Real Estate Risk: Debt Applications

By

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Introduction

The current economic uncertainty and the reports of negative absorption in the first quarter of 2001, concomitant with rising construction numbers, have raised concerns in the debt markets for the future. For several years now, delinquency and default rates have been extremely low and the "best of times" is now in the past. Of course, memories are vivid of the experience in the early 1990s and no one wishes to repeat past mistakes.

As economic uncertainty continues and lenders evaluate their originations and portfolios, it is natural to ask if this historic experience in the debt market has any bearing for the future. More importantly, what is the risk of defaults going forward into the 21st century? To answer these questions, we need a forward looking measure of real estate market risk applied to different loan structures across properties and geography. Put differently, we need a probability distribution of the future cash flows of the collateral over which we can superimpose the financial obligations both for debt services and principal repayment, period by period.

This paper is a further evolution of our thinking [Wheaton, et. al. 1999] on defining, measuring and applying real estate market risk to investment decisions– in this

case commercial mortgage debt. Readers are referred to two companion papers on this topic: the first presents our methodology for how real estate researchers should define, measure and estimate risk for real estate [Wheaton, et. al., 2001a] and the other describes the application of this risk approach to equity risk-return analysis [Wheaton, et. al., 2001b]. This paper presents an application of our methodology to estimating debt default, severity, conditional loss, expected loss and unexpected loss or Value-at-Risk (VaR).

Our work has benefited immeasurably from discussions with fellow academics, researchers, and many clients of Torto Wheaton Research and we are grateful to them.[1]

Estimating a Probability Distribution of Events

In Wheaton et. al. 2001a, we show the steps we follow to estimate the probability distribution of events or outcomes for each of the variables in our forecast models for each property market and geography. This methodology provides the foundation for a forward-looking approach to estimating the default risk and severity associated with real estate debt. Our approach compares the distribution of income and value outcomes for the property to the underwriting standards for a loan on the property. For each period we ascertain the *probability* that there is insufficient value or income to cover the loan, how much expected loss in value is likely to occur, and for given risk standards, what loss is likely to be experienced. We will discuss each of these in turn, but first a quick summary of the methodology that generates the distribution of outcomes.

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[1] Special mention for Tim Riddiough, Robert Gray, Robert Zisker, Ron Redding and Jim Titus

In **Figures 1 and 2**, we reproduce from our earlier paper the probability

distributions or confidence intervals for net operating income (NOI) and value for a

stylized New York office property.[2] The way to think about this is that <u>at each time</u>

<u>period</u> there is a distribution of outcomes for either NOI or value (the two figures) which,

based on the central limit theorem, is approximately normally distributed. We have

demonstrated this as shown by the curve at period six. Using estimates of the expected

value and standard deviation from our econometric models, we can estimate the

probabilities over the distribution. We have nicknamed these probability distributions

"cones." The expected value of the distribution in the cones corresponds to the

"baseline" NOI forecast, which, in this example, shows healthy, increases for four years

as mid-to-late 90's below-market leases roll to market levels.[3]

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[2] The model can generate cash flows or NOI's.
[3] Specific collateral attributes (e.g., rollovers, operating leverage, etc.) can be accommodated in the model.
More below.

