Scattering of waves by submerged shell with oscillator: tail wags dog

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Abstract

When a cylindrical shell with an internal oscillator attached between two diametrically opposed points is submerged in a fluid and subjected to an incident plane pressure wave, scattering takes place and waves radiate in all directions. It is well known that such a device constitutes a strong scatterer of waves of short wavelength (i.e. short in comparison to the physical dimensions of the shell). However, a highly surprising finding is a recent discovery by Guo that this mechanism constitutes also a strong scatterer of waves of long wavelength, indeed, waves that several times longer than the diameter of the shell. However, Guo's theoretical solution relies on wave functions expansions and provides no insight on the mechanics of the problem. In this paper, we present an engineering-type interpretation of this problem, and show under what circumstances this device can exhibit strong fluid-structure interaction effects for excitations of very low frequency.

Waves scattered by submerged shell

Consider one of Guo's problems, namely a thin cylindrical shell of radius $a$ and mass $m$ that is fully submerged in an unbounded, homogeneous, inviscid, compressible fluid with an acoustic wave velocity $c$. The shell contains a simple oscillator of mass $M=3m$ that is attached to the shell with two equal springs of stiffness $r$ each, as shown in Fig. 1. This system is subjected to an incident field of plane harmonic pressure waves of frequency $\omega$, wavelength $\lambda$, wavenumber $k$, which propagate from left to right with an acoustic wave velocity $c$ and then scatter in all directions. These quantities are related through the equation $k=2\pi/\lambda=\omega/c$. Expanding the waves as usual in Fourier series in the azimuth, and considering only large radial distances $r$, one can express the pressure associated with the scattered field as a superposition of cylindrical components of the form:

$$p(r,\theta) = \frac{e^{i(ax-br)}}{\sqrt{r}} \sum_{n=0}^\infty B_n \cos n\theta = \frac{e^{i(ax-br)}}{\sqrt{r}} p_s(\theta)$$

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where the $B_i$ are constants providing the solution for this problem, and $\theta$ is the azimuth. Guo has evaluated the backscattered pressure $p$, for the problem at hand for $\theta = \pi$, and found the results shown in Fig. 2. The solid line depicts the amplitude of the backscattered field for the shell with the oscillator, while the dotted line represents the corresponding field for an empty shell (i.e. lacking the oscillator, but having mass). Considering that $ka = 2\pi a / \lambda$, it follows that the frequency range shown in Fig. 2 corresponds to incident waves of wavelength larger than six times the radius. Notice that very strong radiation of waves takes place even for wavelengths of the order of 60 (!) times the radius of the shell ($ka = 0.1$).

**Engineering interpretation**

Clearly, the results shown in Fig. 2 are not intuitively obvious. Why should the shell respond so vigorously to very low frequency waves (very long wavelengths) and exhibit strong fluid-structure interaction? In what fundamental ways does the oscillator modify the behavior of the empty shell? To answer the previous questions, consider first the empty shell in a vacuum. This system has an infinite — but countable — number of vibration frequencies and modes, which can be found analytically (indeed, they are listed in many standard textbooks on structural dynamics). Fig. 3 depicts the frequency spectrum of the shell (using Guo's thin shell equations, which incorporates a small error that causes the rigid-body mode to have a non-zero frequency value). Fig. 4, on the other hand, shows the modal shapes (simply expressible in terms of sines and cosines) which correspond to Fourier numbers $n=1$ and $n=7$. The first of these is the zero-frequency rigid-body mode, which entails no deformations of the shell; we observe also that while the mode corresponding to $n=0$ — the so-called breathing mode — does participate in the solution, it hardly gets excited because it has a very high resonant frequency. On the other hand, the average lateral vibration of the shell in each mode can be computed by integrating the modal displacement components over the azimuth. Alternatively, since the shell is free and its mass is uniformly distributed, it follows — on account of the principle of conservation of momentum — that this average motion must equal the motion of the centroid of the shell (i.e. the center). It follows that only the mode $n=1$ involves motion of the centroid.

We turn next our attention to an empty shell placed in water and subjected to plane low-frequency pressure waves. While this submerged system no longer has proper modes, it exhibits near-resonances similar to those of the shell in a vacuum, except that its "natural frequencies" are somewhat lower on account of the added ("effective") mass of the surrounding fluid, and the vibrations are damped because of wave radiation. Since at low frequencies the incident pressure field has only very small high-order Fourier components, we conclude that motions of the fluid and the shell are compatible, and that the shell for all practical purposes does not deform. This implies that the higher modes of the shell have very low participation and are virtually not excited. Hence, the shell mostly follows the water's motion, the backscattered waves have low amplitude, and the pressure changes smoothly with frequency.
Consider now the shell in a vacuum with the oscillator mounted inside. This system also has an infinitely countable number of modes, although the modal shapes are now more complicated, because the deformations of the shell can no longer be described by simple sine-cosine functions. Fig. 5 shows a schematic view of an individual mode. Expressing the deformation of the shell in the coupled system by superposition of the modes of the uncoupled system (i.e. the shell alone), we can see that the coupled modes contain a rigid-body translation of the shell plus a deformation about such translation. On account of the principle of conservation of momentum, it can be anticipated that the modal displacements of the shell's centroid and the oscillator's body must occur in opposite directions and be in inverse proportion to their respective masses. Thus, all coupled modes involve translation of the shell's centroid, so they must all elicit participation of the shell's rigid body mode.

Finally, consider the shell in the fluid subjected to the incident plane wave field. Again, this system no longer has proper modes, but it will exhibit near-resonances similar to those of the coupled system in vacuum, albeit with lower natural frequencies and added damping. Hence, all of the modes of the submerged coupled system will involve translation of the shell's centroid. It follows that in reaction to the the incident pressure waves, the shell will attempt to "ride the waves" and experience as a result a displacement of its centroid. This motion will in turn elicit a strong reaction in the oscillator, and excite in the process all of the shell's modes. The result is strong interaction and radiation taking place in that coupled mode whose frequency coincides with the frequency of the incident waves. The radiation patterns observed at these frequencies can be expected to exhibit strong directivity, since they will be dominated by one or more the shell's higher modes.

Reference

Fig. 3  Frequency spectrum of empty shell in a vacuum

Fig. 4  Modes of empty shell

Fig. 5  Modes of internally loaded shell