Massachusetts Institute of Technology
Department of Physics

8.276 Nuclear and Particle Physics **March 22, 2007**

Reading Assignment for 4/3 and 4/5
Particles and Nuclei, Chap. 12, Sects. 13.1-13.5, Chap. 14

Optional reading: Particles and Nuclei, Sect. 9.2
Coughlan and Dodd, Chaps. 7, 8, 15
Cahn and Goldhaber, Chaps. 7,9

Problem Set #6 (due 4/5)

1. *P&N*, 10-4

2. The ratio of the total cross sections for deep inelastic scattering of antineutrinos and neutrinos on the nucleon is measured to be $R = 0.5$ in a particular experiment.

a) Deduce the ratio of antiquark to quark momentum content in the nucleon.

b) Compute the average value of the quantity y (defined in Eq. (10.29)) in neutrino and antineutrino scattering.

3. The charged and neutral current differential cross sections for neutrino- and antineutrino-nucleus scattering may be written

$$\begin{aligned}
\frac{d\sigma^{CC}(\nu)}{dy} &= \frac{1}{2\pi}G^2sQ \\
\frac{d\sigma^{CC}(\overline{\nu})}{dy} &= \frac{1}{2\pi}G^2sQ[1-y]^2 \\
\frac{d\sigma^{NC}(\nu)}{dy} &= \frac{1}{2\pi}G^2sQ[g_L^2 + g_R^2(1-y)^2] \\
\frac{d\sigma^{NC}(\overline{\nu})}{dy} &= \frac{1}{2\pi}G^2sQ[g_L^2(1-y)^2 + g_R^2]
\end{aligned}$$

where s is the square of the center-of-mass energy and Q is the integral of the quark density distributions of the target nucleus. Assume an isoscalar target (i. e., a nucleus with N = Z) with contributions to the scattering from u and d valence quarks only. Use Eq. (11.16) and the information in Table 11.1 to derive the couplings g_L and g_R of the quarks to the Z^0 ($g \equiv \hat{g}$), and show that the total cross-section ratios are

$$\begin{aligned}
\frac{d\sigma^{NC}(\nu)}{d\sigma^{CC}(\nu)} &= \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W \\
\frac{d\sigma^{NC}(\overline{\nu})}{d\sigma^{CC}(\overline{\nu})} &= \frac{1}{2} - \sin^2\theta_W + \frac{20}{9}\sin^4\theta_W.
\end{aligned}$$

4. In the Weinberg-Salam model, the neutral-current cross section for neutrino-electron scattering may be written

$$\left[\frac{d\sigma}{dy}\right]_{\nu e}^{NC} = \frac{1}{2\pi}G^2 mE[g_L^2 + g_R^2(1-y)^2],$$

where m is the electron mass, E is the neutrino energy ($E \gg mc^2$), yE is the electron recoil energy, G is the Fermi coupling constant ($G = 1.16x10^{-5}$GeV^2). The "left-handed" and "right-handed" couplings of the leptons are given in terms of the Weinberg angle θ_W in the following table:

		g_L	g_R
(i)	$\nu_\mu e \to \nu_\mu e$	$-\frac{1}{2} + \sin^2\theta_W$	$\sin^2\theta_W$
(ii)	$\overline{\nu}_\mu e \to \overline{\nu}_\mu e$	$\sin^2\theta_W$	$-\frac{1}{2} + \sin^2\theta_W$
(iii)	$\nu_e e \to \nu_e e$	$\frac{1}{2} + \sin^2\theta_W$	$\sin^2\theta_W$
(iv)	$\overline{\nu}_e e \to \overline{\nu}_e e$	$\sin^2\theta_W$	$\frac{1}{2} + \sin^2\theta_W$

a) Plot the total cross sections σ/E for the scattering of ν_e, $\overline{\nu}_e$, ν_μ and $\overline{\nu}_\mu$ from stationary electron targets as a function of $\sin^2\theta_W$.

b) Where do $\sigma(\nu_\mu)$ and $\sigma(\overline{\nu}_\mu)$ have minimum values?

c) The vector and axial-vector coupling constants are given by

$$c_V = g_L + g_R$$
$$c_A = g_L - g_R.$$

For what value of $\sin^2\theta_W$ is the $\overline{\nu}_\mu$ coupling to electrons purely axial-vector?

5. Recent experiments have shown that neutrinos can "oscillate" from one flavor to another. This can only occur if the neutrinos have non-vanishing masses. Considering only the first two generations of neutrinos, one can write the mass eigenstates ν_1 and ν_2 as linear combinations of ν_e and ν_μ:

$$\nu_1 = \nu_e \cos\vartheta + \nu_\mu \sin\vartheta$$
$$\nu_2 = -\nu_e \sin\vartheta + \nu_\mu \cos\vartheta.$$

a) If at time $t = 0$ electron neutrinos are produced in a weak interaction show that at time t the probability P of detecting a muon neutrino is given by

$$P = \sin^2 2\vartheta \sin^2\left(1.27\Delta m^2\ L/pc\right),$$

where $\Delta m^2 = (m_1^2 - m_2^2)c^4$ is in $(eV)^2$, L is the distance from the neutrino source in meters and p is the neutrino momentum in MeV/c. (The masses m_1 and m_2 may be taken to be so small that $E_i = pc + m_i^2c^3,\ i = 1,2$.)

b) Assuming $\Delta m^2 = 1.5x10^{-4}(eV/c^2)^2$, calculate the oscillation length L for 0.4 MeV, 0.9 MeV, and 14 MeV neutrino energies.