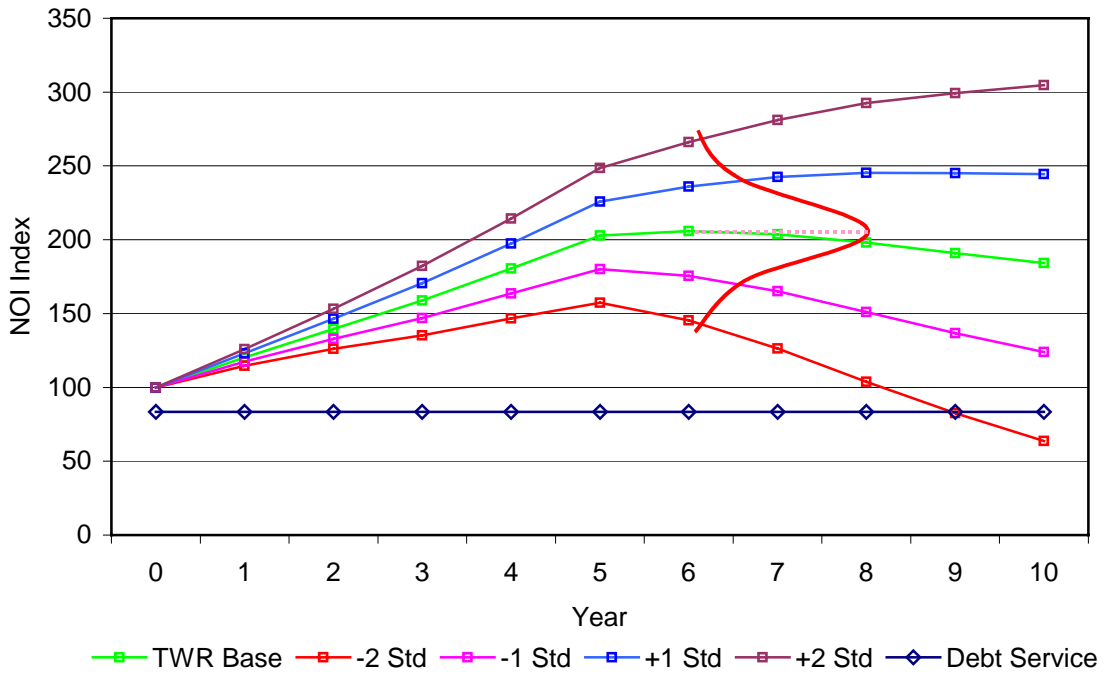**Figure 1. New York Office *NOI* Probability Distributions**
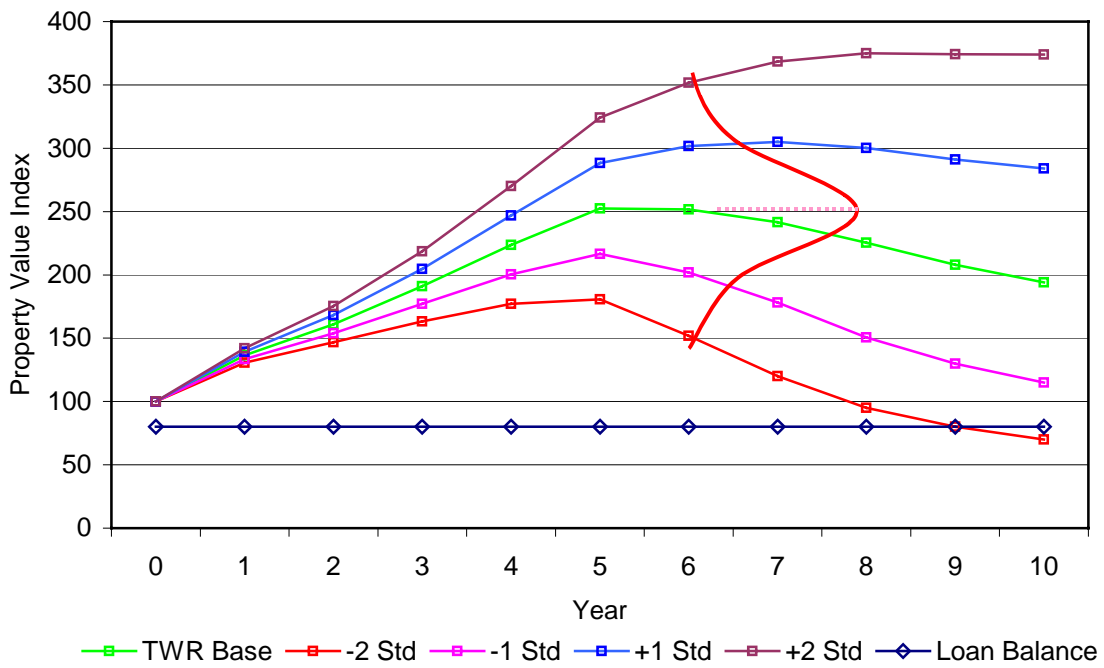

Figure 2. New York Office *Value* Probability Distributions

We have also introduced debt on the property, which for this example is a simple, fixed rate-and-payment mortgage loan which is represented by the horizontal line. (alternative financial structures can be applied.) In **Figure 1**, the horizontal line represents the annual debt service payment of the loan (incorporating both interest and amortization). For this example we assume an initial debt service coverage ratio (hereafter, DSCR) of 1.25.

In **Figure 2**, the line represents the outstanding loan balance. This horizontal line signifies an interest only loan. If we introduced amortization, the loan balance would decline over time. The initial loan to value ratio (LTV, hereafter) for this example is assumed to be .80. Higher horizontal lines signify lower DSCR or higher LTV ratios. Thus any loan structure can be modeled against the underlying probability distribution of the property's NOI and value.

Given **Figure 1**, we could say that there is little probability, in this example, that the New York office NOI will be less than debt service (for the shown 1.25 DSCR) until year eight. By year ten, there is a 7% probability that NOI will not cover the debt payment, or that DSCR will drop below 1.0.

Figure 1 conveys much more information than the simple observation that there is a 7% likelihood of insufficient NOI. There is also, for example, a 2.5% probability that NOI will be insufficient by more than 20%. The approach allows us to estimate *both* the dollar amount and the probability of debt service shortfall.[4]

We can determine similar probabilities for property value versus the loan balance. For instance, what is the probability that the underlying loan collateral or value falls

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[4] The model is flexible and allows us to introduce fixed or other costs associated with shortfalls.

below the outstanding balance, or that LTV rises above 1.0? For this New York example, the probability is 7% in year ten but only 0.02% in year eight.

Imposing a financial structure on the future NOI and value probability distributions gives us a probability for each outcome in each period. This is derived from our forecasts of leasing and capital markets and by plugging these results into a discounted cash flow model to generate NOI and values. With these results we can generate estimates of default and severity to which we now turn.

Estimating Default Probabilities

Modern finance theory suggests that actual defaults (loan restructuring, foreclosure, etc.) do not always occur if NOI is temporarily less than debt service or if collateral value falls below loan balance. Borrowers will typically seek to retain control of the asset until some option "hurdle" or threshold is reached. This is especially true if the borrower believes that the adverse situation is temporary.

