Scattering of Waves by Steel Reinforcement in Concrete
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Abstract

The application of wave-sensing methods to the non-destructive evaluation of concrete structures requires often the use of simple (or at least tractable) models for the solution of the physical problem under consideration. For example, a concrete deck would ideally be represented as a laminated plate. However, steel reinforcement bars embedded in concrete constitute strong scatterers of elastic and acoustic waves of short wavelength. From a rigorous standpoint, this implies that the steel mesh cannot be represented simply as a homogenous layer of material with appropriate physical properties. However, if the wavelengths are not too short and the spacing between rebars not too great, such a strategy is indeed possible. This paper is concerned with the problem of elastic wave scattering by steel reinforcement and examines reflection/transmission patterns of plane waves in a homogeneous mass of concrete reinforced by equidistant steel bars. For this purpose three solutions are used, namely a first order scattering solution, an exact solution, and a solution based on a layered medium with equivalent material properties. It is found that for reasonably long wavelengths, the latter model provides acceptable results.

Introduction

It is well known that ultrasonic and elastic wave propagation methods, when applied to inhomogeneous media such as concrete slabs, give poor results at wavelengths that are comparable to the inhomogeneities present in the concrete mass. In addition, steel reinforcement bars (or rebars, for short) do not form a continuum, but are placed at discrete intervals. This implies that a concrete deck cannot, in general, be represented by means of homogeneous layers of material. However, a renunciation of layered representations of concrete decks and pavements carries with it the loss of rather powerful tools of analysis. This motivated the authors to explore the consequences of ignoring the spatial character of the reinforcing steel on wave reflection and transmission characteristics.

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Elastic solutions

Consider a homogenous, unbounded concrete medium with an infinite array of steel reinforcing rods of radius $a$ placed at equidistant intervals $d$ in the plane $y=0$ (Fig. 1). When this configuration is subjected to a plane incident wave with a wave-front that is parallel to the rebars (the $z$ axis), each bar diffracts and scatters waves in all directions in the $x,y$ plane. Shortly thereafter, the scattered waves impinge on the neighboring bars, they are rescattered in turn, and this process continues until the energy remaining in the plane of the rebars becomes negligibly small. Hence, the motion at any point in the medium can be obtained as the sum of the incident wavefield and the wavefield scattered by the reinforcing bars.

While a rigorous solution to the problem described requires consideration of the cross-interaction between bars, a good engineering approximation can be found if only the primary scattering is considered, that is, if the scattered field of each rebar is computed in the absence of all the others. This is a classical problem for which an exact solution is available (see for example Mow & Pao, 1971; full details are given also in Ghibril & Kausel). When this solution is applied to the problem at hand with incident waves that are not shorter than $5a$ (2.5 times the diameter of the rebars), it is found that the flexibility of the rebars can be neglected, and that they can be modeled as rigid inclusions.

To verify the accuracy of the first order scattering solution, the authors applied also the exact (and rather complicated) solution to the problem of wave diffraction by a row of elastic cylinders (Ghibril & Kausel). This solution was presented by Aviles & Sanchez-Sesma in the context of a study on vibration isolation of foundations by means of a barrier of piles. A comparison of the results obtained by this exact method and those of the first order scattering theory indicate that the latter yields nearly exact results when the incident wave-front is parallel to the plane of the rebars, the center to center spacing is $d=3a$, and the incident wavelength is $l=5a$. Thus, it was judged that the first order scattering solution provided an adequate base of comparison for the more approximate equivalent layer model that follows.

Finally, an approximate model was considered that involves replacing the row of cylinders with a continuous thin lamina with equivalent material properties. The simplest choice consists in taking a lamina of zero thickness, zero stiffness, and having the same mass per area as the excess mass of the rebars (i.e. the mass of the rebars minus the mass of the displaced concrete). Indeed, a comparison of such simple model with a more elaborate thin-layer model demonstrated nearly identical results.

Normal incidence of SH waves

Figures 2 and 3 show the absolute value of the displacements obtained using the first order scattering approximation for a normally incident SH wave with wavelength $l=20a$ and $l=5a$ (if the rebars have radius $a=1cm$, these wavelengths correspond to excitation frequencies of about 10 kHz and 40 kHz, respectively). The rebars are
separated by a distance \( d=10a \). The observation plane is taken to be normal to the plane of the rebars, and passing through the origin of coordinates (at which a rebar is placed). Hence, negative abscissas represent points on the side of the incident wave, while positive abscissas correspond to points in the region of transmitted waves. Figures 4 and 5 depict the same data, but computed with the thin lamina model.

Comparison of figures 2, 4 and 3,5 show that the thin layer model produces results that are both quantitatively and qualitatively similar to those of the first order scattering model; nevertheless, there are also clear differences, particularly for shorter wavelengths. Inspection of these figures reveals also that the thin layer model predicts an oscillating amplitude on the side of wave incidence, and a constant amplitude beyond. The oscillations are the result of interference between the incident and the reflected waves, while the transmitted waves are not subjected to wave interference. The more exact first order scattering model evidences similar features.

Conclusions

The representation of reinforcing bars by means of a homogenous layer of material with equivalent properties seems to be an acceptable engineering strategy when the excitation involves wavelengths that are not shorter than about five rebar diameters, and the distance between the rebars is comparable to or less than the dominant wavelength. Shorter wavelengths elicit local effects that cannot be captured with simple models.

References


Fig. 1