MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

8.276 Spring 2007 **February 15, 2007**
Solution to Problem set #1

1 (5 points) see text (#4-1) (note: $F \equiv A$ the detector area)
2 (5 points) see text (#4-2)
3 (10 points)

(a)

$$\rho_{\mathrm{Cu}} = 9\mathrm{g/cm}^3 \qquad A = 63.5 \tag{1}$$

$$\begin{aligned}(\text{the number of scattering center}) &= \frac{6\times 10^{23}}{63.5\mathrm{g}} \times \mathrm{g/cm}^3 \times 0.1\mathrm{cm} \times 4\mathrm{cm}^2 \\ &= \boxed{3.4\ \times 10^{22}}\end{aligned}$$

(b)

$$\begin{aligned}(\text{the fraction scattered}) &= \sigma \cdot (\text{the number of scattering center}) \\ &= 10^{-26}\mathrm{cm}^2 \times 8.5\times 10^{21}\mathrm{cm}^{-2} \\ &= \boxed{8.5\ \times 10^{-5}} \qquad\qquad 0.008\%\end{aligned}$$

4 (10 points) Rutherford scattering cross section

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{z^2 Z^2 \alpha^2 (\hbar c)^2}{(4T)^2 \sin^4\frac{\theta}{4}} \\ &= \left[\frac{2\times 82\times 197\mathrm{MeV}\cdot\mathrm{fm}}{137\times 40\mathrm{MeV}\times 0.5}\right]^2 \\ &= 1.39\times 10^{-24}\mathrm{cm}^2\end{aligned}$$

$z = 2$ (charge of α-particle)
$Z = 82$ (charge of Pb nucleus)
$T = 10\mathrm{MeV}$
$\theta = \pi/2$
$\alpha = \frac{1}{137}$

$$\frac{N}{\sec} = 10^6\sec^{-1} \times \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{cm}^2 \times \frac{6\times 10^{23}}{208\mathrm{g}} \times \frac{11.3\mathrm{g}}{\mathrm{cm}^2} \times 0.1\mathrm{cm} \times \frac{1\mathrm{cm}^2}{(100)^2\mathrm{cm}^2} = \boxed{0.453\sec^{-1}}$$

5 (10 points) Although the α-particle energy wasn't specified at the beginning of the problem, it is safe to assume it to be non-relativistic, since $m_\alpha \approx 4\mathrm{GeV/c}^2$. Moreover, the maximum deflection angle of $\alpha \approx m_e/m_\alpha \approx 0.5/4000 \approx 0$.

So we can use 1-dimensional kinematics.

From the energy and momentum conservation,

$$v'_e = \frac{2m_\alpha v_\alpha}{m_\alpha + m_e} \approx 2v_\alpha.$$

The fractional momentum transfer:

$$\frac{p'_e}{p_\alpha} = \frac{2m_e v_\alpha}{m_\alpha v_\alpha} \approx \frac{2m_e}{m_\alpha} \approx \boxed{2.5 \times 10^{-4}} \qquad \text{(small)}$$

The energy transfer:

$$E'_e = \frac{1}{2} m_e v'^2_e = \frac{4m_e T_\alpha}{m_\alpha} = \boxed{5 \times 10^{-4} T_\alpha} \qquad \text{(small)}$$

For $T_\alpha = 10\text{MeV}$,

$$E'_e = \frac{4 \times 0.5 \times 10}{4000\text{MeV/c}^2} \text{MeV/c}^2\text{MeV} \approx \boxed{5 \times 10^{-3}\text{MeV}}$$