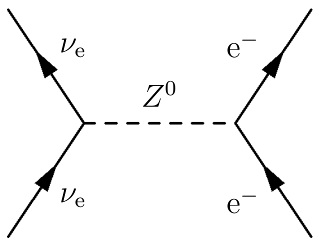
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

8.276 Spring 2007 **March 22, 2007**
Solution to Problem set #5

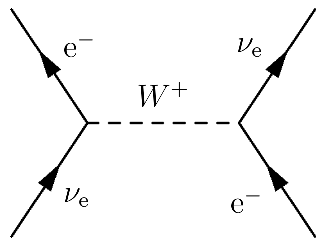
1. (10 points) see text

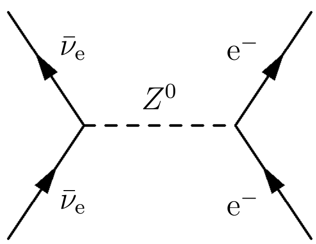
2. (5 points)

$\nu_e e^- \rightarrow \nu_e e^-$ and $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ can proceed through either charged- or neutral-current interaction:

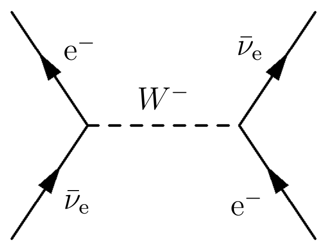


or

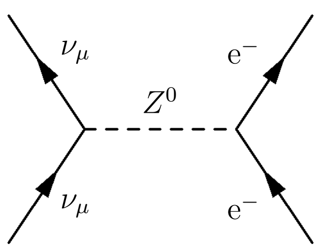




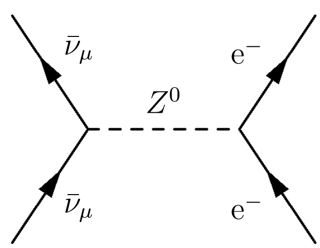
or



For $\nu_\mu e^- \rightarrow \nu_\mu e^-$ and $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$, L_μ and L_e conservation forbids the charged-current interaction. (e.g. $\nu_\mu \nrightarrow e^- + W^+$) So, these processes can only occur through the neutral current interaction:



or



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3. (10 points)
Use 4-momentum conservation.

$$
\begin{aligned}
&p_\nu + p_\mathrm{e} = p'_\nu + p'_\mathrm{e} \\
&p_\nu^2 + 2p_\nu \cdot p_\mathrm{e} + p_\mathrm{e}^2 = {p'_\nu}^2 + 2p'_\nu \cdot p'_\mathrm{e} + {p'_\mathrm{e}}^2 \\
&\implies \quad p_\nu \cdot p_\mathrm{e} = p'_\nu \cdot p'_\mathrm{e}, \quad \text{since} \quad p_\nu^2 = {p'_\nu}^2 = 0 \\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad\; p_\mathrm{e}^2 = {p'_\mathrm{e}}^2 = m^2 \\
&p_\nu \cdot p_\mathrm{e} = (p_\nu + p_\mathrm{e} - p'_\mathrm{e}) \cdot p'_\mathrm{e} \\
&\qquad\quad\; = p_\nu \cdot p'_\mathrm{e} + p_\mathrm{e} \cdot p'_\mathrm{e} - m^2
\end{aligned}
$$

In the lab system, $p_\nu = (E_\nu, \boldsymbol{p}_\nu)$; $p_\mathrm{e} = (m, 0)$; $p'_\mathrm{e} = (E, \boldsymbol{p})$.
So we have

$$
\begin{aligned}
E_\nu m &= E_\nu E - |\boldsymbol{p}_\nu||\boldsymbol{p}| \cos\theta + mE - m^2 \\
&\approx E_\nu E(1 - \cos\theta) + mE \quad \text{since} \quad |\boldsymbol{p}| \approx E \\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad\;\; m^2 \approx 0
\end{aligned}
$$

$$
\begin{aligned}
&1 - \cos\theta = \frac{(E_\nu - E)m}{E_\nu E} \\
&2\sin^2\frac{\theta}{2} = m\left(\frac{1}{E} - \frac{1}{E_\nu}\right) \\
&\sin\frac{\theta}{2} \approx \sqrt{\frac{m}{2E}} \approx \frac{\theta}{2} \\
&\implies \theta \approx \sqrt{\frac{2m}{E}}
\end{aligned}
$$

$\sin\frac{\theta}{2}$ max when the second term is negligible, i.e. $E_\nu \ll E$ (as is generally the case).

4. (10 points)

a) $\Gamma \propto m^5$ (Eq. (10.5));

$$\frac{\Gamma(\tau \to \mathrm{e} + \nu + \bar{\nu})}{\Gamma(\mu \to \mathrm{e} + \nu + \bar{\nu})} = \left(\frac{1784}{105}\right)^5 = 1.42 \times 10^6$$

$$\Longrightarrow \Gamma(\tau \to \mathrm{e} + \nu + \bar{\nu}) = \frac{1.42 \times 10^6}{2.2 \times 10^{-6}} = 6.44 \times 10^{11}\mathrm{s}^{-1}$$

$$\Longrightarrow \text{Total } \tau \text{ decay rate} = \frac{6.44 \times 10^{11}}{0.17} = 3.79 \times 10^{12}\mathrm{s}^{-1}$$

$$\Longrightarrow \text{Lifetime} = \frac{1}{3.79 \times 10^{12}} = 2.64 \times 10^{-13}\mathrm{s} = \tau_0.$$

b) τ_0 is the lifetime in the rest frame of the τ lepton. In the laboratory, the lifetime will be $\tau = \gamma\tau_0$ (the relativistic time dilation), where $\gamma = \dfrac{E_\tau}{m_\tau c^2}$.

Momentum $\quad p_\tau = \beta\gamma m_\tau c$

Distance traveled $\quad d = vt = \beta c \cdot \gamma\tau_0 = \dfrac{p_\tau \tau_0}{m_\tau}$

$$= \frac{5 \times 2.64 \times 10^{-13}}{1.784} \times 3 \times 10^8\mathrm{m}$$

$$= 2.22 \times 10^{-4}\mathrm{m}$$