Lenders recognize that some borrowers, who are experiencing difficulties in paying their loan obligations, have more experience with the property, and it is in the lender's interest at times to provide some forbearance rather than foreclosing, especially when the experience is considered temporary. Reinforcing this behavior is the title transfer costs of foreclosure.

Researchers modeling credit risk for real estate have long sought to use databases of default experience to calculate the probabilities of default *given* certain market, property and loan characteristics. There have been only a few historical studies and each has been limited in one way or another: a limited number of properties, a limited time

period of coverage and limited or missing variables. The few research studies that have been done have produced results that are methodologically consistent with our approach here, but with different magnitudes of risk.

In general we can identify two approaches to estimating default probabilities: historical and forward looking. The historical approach is to identify certain loan, property and market characteristics that have lead to defaults in the past and to assume that these same parameters will lead to future default experiences. This approach essentially believes that the past, mostly the experience of the 1980s, will be repeated in the future.

The second approach is forward looking. This approach estimates the probabilities of future market outcomes, using econometric forecasts of the future, and then matches these against contractual debt obligations, to estimate the probability of a particular loan defaulting in a particular period. In this forward approach we use historical market experience to estimate the forecasting model, and in a limited sense, to calibrate how default probabilities are linked to market outcomes. Actual future default probabilities thus depend on the combination of a forecast, and the calibrated default model.

We have reviewed the literature on commercial mortgage default, and have obtained access to a very limited loan database. Combining sources, we have been able to *calibrate* empirically a default probability model. Since we readily admit that more research is needed for this function, we have built our model to allow different weightings for how DSCR and LTV values can lead to defaults. This will allow our

users to take different courses of action at different times in the cycle. The default
function we are currently using is shown in **Figure 3**.

Figure 3. Default Probability Function
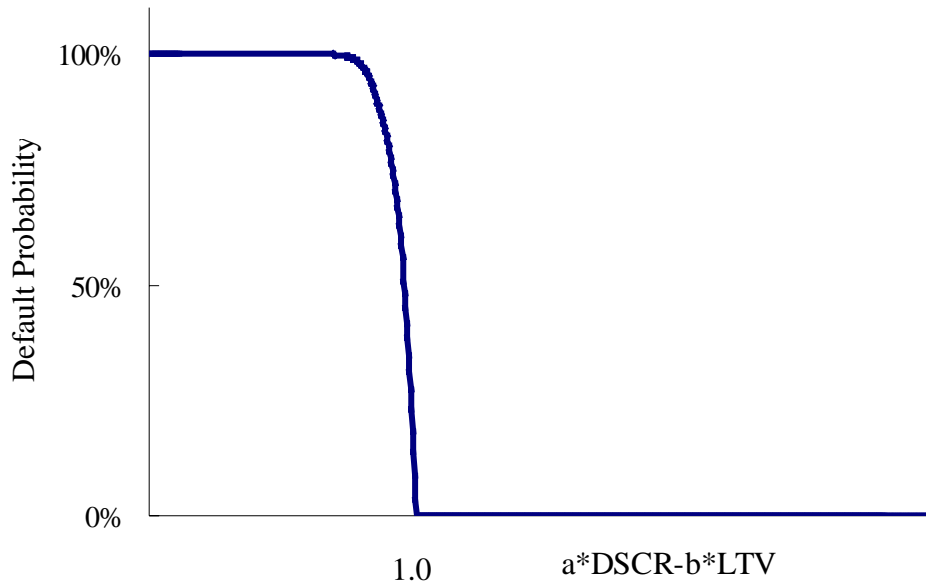
Our empirical evidence suggests that actual default probabilities go from zero to
one as LTV ranges from 0.95 to 1.25, or DSCR from 1.0 to .8. The probability of default
is also more sensitive to LTV than to DSCR, most likely because the latter can be a
temporary occurrence while LTV can reflect a more permanent or longer lasting
situation. Thus, given each value of LTV and DSCR, we can calculate a default
probability and then sum this probability times the likelihood that LTV and DSCR take
on these values.[5]

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[5] We use a logistic default probability model that is conditioned on values of LTV and DSCR, which in
turn have their own probability distributions. Integrating, we get the overall default probability for the loan
– each period into the future.

It is important to recognize that the probability portrayed in **Figure 3** is an instantaneous event probability that does not reflect the history or experience of a specific loan. To be used with a particular loan, the instantaneous probabilities must be analytically converted into a full hazard function that conveys the marginal and cumulative default probability of the loan, each period throughout its life.[6] With a more extensive database covering full loan histories, it would be possible to estimate directly the hazard function econometrically. Applying the logistic function in **Figure 3** to each year's NOI and value distributions in the New York example, (Figures 1 and 2) we get the loan's default hazard each year, and cumulatively over its life. This is shown in **Figure 4**.

