Massachusetts Institute of Technology
Department of Physics

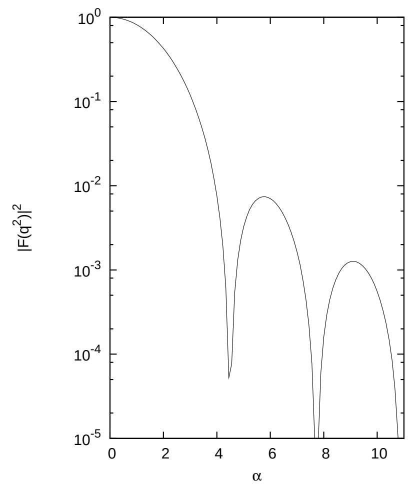
8.276 Spring 2007 **February 27, 2007**
Solution to Problem set #2

1. (10 points)

$$\begin{aligned}
F(q^2) &= \int e^{i\vec{q}\cdot\vec{r}}\rho(r)\mathrm{d}^3r \qquad \text{with } \hbar = 1 \\
&= \int_0^\infty \rho(r)r^2\mathrm{d}r \int_0^\pi e^{iqr\cos\theta}\sin\theta\mathrm{d}\theta \int_0^{2\pi}\mathrm{d}\phi \\
&= 2\pi\int_0^\infty \rho(r)r^2\mathrm{d}r\int_{-1}^{1} e^{iqrx}\mathrm{d}x \qquad (x=\cos\theta) \\
&= 2\pi\int_0^\infty \rho(r)r^2\mathrm{d}r\frac{1}{iqr}(e^{iqrx}-e^{-iqrx}) \\
&= \frac{4\pi}{q}\int_0^\infty \rho(r)r\sin(qr)\mathrm{d}r
\end{aligned}$$

$$\text{with} \quad \rho(r) = \begin{cases} \dfrac{3}{4\pi R^3} & 0<r<R \\ 0 & r>R \end{cases}$$

$$\begin{aligned}
F(q^2) &= \frac{3}{\alpha^3}\int_0^\alpha \alpha'\sin\alpha'\mathrm{d}\alpha' \qquad \alpha' = qr,\ \alpha = |\vec{q}|R \\
&= \frac{3}{\alpha^3}(\sin\alpha - \alpha\cos\alpha)
\end{aligned}$$



(see Fig. 5.6) minima at $\alpha \approx 4.5, 7.7, 10.9$

2. (15 points)

The momentum transfer: $$|\vec{q}| = 2p\sin\frac{\theta}{2} = 2(500)\sin 5^\circ = 87.2\mathrm{MeV/c}$$

The Mott cross section:
$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{4Z^2\alpha^2(\hbar c)^2}{(qc)^4}E^2\cos^2\frac{\theta}{2} \\
&= \frac{4(20)^2(197.3)^2(500)^2}{(137)^2(87.2)^4}\cos^2 5^\circ \\
&\approx 14.2\mathrm{fm}^2/\mathrm{sr}
\end{aligned}$$

The Form factor: $R = 1.2A^{1/3} = 1.2(40)^{1/3} = 4.10\text{fm}$

$$\alpha = \frac{|\vec{q}|R}{\hbar} = \frac{87.2 \cdot 4.10}{197.3} = 1.81$$

$$F(\vec{q}^2) = 3\frac{(\sin\alpha - \alpha\cos\alpha)}{\alpha^3} = 0.71$$

$$|F(\vec{q}^2)|^2 = 0.50 = \text{the factor by which cross section reduced}$$

3. (10 points) electron-nucleus scattering kinematics with recoil

a)

$$Q^2 = \frac{EE'}{c^2}\sin^2\frac{\theta}{2} = 2M(E - E') \qquad \text{(see text)}$$

where

$$E' = E\left[1 + \frac{E}{Mc^2}(1 - \cos\theta)\right]^{-1}$$

$$Q^2 = Q^2_{\text{max}} \text{ at } \theta = 180°$$

$$\boxed{Q^2_{\text{max}} = \frac{4E^2M}{Mc^2 + 2E}}$$

b)

$$\nu = E - E' = \frac{2E^2}{Mc^2 + 2E} \quad \text{at } \theta = 180°$$

$$\text{and} \quad \boxed{E'_{\text{recoil}} = Mc^2 + \nu}$$

$$\boxed{|\boldsymbol{P'}| = \sqrt{Q^2_{\text{max}} + \frac{\nu^2}{c^2}}}$$

For ^{12}C with $E_e = 1\text{GeV}$

$$Mc^2 = (12)(0.9315) = 11.2\text{GeV}$$

$$\text{and} \quad \nu = \frac{2}{11.2 + 2} = 0.15\text{GeV}$$

$$\text{So} \quad \boxed{E'_{\text{recoil}} = 11.35\text{GeV}}$$

$$\boxed{|\boldsymbol{P'}| = 1.85\text{GeV/c}}$$

$$Q^2_{\text{max}} = \frac{4 \cdot 1(11.2)}{11.2 + 2} = 3.39(\text{GeV/c})^2$$

$$\boxed{Q_{\text{max}} = 1.84\text{GeV/c}}$$

4. (10 points) see text (5-4)