1. Short Answer Section (16 pts)

1. The nuclear radius of $^{226}\text{Ac}$: $R_{\text{nuc}} = 1.25 (226)^{1/3} = 7.6140 \text{F}$

2. What is the depth of the nuclear well for individual nucleons in $^{199}\text{Au}$? Standard nuclear well depth is, $V_{\text{nuc}} = 50 \text{MeV}$

3. Estimate the magnitude of the Compton scattering cross section for a 1 MeV gamma ray: Compton cross section is in attached formulae as, $\sigma_{\text{Comp}} = \sigma_{\text{TM}} \frac{m_e c^2}{\hbar} \sim \frac{1}{3} \text{barn}$

4. Name the three processes responsible for gamma absorption as the gamma ray energy is raised from 10s of Kev to several MeV: Photoelectric absorption for $E_\gamma \ll m_e c^2$, Compton scattering for, $E_\gamma \sim m_e c^2$, and pair production for, $E_\gamma \geq 2m_e c^2$

5. What is the spin of the deuteron? What is the degeneracy of this state: Deuteron spin, $S = 1$, degeneracy = 3.

6. Water (which contains hydrogen) is much more effective than Actinium in thermalizing neutrons even though the scattering cross section in Actinium is more than ten times larger than the neutron-hydrogen cross section. Why? Thousands of collisions in a heavy element like Actinium are required for thermalization (see table attached), while only a dozen or so are required for hydrogen. This more that makes up for the difference in cross sections.

7. How many states are in the $1j_{15/2}$ level? In other words what is the degeneracy of the $1j_{15/2}$ level: There are $2j + 1 = 16$ states in the $15/2$ level. This is the degeneracy.

8. Compute the spin-orbit splitting of the $1h$ level in $^{209}\text{At}$. Assume a spin-orbit coupling of,

$$V_{\text{nuc}} = V_0 + V_{\text{SO}} \frac{\hat{I} \cdot \hat{s}}{\hbar^2}$$

**Raw Text**

$$V_{\text{nuc}} = \frac{\hbar^2}{2m_e c^2} \frac{2 \pi}{\sin \frac{\pi}{2}} \approx 50 \text{MeV}$$

**Solution**

$$V_{\text{nuc}} = \frac{\hbar^2}{2m_e c^2} \frac{\pi}{2} \approx 50 \text{MeV}$$

**Solution**

$$V_{\text{nuc}} = \frac{\hbar^2}{2m_e c^2} \frac{\pi}{2} \approx 50 \text{MeV}$$
causes the $1h_{11/2}$ level to split down and the, $1h_{9/2}$ level to split up. Most of this answer can be garnered from the shell model diagram in the back. Here we give the underlying analysis. The $h$ level is, $l = 5$, so it couples with spin $1/2$ to produce, $j = 9/2$ and, $j = 11/2$ levels, with occupancies, $D = 10$, and $D = 12$ respectively. The computation of $\hat{I} \cdot \hat{s}$ follows from adding the $l$ and $s$ angular momenta,

$$
\hat{j} = \hat{l} + \hat{s} \\
\hat{j}^2 = \hat{l}^2 + \hat{s}^2 + 2\hat{l} \cdot \hat{s} \\
\hat{l} \cdot \hat{s} = \frac{\hbar^2}{2} \left( j^2 - \hat{l}^2 + \hat{s}^2 \right) = \frac{\hbar^2}{2} \left( (5 \pm 1/2) (6 \pm 1/2) - 5 \cdot 6 - \frac{3}{4} \right) \\
\rightarrow \frac{5}{2} \hbar^2 \text{ or } -3\hbar^2
$$

Thus the, $j = 11/2$, state is split down (deeper effective well) and the, $j = 9/2$, state is split up. The magic number 82 results from adding the 12 states from the $1h$ splitting to $1h_{11/2}$ to the 70 cumulative occupancy states from oscillator levels through, $N = 4$.