Figure 4. New York Office Marginal and Cumulative Default Hazards

Cumulative Probability — Marginal Probability

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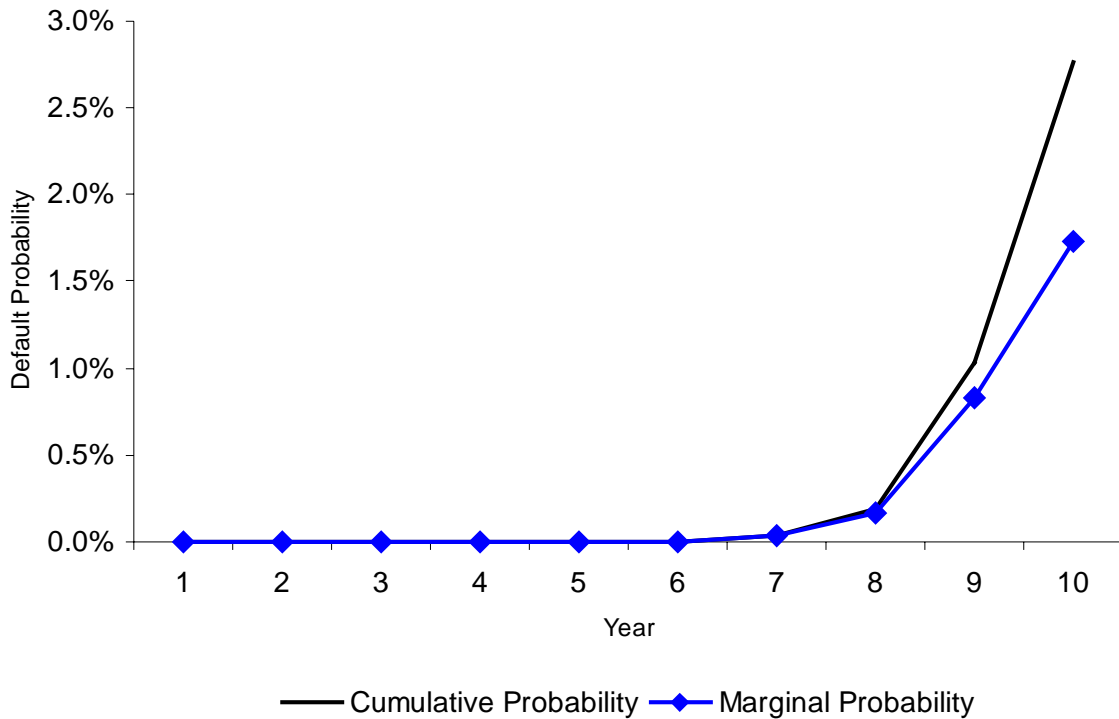
[6] Let L_i denote the logistic probability in period i, H_i the annual or marginal hazard probability, and CH_i the

The hazard default probability is a true "competing risk" likelihood that a *default occurs in that particular period, and not others.* Thus, they are mutually exclusive events and can be directly added into a cumulative probability of default. The cumulative probability is the likelihood that a *default event happens anytime up to the designated period.* In **Figure 4** we are showing that essentially the probability of a market credit loss is practically zero through period seven and then each period thereafter there is a rising marginal probability, which cumulates up to three percent by period ten.

Before proceeding further, some additional comments are needed on the default function. While we were able to review the database of loan defaults compiled by GMACCM through its subsidiary, Mortgage Analytics, we did not econometrically estimate a default function. Rather we have used their experience to calibrate a function based on the forward period values of the property's LTV and DSCR. Each of these variables is essentially weighted in the function to reflect what we consider important in the behavior of borrowers as to exercising the option to "put" a mortgage. Our future research agenda will continue to address these issues, but our approach allows the user considerable flexibility in applying the model and software.

For instance, we can alter the coefficients on the LTV and DSCR in the default function for each property type and, if one wishes, each market, to reflect the experiences or preferences of a particular lender. Alternatively, we can alter the weightings between the two variables to reflect expectations of *different* borrower behavior at *different* points in the cycle. We need not belabor the points now, but the key factor is that the model and software have enormous built in flexibility reflecting the complexities of real situations in public and private debt.

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cumulative hazard. $H_i = (1-CH_{i-1}) L_i$, and $CH_i = CH_{i-1} + H_i$, with $CH_0 = 0$.

Calculating Loss Severity

To this point we have generated probability distributions of future NOI and value, superimposed a loan structure, and then developed a default function which estimates the probability of default for each period. Applying the default function to the probability distribution of NOI and values, we can estimate the marginal and cumulative probabilities that a particular loan will default. Now we turn to the severity question: if there is a default, how much will be lost?

There are various options for calculating debt severity – conditional on a default occurring. Our analysis of the data at our disposal and of the literature, suggests that once a default event has happened, the severity of the loss experienced is closely related to the income and value available in the collateral at that period. Further, default events typically occur after several periods, during which only partial or delinquent debt payments have been made because of insufficient income. In addition, principal losses depend on the shortfall between collateral value and outstanding loan balance at the time of the default, plus some friction or administrative costs. Each of these considerations is part of the TWR model making it straightforward to estimate the net impact of these losses and hence a total loss severity.

First, we assume that *actual* debt payments are always equal to the lesser of NOI or the scheduled debt payment. Thus, when (with some probability) NOI is less than debt service, the difference is a shortfall or deferred payment. For each period we calculate the expected value of such delinquencies – over only that part of the NOI probability distribution where NOI is in fact less than debt service.

