MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

8.276 Spring 2007 **March 8, 2007**
Solution to Problem set #3

1. (5 points) The Rosenbluth formula is

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(\alpha^2)\tan^2\frac{\theta}{2}\right]$$

$$\text{with} \quad \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{4Z^2\alpha^2(\hbar c)^2 E'^2}{(Qc)^4}\cos^2\left(\frac{\theta}{2}\right)\frac{E'}{E}; \qquad \tau = \frac{Q^2}{4M^2c^2}.$$

At $\theta = 180°$, $\cos^2\frac{\theta}{2} = 0$, but $\cos^2\frac{\theta}{2}\tan^2\frac{\theta}{2} = \sin^2\frac{\theta}{2} = 1$, so only the magnetic scattering term (second term in []) contributes. The electron is scattering from magnetization density in the targets. Charge scattering vanishes at 180°.

$$\frac{d\sigma(180°)}{d\Omega} = \frac{4Z^2\alpha^2(\hbar c)^2}{(Qc)^4}\frac{E'}{E}\left(2\tau G_M^2(Q^2)\right)$$

$$Q^2c^2 = 4E'E$$

$$E' = \frac{E}{1 + \frac{2E}{Mc^2}}$$

$$\tau = \frac{E'E}{(Mc^2)^2}$$

2. (10 points) Writing the Rosenbluth formula in terms of K_1 and K_2,

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{\tau K_2}{Q^2} + \frac{2\tau}{Q^2}K_1\tan^2\frac{\theta}{2}\right]$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{K_2}{(2Mc)^2} + \frac{2K_1\tan^2\frac{\theta}{2}}{(2Mc)^2}\right] \qquad \text{with } Q^2 = -q^2, \tau = \frac{Q^2}{(2Mc)^2}$$

For a point proton, $K_1 = Q^2$ and $K_2 = (2Mc)^2$, so

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[1 + \frac{2Q^2}{(2Mc)^2}\tan^2\frac{\theta}{2}\right].$$

Since $Q^2 = \frac{4E'E}{c^2}\sin^2\frac{\theta}{2} \approx \frac{4E^2}{c^2}\sin^2\frac{\theta}{2}$, $Q^2 \ll (Mc)^2$ for $E \ll Mc^2$ and thus second term $\ll 1$. [Since $\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$ contains a factor of $\cos^2\frac{\theta}{2}$, there is no danger of the second term "blowing up" as $\theta \to 180°$.]

3. (15 points)

For pointlike protons, $\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \frac{4\alpha^2(\hbar c)^2 E'^2}{(Qc)^4}\cos^2\left(\frac{\theta}{2}\right)\frac{E'}{E} \quad (Z=1).$

$$E' = \frac{E}{1+\frac{E}{Mc^2}(1-\cos\theta)} \qquad \big(E = 15\text{GeV}, \theta = 0.1, Mc^2 = 0.94\text{GeV}\big)$$

$$= \frac{15}{1+\frac{15}{0.94}(0.005)} = 13.9\text{GeV}$$

$$(Qc)^2 = 4EE'\sin^2\frac{\theta}{2} = 2.08\text{GeV}^2$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \frac{4(0.197)^2(13.9)^2}{(2.08)^2(137)^2}(0.998)\frac{13.9}{15} = 3.42\times10^{-4}\text{fm}^2/\text{sr} = 3.42\times10^{-30}\text{cm}^2/\text{sr}$$

To compute the finite-size cross section, we'll need

$$\tau = \frac{Q^2}{(2Mc)^2} = \frac{2.08}{(2\times0.94)^2} = 0.59,$$

$$G_E(Q^2) = \left(1+\frac{2.08}{0.71}\right)^{-2} = 0.065,$$

$$\text{and} \quad G_M(Q^2) = 2.79 G_E(Q^2) = 0.181.$$

For finite-size protons,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = (3.42\times10^{-4})\left[\frac{(0.65)^2+0.59(0.181)^2}{1.59} + 2(0.59)(0.181)^2\tan^2(0.05)\right] \quad \text{fm}^2/\text{sr}$$

$$= (3.42\times10^{-4})\,[0.0148+0.000097]\,\text{fm}^2/\text{sr}$$

$$= 5.09\times10^{-6}\text{fm}^2/\text{sr} = 5.09\times10^{-32}\text{cm}^2/\text{sr}.$$

$$\text{count rate} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\times(\triangle\Omega)\times\frac{N}{\text{cm}^2}\times\phi, \qquad \triangle\Omega = 10^{-4}\text{sr}, \quad \phi = 10^{14}\text{s}^{-1}$$

$$\frac{N}{\text{cm}^2} = 6.02\times10^{23}\frac{\text{atoms}}{\text{mole}}\times 1\frac{\text{mole}}{\text{g}}\times 6\times10^{-2}\frac{\text{g}}{\text{cm}^3}\times 100\text{cm} = 3.6\times10^{24}$$

The number of electrons scattered per second:
(a) for pointlike protons $(3.4\times10^{-30})(10^{10})(3.6\times10^{24}) = 1.23\times10^5\text{s}^{-1}$
(b) for protons of finite-size $(5.1\times10^{-32})(10^{10})(3.6\times10^{24}) = 1.83\times10^3\text{s}^{-1}$

4. (10 points)
From textbook (7.1),

$$W^2c^2 = P'^2 = (P+q)^2 = M^2c^2 + 2M\nu - Q^2,$$
$$\text{where} \quad Q^2 = -q^2 = 4\frac{EE'}{c^2}\sin^2\frac{\theta}{2}$$
$$\text{and} \quad \nu = E - E'.$$

In this problem, $E = 10\text{GeV}$, $E' = 7\text{GeV}$ (so $\nu = 3\text{GeV}$), $\theta = 10°$.

$$Q^2 = 4\frac{10 \times 7}{c^2}\sin^2 5° = 2.13\text{GeV}^2/c^2$$
$$W^2 = (0.938)^2 + 2(0.938)(3) - 2.13 = 4.38\text{GeV}^2/\text{c}^4$$
$$\text{so} \quad \boxed{W = 2.09\text{GeV}/c^2}.$$

5. (5 points) see textbook

6. (15 points) see textbook