2 Weak Interactions (20 pts)

Supply the missing component(s) in the following processes:

1. (2 pts.) $\nu + ^{99} Tc \rightarrow ^{99} Ru + e^-$
2. (2 pts.) $e^+ + ^{23} Ne \rightarrow ^{23} Na + \nu$
3. (2 pts.) $n \rightarrow p + e^- + \nu$
4. (2 pts.) $p + \nu \rightarrow n + e^+$
5. (2 pts.) $^{25} Al \rightarrow ^{25} Mg + e^+ + \nu$
6. (2 pts.) $^{15} O + e^- \rightarrow ^{15} N + \nu$
7. (2 pts.) $^{40} K \rightarrow \bar{\nu} + ^{40} Ca + e^-$
8. (2 pts.) Make a quantitative estimate of the cross section for the reaction of neutrinos interacting with $^{99} Tc$, process #1 above. What is the mean free path for this reaction for neutrinos entering solid technetium with an atomic density of, $n_{atom} = 10^{23} \text{ cm}^{-3}$? The cross section can be estimated as the cross sectional area of the nucleus, $^{99} Tc$, times the weak interaction coupling coefficient, $G^2 = 10^{-24}$, i.e. $\sigma \approx \pi R_{Nuc}^2 G^2$ or $\sigma \approx \pi (1.25)^2 (99)^{2/3} \times 10^{-24} \text{ F}^2 \approx 10^{-48} \text{ cm}^2$. This implies a mean free path of, $\lambda \approx \frac{1}{n_{atom} \sigma} \approx 10^{25} \text{ cm} \approx 10^7 \text{ light years}!$
9. (4 pts.) The figure below indicates the energy, as computed from the semi-empirical mass formula, for a range of elements, all having mass number, $A = 63$. Draw arrows on
the plot to indicate possible decays starting from both, $^{63}Fe$, and, $^{63}As$, and state the decay mode. What is the final, stable element at this mass number? (Please redraw this figure in your exam book.): Beta decays on left from $Fe$ to $Cu$, and positron beta decays and/or electron captures on right from As, to Cu. Final stable element is Copper.

3  Shell Model (12 pts)

1. (6 pts.) Use the shell model level diagram attached to the back of the exam to determine the spin and parity assignments of the following nuclei:

$^{83}Kr, ^{45}Sc, ^{11}B, ^{113}In, ^{84}Kr, ^{189}Os$
Answers:

\[ ^{83}Kr = \frac{9}{2}^+ \]
\[ ^{45}Sc = \frac{7}{2}^- \]
\[ ^{11}B = \frac{3}{2}^- \]
\[ ^{113}In = \frac{9}{2}^+ \]
\[ ^{84}Kr = 0^+ \]
\[ ^{189}Os = \frac{13}{2}^+ \]

2. (3 pts.) Now reconsider the nuclide, \(^{189}Os\). Note that the observed spin-parity is actually, \(\frac{3}{2}^-\). Give an explanation for this observation using properties of the pairing force. : The simple shell analysis puts an unpaired neutron in the \(1i_{13/2}\) level, while all neutrons in the \(3p_{3/2}\) level beneath are paired. Given the increased pairing strength for higher angular momentum states the observation suggests that ground state actually has broken pair from \(3p_{3/2}\) level with neutron raised -- and paired yielding more binding energy -- to the \(1i_{13/2}\) level. Why it is the \(3p_{3/2}\) level and not the \(3p_{1/2}\) level is unclear. (Drawing would clarify)

3. (3 pts.) Make a spin-parity prediction for the nuclide, \(^{122}Sb\), even though your answer may not be unique. : This depends on coupling of the unpaired proton \((Z = 51)\) and neutron \((N = 71)\) which reside in the \(1g_{7/2}\) and \(1h_{11/2}\) respectively. The parity is odd from the parity product, while the spin is the result of adding, \(j = 7/2\), and, \(j = 11/2\), which would give, \(J = 2, 3, 4, 5, 6, 7, 8, 9\). Pick, \(J = 2\), as lowest possible coupled spin to yield, \(2^-\) as prediction.