Second, if a default occurs in period t, the loan payments, which have been delinquent (with some probability) for all periods prior to t, if applicable, are summed into what we call *delinquency severity.*

Third, if a default occurs in period t, we assume that the loss of principal equals the difference between property value[7] and the loan balance. This is called *principal loss.*

Total loss severity, if a default event occurs, is then the sum of cumulative income delinquencies plus principal loss. Of course, both the probability of a default and its severity will vary by period over the term of the loan. We will present an application example in the next section.

Expected Loan Loss.

The next issue is to measure the overall real estate "risk" of a given loan over a particular interval of time. We use a concept that exists widely in the risk and insurance literature: *Expected loan loss.*

Expected loan loss is the product of the default probability times the severity. This can be calculated for each period, and since the periodic defaults are mutually exclusive events, the periodic expected losses are summed into an overall, lifetime expected loss. In theory, a risk neutral investor would pay the estimated expected loss if they wished to be *insured* against loan loss.

Table 1 shows the expected loss calculation for the New York office market example. The default probabilities in columns two and three correspond to **Figure 4** and

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[7] To understand how we estimate future values of the property see our first paper in this series, Wheaton et. al. 2001a

show the marginal and cumulative default probabilities. We also show the survival rates in this table.

If a default was to occur, the loss for that period is shown as the Annual Severity. We have *not* shown individually the income and principal losses with their corresponding probability, but rather the total severity. Finally, annual expected loss is the product of severity and default probability[8]. The final column in **Table 1** sums these losses *cumulatively* by year to get the expected loss *over* the holding time up to that period.

In our example, the New York office property loan, given the financial and collateral specifications, has a 2.8% cumulative probability of default over ten years. The marginal probabilities are essentially zero in the first seven years and begin rising in the eight year reaching a high of 1.7% in year ten. The calculations show that *if* the loan did default in year ten (it has a 97.2% probability it will survive beyond then.) the loss severity is estimated at $316,577.[9] The expected loss is $5,493 in year ten, and the loss over the full ten year holding period is $8,070 - representing .81% of the loan's original value.

Most of this risk occurs in years eight through ten when there is considerable uncertainty over outcomes in the New York market. While we are using New York as a stylized example, the probability distribution of outcomes does reflect the historic volatility of this market. There were huge defaults in 1975 and 1991, and, of course, in 1933. Looking forward, record rents are prompting future developments, and while New York is one of the best office markets currently, the results in **Table 1** reflect both the

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[8] Because of rounding issues the table results are exact but are not the product of the columns as reported.
[9] Again, this loss is the sum of a delinquency loss plus a principal loss.

lease rollovers in the outer years, when more supply will be available, combined with traditional demand uncertainty.

Table 1: New York Office Expected Loss and Severity

Year	Default Hazard	Cumulative Default Probability	Holding Period Survival Rate	Annual Severity	Annual Expected Loss	Holding Period Expected Loss	As a % of Original Loan Amount
1	0.0%	0.0%	100.0%	$0	$0	$0	0.00%
2	0.0	0.0	100.0	0	0	0	0.00
3	0.0	0.0	100.0	0	0	0	0.00
4	0.0	0.0	100.0	0	0	0	0.00
5	0.0	0.0	100.0	0	0	0	0.00
6	0.0	0.0	100.0	0	0	0	0.00
7	0.0	0.0	100.0	153,473	46	46	0.00
8	0.2	0.2	99.8	192,249	317	363	0.04
9	0.8	1.0	99.0	265,157	2,214	2,577	0.26
10	1.7	2.8	97.2	316,577	5,493	8,070	0.81

Applications of the Expected Loss Finding

To review, we have developed a model (and software) that takes the forecasts we prepare for each leasing market, and their confidence intervals, and passes these through a discounted cash flow model, to generate NOI (or cash flow) and value probability distributions of outcomes. Superimposing a specific debt structure on the income and value outcomes, we use a logit model to estimate default probabilities and if a default occurs, loss severity is based on the collateral income and value available. Expected loss is the product of severity and default probability.

There are many ways to *express* these results and there are two ways to *generate* the results. We will turn first to the latter.

To generate results like those above, the discounted cash flow model can either use generic property characteristics, or collateral specific characteristics. For example, one could use average lease rollover assumptions for generic properties or use *collateral*

specific lease rollover *as known* for the property. In the former case we would assume the property is "average" in the market and compare "average" properties across type and geography.[10] In the latter case we would use known details of the property such as existing rent roll, operating leverage, tenant improvements, etc. The more specific the information on the collateral, the better the analysis.

The important consideration as to which approach to employ depends on the goals of the analysis. Is the goal to measure 1.) relative risk or 2.) absolute risk?[11] The generic results, where uniform assumptions of property and loan characteristics are made across markets, can provide good *relative* measures of risk. Running the analysis across all markets and geography would allow a ranking of which markets/property types are generally at the higher risk.

However, for measuring absolute risk of default, and expected loss, using collateral specific knowledge (as well as loan specific knowledge) will be paramount. For instance, a property that has a twenty-year lease from an AAA tenant will reflect far less risk than that experienced by a prototypical property analyzed under generic assumptions. *The fact that our model can do both generic and collateral specific modeling of credit risk makes it extremely flexible and useful.*

For those familiar with the debt world it is obvious that there are many helpful and different ways to arrange the output of our analysis. The calculations can be expressed as yield degradation, risk adjusted yield, risk adjusted LTVs or risk adjusted DSCR for current and/or future periods. Additionally, the results can be expressed in

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[10] Our database allows us to generate class specific, generic NOIs. For example, we can run generic analysis for class A vs. Class B in a given geography.

