and O. Piccioni, *Phys. Rev.*, **100**, 1487 (1955).]

c) $\mathrm{K}_1 \to 2\pi$, $\mathrm{K}_2 \to 3\pi$, (except for small amount of CP violation). If see any 2π decays, know that beam has K_1 admixture.

6. (10 points) see text

Note that the text is several years out of date. The "B-factory" at SLAC referred to in part b) has been built and has been operating for five years, with beam energies $E_{e^-} = 9.0\text{GeV}$ and $E_{e^+} = 3.1\text{GeV}$.