4. **The Deuteron (10 pts)**

Show that the deuteron has exactly one bound state (ignoring spin dependence of nuclear force). Use the proper rest masses of the proton and neutron (but take them to be the same), i.e., \(m_p \approx 938\ MeV\), and \(m_n \approx 938\ MeV\). Take a square nuclear well with range, \(R_0 = 2.1\ F\), and depth, \(V_{Nuc} = 35\ MeV\) and show that a bound state exists by calculating the minimum well depth to bind proton and neutron within well of range, \(R_0\). Planck’s constant may be taken to be, \(hc = 197\ MeV - F\). (6 pts.)
Fitting wavefunction is well requires turnover of oscillatory form inside well to decaying form outside. (picture would be good). Leads to quarter wave rule,

\[
\frac{1}{4} \lambda = R_0 \\
\frac{2\pi}{\lambda} = \frac{\pi}{2R_0}
\]

Kinetic energy associated with this wave is,

\[
E_{\text{kin}} = \frac{\hbar^2 k^2}{2\mu} = \frac{\hbar^2 c^2 k^2}{2\mu c^2} = \frac{197^2 \text{ MeV}^2 - F^2}{938 \text{ MeV}} \cdot \frac{\pi^2}{4 \cdot 2.1^2 F^2} = \frac{197^2 \cdot \pi^2}{938 \cdot 4 \cdot 2.1^2} \text{ MeV} = 23.149 \text{ MeV}
\]

which is sufficient to be bound in 35 MeV potential well.

Show that no finite angular momentum excited states exist, by estimating minimum increase in kinetic energy needed allow, \( l = 1 \), state. (3 pts.)

Effective potential energy associated with angular motion is,

\[
V_{\text{eff}} = \frac{\hbar^2 l (l+1)}{\mu r^2}
\]

which can be bounded from below (inside the well) as,

\[
V_{\text{eff}} \geq \frac{\hbar^2 l (l+1)}{\mu R_0^2}
\]

putting in numbers gives,

\[
V_{\text{eff}} \geq \frac{197^2 \cdot 2}{469 \cdot 2.1^2} = 37.528 \text{ MeV}
\]

This is significantly more than the \( \sim 10 \text{ MeV} \) left above existing bound state.

For a complete answer, also show that no excited radial, \( l = 0 \), states can be bound.(1 pt.)

Next higher radial mode requires 3 half wavelengths inside well so that eigenfunction can cross axis once and match to decaying form outside the well. This implies 9 times as much kinetic energy as ground state or, \( \sim 9 \cdot 23.149 = 208.34 \text{ MeV} \) well higher than remaining binding potential.

Extra credit: Now consider two fictitious cases. In case 1, the proton and neutron are four times as massive \( (m_p c^2 = m_n c^2 = 3752 \text{ MeV}) \) as the real nucleons. In case 2, the proton
and neutron are one quarter as massive \((m_p c^2 = m_n c^2 = 234.5 \text{ MeV})\). In both cases, the nuclear force is the same \((V_0 = -35 \text{ MeV}, R_0 = 2.1 F)\). In which of these cases does the deuteron still exist as a bound state? In the case that a bound state deuteron does exist, describe any differences in structure (extra bound states, spin, etc.) between this fictitious case and the real deuteron. If spin dependence of the nuclear force is included, how would the spin differ in the fictitious cases?

5 Binding Energy (10 pts)

The so-called semi-empirical mass formula can be written schematically as a formula for the binding energy, \(B\), of a nucleus, as a function of mass number, \(A\), and proton number, \(Z\),

\[
B(A, Z) = \text{volume term} + \text{surface term} + \text{Coulomb term} + \text{symmetry term} + \text{pairing term}
\]

Write this formula by providing the proper, \(A\), and, \(Z\), dependence of each term with an unspecified coefficient. Be sure to indicate the proper sign for each term. For the pairing term, you only need indicate its sign, accounting for the three possibilities of, odd \(A\) nuclides, odd-odd nuclides, and even-even nuclides.

\[
B = a_v A - a_s A^{2/3} - a_{Coul} \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{1}{A} (A - 2Z)^2 + a_{pair} A^{-3/4} \begin{cases} + \text{ (for even-even)} \\ 0 \text{ (for odd } A) \\ - \text{ (for odd-odd)} \end{cases}
\]

**Extra credit:** Consider the nuclide, \(^{208}\text{Pb} \). Evaluate the contributions to the binding energy per nucleon (that is \(B/A\)) from the surface term and the symmetry term and determine which is larger in magnitude.