[11] Of course, absolute risk can be used to measure relative risk as well.

dollars, present value terms and as percentages of the original loan or the loan value at any time period. We are preparing several papers showing various applications.

Value-at-Risk and Unexpected Loss

In recent years, an additional measure of exposure to risk has emerged, which is sometimes applied in the regulation of financial institutions. The concept is to estimate the distribution of losses that the holder of a debt instrument (or a portfolio of such instruments) is likely to experience over some time horizon. The analysis determines a level of risk (say 5% or 1%) and estimates the loss that could occur with this probability. The presumption is that "prudent" institutions will set aside reserves to cover this level of risk or amount of loss.

The first requirement in Value-at-Risk(hereafter, VaR) analysis is to select an "appropriate" risk level. Is it .5%, 1%, 5%? This choice can make a significant difference in that there is always *some* probability that one will loose very large sums. For real estate, what level of event was the 1991 recession, or the 1975 recession, or the great depression? Do we really expect financial institutions to insure fully for such rare events?

The second issue is how to derive the distribution of losses. In particular, should we estimate *potential or realized* losses? Consider the case of a bond that is subject to interest rate fluctuation over a finite holding period. The *potential* distribution of losses involves simulating daily or weekly interest rate fluctuation, converting each simulated interest rate event into a bond price, and then adding up the probabilities over time for

each level of loss. One does not care about how the investor reacts to these losses, whether they sell or hold. These are *potential losses.*

Suppose, however, that the investor has various rules for selling and wants to determine a full distribution of *realized loss* using these rules. This is much more difficult and often necessitates simulating the investor's behavior over the distribution of possible outcomes. Simulating *realized losses* requires more than simply forecasting interest rates and bond pricing. For each simulated interest rate path, one would have to determine the first exposure to a loss that was sufficient to warrant sale, then sell the loan at that loss, which then, in turn, prevents exposure to further interest rate risk (either up or down). The rule, and the parameters within the rule, can completely change the loss distribution.

The question of whether to use *potential or realized* debt losses in real estate for the measurement of VaR is particularly important, because institutions write down loans and/or dispose of them – as opposed to keeping them to full term. In our expected loss analysis we carefully model *realized* losses – since a loan can default only once and this likelihood is based on the competing risk hazard function. In our VaR analysis, therefore, we must take the same approach. To be consistent, the distribution of losses in the VaR analysis must have a first moment whose value is our measure of expected loss.

To examine the full distribution of *realized losses* over a holding period, we proceed in the following steps.

1). Simulate each period in the life of a mortgage with 10,000 Monte Carlo (random) draws (of NOI and value) from the probability distributions.

2). When the draw generates a default (using the logit model probabilities), the loss associated with that draw is recorded and the loan is eliminated. All of the realized losses for that year are then sorted and their probability distribution is derived.

3). Surviving loans (occurring with one minus the logit probability) are then simulated in the next year with a new round of 10,000 random draws. Here again, losses are recorded and their distribution determined.

4). Since only surviving loans are subjected to the next year's event draw, the overall probability of default in each particular year is equal to the marginal probability as determined from the hazard model.

5). This survival criteria also allows the loss distribution for each year to be aggregated into a cumulative loss distribution over a "holding" period. Thus over a five year "window," those losses (out of 50,000 draws) that occurred in any year are combined, sorted and their distribution determined.

Following this methodology, we obtain a full probability distribution of realized losses over any holding period that is completely consistent with our expected loss calculation over the same period. Expected loss simply equals the mean of the VaR loss distribution – including of course the (generally large) probability of zero loss or no loss.

The VaR for New York office property is shown in **Table 2**. The first step is to select a confidence level, which is one-minus-the risk level (e.g., 99% confidence equals 1% worst risk). Next read the minimum loss likely to happen with this risk level--over

the holding period. For illustration, this frequency distribution of losses also is shown for

the full ten year holding period, in **Figure 5**.

Table 2. VaR for Various Confidence Levels and Holding Periods

Holding Period	99.9%	99.5%	99.0%	98.0%	95.0%	90.0%	85.0%
1	$0	$0	$0	$0	$0	$0	$0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
8	179,225	0	0	0	0	0	0
9	468,409	222,915	33,218	0	0	0	0
10	688,886	451,697	313,698	155,902	0	0	0

Figure 5. VaR Across Confidence Levels
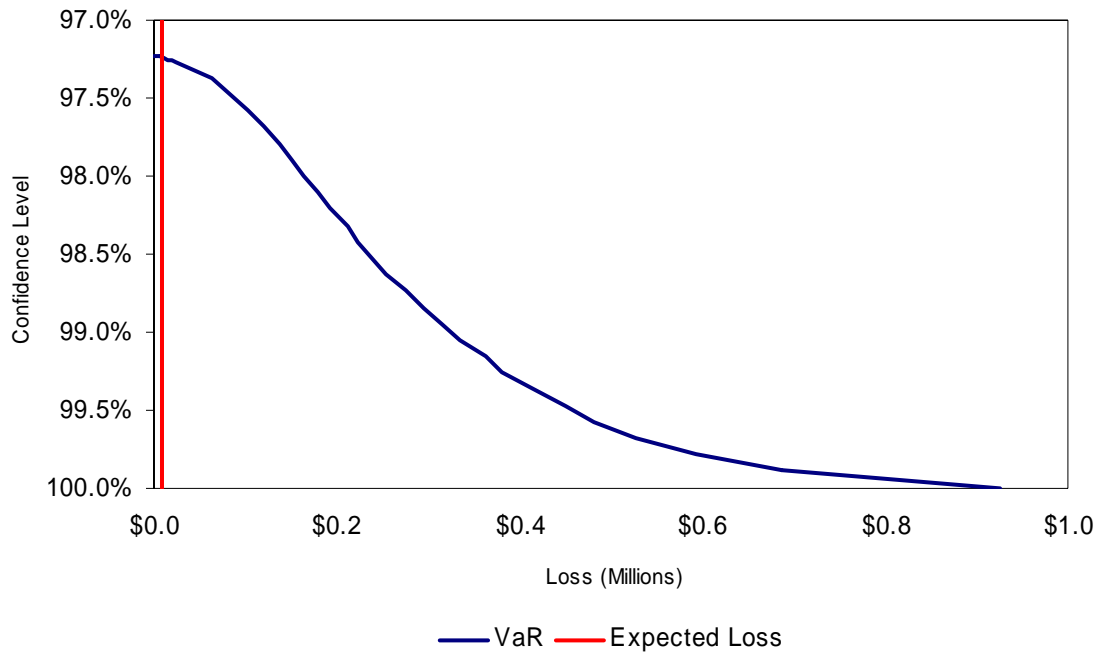
There are several observations to be made from **Table 2** and **Figure 5**. First,

there is about a 97% probability of no loss at all over the full ten year horizon, and close

to a 100% probability in the first seven years. Secondly, this large likelihood of zero loss creates an expected loss value that is quite small and close to the origin in **Figure 5**. Third, the distribution in **Figure 5** exhibits a truncated tail – rather than a more traditional normal distribution tail. This is because there is a limit to how much loss can occur in a mortgage – no matter how adverse the event may be. Our entire approach is this example limits total loss to interest and principal. We could, of course, add constant dollars or percentages to reflect administrative or friction costs.

Finally, the distribution intersects the horizontal axis very sharply, rather than rising gradually. What this means is that once losses start to occur, they build up very rapidly. This reflects the shape of the confidence intervals in this stylized New York example. They widen very rapidly after year eight.

One Market: Many Loans

One important flexibility of the model is its ability to generate asset specific cash flows or NOIs. This feature allows us to estimate an expected loss or unexpected loss for multiple loans in the same market. For example suppose that one has a portfolio of ten office loans in New York of $100,000 rather then one loan of $1,000,000. Accounting for asset specific features will give us ten different expected values (and unexpected losses) and allow us to look at the ten loans in New York separately. While each of the ten loans will have a similar exposure to the events in the marketplace, each will have a different time period exposure, depending on specific lease rollovers and different rents, occupancy and operating leverage.

For instance in the early 1990s every office building in New York was exposed to the economic downturn, but not every mortgage in New York defaulted. Some properties had sufficient income, because of existing leases or because of good operating leverage or because of no rollovers when rents were falling. With the TWR model these myriad of asset specific factors can be adjusted to reflect the idiosyncratic nature of the real estate asset in a market environment.

To give an example of this for New York, we ran a second office loan under the assumption that the property was exposed to tenant turn over of 50% per year for each of the next two years and assumed rents in place equal to today's average rental.[12] With the market currently topping, this assumption puts the property at greater exposure to current leasing fundamentals than the earlier example.

With greater exposure to the market due to these lease rollovers the expected loss for this loan is dramatically higher. The results for this "Market Exposed" property, rather than the "Seasoned Property" example as was shown in Table 1, are presented in **Table 3.** With current rents at market in the "Market Exposed" property and with leases rolling 50% per year the expected loss is $20,758 versus $5,493 in period ten with the expected default frequency rising to 7.7% in that year versus 1.7% for the "Seasoned Property" .[13]

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[12] Recall that the stylized example used before for New York assumed rollovers of 20% per year and rents in place reflecting the average of the previous five years.
[13] We are assuming a loan of similar structure as for the "Seasoned Property."

Table 3: New York Expected Loss and Severity for "Market Exposed" Property

Year	Default Hazard	Cumulative Default Probability	Holding Period Survival Rate	Annual Severity	Annual Expected Loss	Holding Period Expected Loss	As a % of Original Loan Amount
1	0.00%	0.00%	100.00%	$0	$0	$0	0.00%
2	0	0	100	0	0	0	0
3	0	0	100	48,230	19	19	0
4	0.2	0.2	99.8	91,301	155	174	0
5	0.5	0.7	99.3	105,575	486	660	0.1
6	1.3	2	98	120,107	1,537	2,197	0.2
7	2.5	4.5	95.5	202,735	5,068	7,265	0.7
8	4.9	9.4	90.6	227,748	11,205	18,470	1.8
9	6.2	15.6	84.4	250,101	15,581	34,051	3.4
10	7.7	23.3	76.7	268,188	20,758	54,809	5.5

We also ran the analysis for new construction in New York where we assumed that the property had no existing leases and that lease up would take about two years from today. Under this analysis the expected loss rises to $49,313 or 13.6% of the loan value by year ten and the default probability is in double digits for years nine and ten. Again, with the market topping currently, new construction, even in a strong office market like New York, has significant risk.