6 Gamma Decay (12 pts)

The scandium isotope, \(^{43}\text{Sc} \), with ground state spin and parity, \(7/2^-\), has the excited states shown below in the figure (the energy numbers are approximate).
Determine the angular momentum and parity and type (magnetic or electric) of the gamma ray photons that are emitted to get from the $3/2^-$ excited state at 749 KeV to the ground state. (6 pts.)

*Angular momentum of gamma ray photon must be, $l = 2, 3, 4, 5$, and it must carry even (+) parity. Of all the possible transitions, $E2, M3, E4, M5$, we expect $E2$ to dominate.*

Estimate the lifetimes of the three transitions shown. (5 pts.)

*Lifetime estimates based on the formula,*

$$
\lambda_\gamma \sim \frac{e^2}{\hbar c} |\mathbf{k} \cdot \mathbf{r}_{mn}|^{2l+1} \omega_{mn}
$$

*which can be rewritten in practical units ($E = \hbar \omega, \omega = k_c r_{mn} \approx 1.25 A^{1/3} F$, etc.),

$$
\lambda_\gamma \approx \frac{e^2}{\hbar c (\hbar c)^{2l+1}} A^{2l+1} 1.25^{2l} F^{2l} c
$$

$$
= \frac{1}{137^{4/3}} \frac{E_{MeV}^5 A^{4/3}}{1975^{5} MeV^5 - F^5} \cdot 1.25^{2l} F^{2l} \cdot 3 \times 10^{23} F/sec
$$

$$
= \frac{1}{137^{4/3}} 2.4414 \cdot 3 \times 10^{23} E_{MeV}^5
$$

*For the, $l = 2$, transitions (we are not distinguishing magnetic from electric transitions) this becomes,*

$$
\lambda_\gamma \approx \frac{1}{137^{4/3}} \frac{E_{MeV}^5 43^{4/3}}{1975^{5} MeV^5 - F^5} 2.4414 F^4 \cdot 3 \times 10^{23} F/sec
$$

$$
= 2.7144 \times 10^{12} E_{MeV}^5 \ sec^{-1}
$$
while for the, \( l = 1 \), transition this is,

\[
\lambda_\gamma \approx \frac{1}{137} \frac{E_{MeV}^3 A^{2/3}}{197^2 M e V^3} - F^3 1.25^2 \cdot 3 \times 10^{23} F/\text{sec}
\]

\[
= \frac{1}{137} \frac{43^{2/3}}{197^3} 1.25^2 \cdot 3 \times 10^{23} E_{MeV}^3 \text{sec}^{-1}
\]

\[
= 5.492 \times 10^{15} E_{MeV}^3 \text{sec}^{-1}
\]

Which makes for the estimates \( (\tau = 1/\lambda) \),

\[
\begin{align*}
\lambda (749 \text{ KeV}) &= 6.398 \times 10^{11} \text{ sec}^{-1} \\
\tau (749 \text{ KeV}) &= 1.5628 \times 10^{-12} \text{ sec} \\
\lambda (310 \text{ KeV}) &= 7.771 \times 10^{9} \text{ sec}^{-1} \\
\tau (310 \text{ KeV}) &= 1.286 \times 10^{-10} \text{ sec} \\
\lambda (439 \text{ KeV}) &= 4.6472 \times 10^{14} \text{ sec}^{-1} \\
\tau (439 \text{ KeV}) &= 2.151 \times 10^{-15} \text{ sec}
\end{align*}
\]

Which of the two transitions from the \( 3/2^- \) state do you think will be faster? (1 pt.)

The transition to the ground state will tend to be faster because it is driven by higher free energy, the transition to the \( 3/2^+ \) excited state will tend to be faster because it is an \( l = 1 \), transition. We expect the \( l = 1 \) transition to be faster as the above numbers clearly show.

7 \ Fission (8 pts)

Consider the induced fission reaction,

\[
n + ^{235} U \rightarrow ^{93} Rb + ^{141} Cs + 2n + 224.3 \text{ MeV}
\]

where the incoming neutron is thermal and its kinetic energy can be ignored. Calculate the height of the Coulomb barrier between the fission products, \( ^{93} Rb \), and, \( ^{141} Cs \), separated by the sum of their nuclear radii. Use this calculated result to determine the fission activation energy, \( E_{act} \). (6 pts.)

\[
\text{Coulomb barrier height is,}
\]

\[
V_{Coul} = \frac{e^2 Z_{Rb} Z_{Cs}}{(R_{Rb} + R_{Cs})} = \frac{e^2}{\hbar c} \frac{\hbar c \cdot 37 \cdot 55}{1.25 (93^{1/3} + 141^{1/3})} = 197 \times 37 \cdot 55 \text{ MeV} = 240.5 \text{ MeV}
\]
which implies an activation energy,

\[ E_{\text{act}} = 240.5 - 224.3 = 16.2 \text{ MeV} \]

If the excitation energy,

\[ E_{\text{ex}} \equiv c^2 \left( m^{(236)\text{U}}^* - m^{(236)\text{U}} \right) = c^2 \left( m^{(235)\text{U}} + m(n) - m^{(236)\text{U}} \right) \]

is, \( E_{\text{ex}} = 6.2 \text{ MeV} \), does this calculation predict induced fission for \(^{235}\text{U}\) from thermal neutrons? (2 pts.)

*Induced fission is not predicted by this calculation since the excitation energy is less than the activation energy. Of course, the real world is somewhat different from this calculation, of course, and \(^{235}\text{U}\) does undergo induced fission from thermal neutrons.*

8 Radiation Interactions with Matter (12 pts)

A radioactive material is shielded in a canister that consists of a cylindrical tube of lead that is 0.1 cm thick. The material is emitting a continuous stream of 4 MeV alpha particles and 1 MeV beta electrons, as well as intermittent emissions of 1 MeV gammas. Make a quantitative estimate of the mean free paths in lead of these three forms of radiation and make a guess as to which of the radiations will be sufficiently well absorbed to allow human handling of the canister. (You may assume the density of lead (Pb) is \( 3.3 \times 10^{22} \text{ atoms/cm}^3 \) and that the fluxes of the various radiations need to be reduced by order \( \sim e^{-10} \) in order to be safe). (4 pts. for each radiation calc.)

*For alpha radiation the stopping mechanism in Coulomb scattering off atomic electrons, but with small fractional energy loss effective cross section needs to be reduced by, \( \Delta E/E \approx 4m_e/m_\alpha \approx 0.511/938 = 5.4478 \times 10^{-4} \approx 1/2000 \). The alpha particle speed is, \( \beta^2 = 2E/(4 \cdot 938) = 2/(938) = 2.1322 \times 10^{-3} \approx 2/1000 \)

\[
\sigma_{\text{Coul}} = 2\pi r_e^2 \frac{Z^2}{\beta^4} \ln \Gamma = 0.5 \text{ barns} \quad \frac{4}{2 \times 10^{-6} 16} = 1.6 \times 10^7 \text{ barns}
\]

From which we estimate the mean free path to be,

\[
\lambda \approx \frac{1}{Z n_{\text{atom}} \sigma_{\text{Coul}} \Delta E/E} = \frac{2000}{82 \cdot 3.3 \times 10^{22} \cdot 1.6 \times 10^7 \cdot 10^{-24}} \approx 4.6 \times 10^{-5} \text{ cm}
\]

Which puts 10 mean free path at a few microns so the 0.1 cm lead shield is surely sufficient shielding.

*For beta electrons we also take Coulomb scattering from atomic electrons and ignore the irregularity of the path (this underestimates the shielding required, of course). There is
no $\Delta E/E$ factor since this is order unity for electrons hitting electrons. We also take, $\beta \approx 1$ and ignore relativistic effects.

$$\sigma_{\text{Coul}} = 2\pi r_e^2 \frac{Z^2}{\beta^4} \ln \Gamma = 0.5 \text{ barns} \cdot 16 = 8 \text{ barns}$$

$$\lambda \approx \frac{1}{82 \cdot 3.3 \cdot 10^{22} \cdot 8 \cdot 10^{-24}} \approx 4.6194 \times 10^{-2}$$

Which is marginally less than the 0.1 cm lead shield thickness and not 10 mean free paths thick. Human handling not recommended for this radiation.