Portfolio Risk

Clearly, most lenders have portfolios composed of multiple loans that span across both property types and geography. Thus, the analysis we have developed here needs to translate the risks associated with individual loan to the portfolio level. To apply the TWR analysis to the portfolio level is straight-forward and requires estimates of three inputs: each loan's expected loss, each loan's VaR (at some level), and then a correlation matrix across loans.

The first calculation for a portfolio is the *portfolio's* expected loss. This is simply the *weighted average of the expected losses of each loan in the portfolio*. If the losses are expressed in dollars, these are simply added. If the losses are expressed as a percentage of each loan's value, then value weights must be applied to each loan before aggregating.

To measure a "portfolio VaR", we adopt the convention of determining the "unexpected loss" for each loan and then the same estimate for the portfolio as a whole. Unexpected loss is simply the difference between VaR (at some level) and expected loss, and is similar to the statistical measure of variance. Because of this similarity, the unexpected loss for a portfolio cannot be derived simply by adding up the unexpected loss for each individual loan. *One must also consider the covariance or correlation between loans, and then apply the standard matrix formula for aggregating variances across assets*.

To derive such a correlation matrix, we assume that the correlation between two loans is the same as the correlation between the two geography-specific property types that represent the loans' collateral [e.g. New York Office and Dallas Apartments]. The variable that we use for the correlation is annual rent. In developing these rent correlations we examine both the twenty year history and ten year forecast for each property market and geographic area. We have applied this approach quite successfully to equity risk analysis [Wheaton, et al, 2001b].

As an example we analyzed sixteen office markets, arbitrarily chosen, and calculated the expected loss for each and for a portfolio of these loans. **Figure 6** summarizes the results and shows that the expected loss is as high a $90,000 in Indianapolis and rather trivial in Atlanta. The full range or results for each is given as well as the portfolio's expected loss of approximately $20,000.

Figure 6: Portfolio Expected Loss
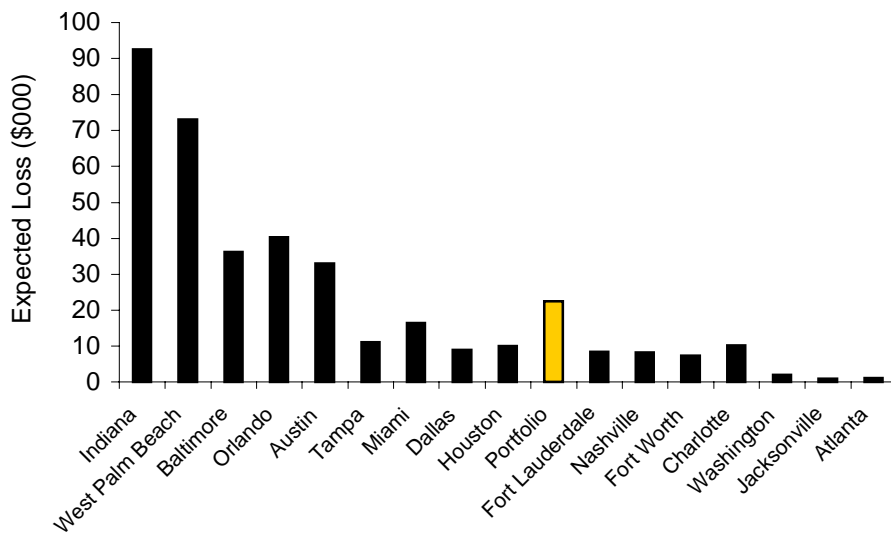
Currently, we have software available that can within several minutes, take a portfolio of several hundred loans, with expected and unexpected loss for each, apply the relevant portions of the correlation matrix and derive the comparable expected and unexpected loss figures for the portfolio as a whole. The results for sample portfolios are just as one would expect. When loans are spread across property markets that have very low (or negative) correlations, the portfolio unexpected loss can be quite a lot less than the average unexpected loss of the loans in the portfolio.

Conclusions: Sources of Investment Risk.

We have outlined an approach for assessing the real estate risk to loans on commercial properties. The approach is based on the premise that the most important source of this risk is the market's fundamentals, rent and vacancy. It is this, which in turn generates the risk that the loan defaults, that the timely payment of interest is lost, as is the value of the loan's principal.

This approach yields risk measures that in some markets seem low by historic performance data. This is because it accounts for only one source of investment risk: market risk. In the past, actual historic loan losses have come not only from market risk, but also from idiosyncratic property risk, (e.g. location, structure, management, etc.) as well as from broader capital market risk (e.g. interest rates). Thus the market and credit risk measure developed here is in some sense a lower bound on the total risk facing any commercial mortgage.

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