For the gammas, we use the estimate for Compton scattering, again from atomic electrons,

$$\sigma_{\text{Comp}} = \sigma_T \frac{m_e c^2}{\hbar \omega} \approx \frac{21}{32} \text{ barn} = \frac{1}{3} \text{ barn}$$

giving,

$$\lambda \approx \frac{1}{82 \cdot 3.3 \cdot 10^{22} \cdot 0.3333 \cdot 10^{-24}} = 1.1 \text{ cm}$$

Making the shielding woefully inadequate for gamma radiation.
Appendix: Some helpful formulas

\[ \psi \sim e^{ikx} + f(\theta) \frac{e^{ikr}}{r} \]  \hspace{1cm} (1)

\[ B = a_v(\text{Volume}) + a_s(\text{Surface}) + a_c(\text{Coulomb}) + a_{sym}(\text{Symmetry}) + \text{Pairing Term} \]  \hspace{1cm} (2)

\[ a_v = 15.5 \text{ MeV}; \ a_s = 16.8 \text{ MeV}; \ a_c = 0.72 \text{ MeV}; \ a_{sym} = 23 \text{ MeV} \]  \hspace{1cm} (3)

\[ \frac{d\sigma}{d\Omega} = |f(\theta)|^2 \]  \hspace{1cm} (4)

\[ f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l + 1)e^{i\delta_l} \sin \delta_l P_l(\cos \theta) \]  \hspace{1cm} (5)

\[ D(N) = \frac{1}{2}(N + 1)(N + 2) \]  \hspace{1cm} (6)

\[ N = 2(n - 1) + l \]  \hspace{1cm} (7)

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<thead>
<tr>
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<th>1</th>
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\[ V_{eff} \to V_{eff} + V_{SO}\hat{l} \cdot \hat{s} \]  \hspace{1cm} (8)

\[ E = \sqrt{m^2c^4 + p^2c^2} \]  \hspace{1cm} (9)

\[ \lambda_\gamma \sim \frac{e^2}{\hbar c} |k \cdot r_{mn}|^2 \omega_{mn} \]  \hspace{1cm} (10)

\[ \sigma_{\text{Coul}} = 2\pi r_e^2 \frac{Z^2}{\beta^4} \ln \Gamma \]  \hspace{1cm} (11)

\[ 4\pi r_e^2 = 4\pi \left( \frac{e^2}{m_c c^2} \right)^2 \sim 1 \text{ barn} \]  \hspace{1cm} (12)

\[ \sigma_{Br} = \sigma_{\text{Coul}} Z^2 \frac{\omega_{mn}}{\hbar c} \frac{T + m_c c^2}{m_c c^2} \]  \hspace{1cm} (13)

\[ \lambda_\beta \sim G^2 \frac{m_c c^2}{\hbar} |M_{mn}|^2 \]  \hspace{1cm} (14)
\(\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{2}{3}\) barn \hfill (15)

\[\sigma_{\text{Comp}} = \sigma_T \frac{m_e c^2}{\hbar \omega}\] \hfill (16)

<table>
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<th>Nucleus</th>
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<tr>
<td>(^1\text{H})</td>
<td>18</td>
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<td>(^2\text{H})</td>
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<tr>
<td>(^4\text{He})</td>
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<tr>
<td>(^{12}\text{C})</td>
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<td>(^{238}\text{U})</td>
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\(\sigma_{\text{fus}} = \frac{S}{E_{\text{KeV}}} \exp\left[-\frac{G}{E_{\text{V/2}}^2}\right]\) \hfill (17)
\[ N = 2(n-1) + 2 \]

Oscillator Levels

<table>
<thead>
<tr>
<th>( n )</th>
<th>Intermediate form</th>
<th>Intermediate form with spin orbit</th>
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<td>6</td>
<td>4s - 2s - 2p - 3p - 4p - 5s - 6s</td>
<td>1s_{1/2} - 2s_{1/2} - 2p_{1/2} - 2p_{3/2} - 3p_{1/2} - 3p_{3/2} - 4p_{1/2} - 4p_{3/2} - 5s_{1/2} - 6s_{1/2}</td>
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<td>1s_{1/2} - 2s_{1/2} - 1p_{1/2} - 2p_{1/2} - 3p_{1/2} - 4p_{1/2} - 5s_{1/2} - 6s_{1/2}</td>
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</table>

\[ 2(2\ell + 1) \sum (2\ell + 1) \]

\[ 2 \ell + 1 \sum (2\ell + 1) \]