NEW BOX CONTAINER SYSTEM FOR WASTE DRUMS: DYNAMIC TESTS AND QUALIFICATION

H. C. Flessner, E. Kausel, and F. H. Timpert
University of Hamburg, Department of Computer Applications, Vogt-Köln-Str. 30, D-22527 Hamburg, Germany
Massachusetts Institute of Technology, Department of Civil Engineering, (Room 1–271), Cambridge, MA 02139, USA
CORROBESCH VGmbH, and STM Safety Technology Management Co. Ltd
Klosterwall 2, D-20095 Hamburg, Germany

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Abstract — A first technical report is presented on a new 20 ft box container, designed by the firm CORROBESCH/STM as a Type A package for the transport of radioactive and other dangerous materials, and having a carrying capacity of 22 tons. The container itself weighs only 4 tons, and it incorporates a proprietary corrosion finish that is highly resistant to mechanical wear, deformation, and radioactive contamination. The Type A Package, which also can be used as industrial package Type 2 and Type 3, is designed to withstand accelerations of up to 6g. This design criterion was established based on the European Railroad Associations requirement of considering dynamic loads of 4g arising during routine transport times a safety factor of 1.5. In contrast, the ISO Norm 1496, part 1, does not require explicit consideration of dynamic loads but only requires a static load test. Therefore, the main motivation behind the dynamic load design criteria was the complete lack of freight containers capable of withstanding dynamic loads of up to 6g. The container was subjected in Germany to a series of collision and drop tests, as specified by the International Atomic Energy Agency (IAEA), and it passed these tests to complete satisfaction. As a result, the container received certification by agencies such as the Germanischer Lloyd (GL), the Deutsche Bahn AG (DB), and the Bundesanstalt für Materialprüfung (BAM). At the same time, the authors have also developed a mathematical model of the container to predict its dynamic behaviour during service loads, and have been able to make motion predictions that are in good agreement with signatures recorded during tests. The test facility of the German Railroads in Minden/Germany is only designed to take up ram tests of up to 4g dynamic loads. Therefore, the container design criterion of 6g dynamic loads was checked with the benchmarked computer model presented here. Based on the computer code results it is demonstrated that the container withstands the forces of up to 510 kN of end walls resulting from dynamic loads of 6g.

BOX CONTAINER

The German firm CORROBESCH/STM has developed a 20 ft ISO box container, qualified as a Type A package which can also be used as industrial package Type 2 and Type 3, intended for the transport of solid low level radioactive wastes and other hazardous materials normally stored in 200 litre drums. While the container weighs only 4 tons, it can carry up to 22 tons of cargo in various configurations. The container is made of stiffened steel plates, and is closed at one end by two doors with latches and bolts (see Figure 1). All interior and exterior surfaces are covered with a paint developed and patented by CORROBESCH, which is resistant to chemical corrosion, to mechanical deformation and wear, to scratches and tears, and to radioactive contamination. The container is designed to withstand longitudinal accelerations of up to 6g.

SUPPORT TRAYS AND MOTION-RESTRAINING DEVICES

One of the aims in the development of this container was to achieve a system that could transport cargoes with variable numbers of drums stacked in changeable configurations — including an only partially filled container while ensuring the safety of the container against accidental dynamic forces and vibrations that could conceivably arise during transport. This goal led CORROBESCH/STM to the design of an interchangeable tray system for the drums that can easily be handled

Figure 1. A new 20' ISO box container qualified as a Type A package.
and loaded into the container, together with passive restraining devices that prevent this assembly from moving inside the container when subjected to dynamic forces. Another important consideration was the fact that after loading and sealing of the container, it is not possible to inspect or adjust the contents until after it has arrived at its destination. Since the transport can last days, weeks or even months, it was deemed necessary to avoid the use of restraining devices on the drums that rely on initial set forces, such as tension straps, because with time such devices can lose their tension, and thus their effectiveness. Hence, CORROBESCH/STM developed passive restraining devices that adjust themselves as the drums vibrate back and forth.

The drums are stacked in two layers, and rest on light flexible trays within grooves that have been carved out in the shape of the drum heads. There are three layers of trays: one on the bottom surface, one between the two drum layers, and one on the top covering the drums. These trays are not continuous at a given elevation, but consist of individual trays having the width of the container, with space for up to three drums; these are placed, one at a time, into the container. Between each individual tray lies a removable restraining bar that is anchored elastically to the side walls of the container. A remaining clearance space above the top tray layer of about 50 cm, which is needed to load or unload the drums using forklifts, is then secured with passive restraining devices consisting of masses and springs that lock the trays into place, and which adjust themselves as the container is shaken by vibrations during transport (see Figure 2).

DYNAMIC TESTS

On 29 June 1994 the fully loaded container with a total weight of 26 ton was subjected to a series of controlled collisions tests at the experimental facilities of the German Railroads (Deutsche Bahn AG) in Minden, FRG. The container was loaded and anchored onto a 20 ton flat car (Figure 3), which was subsequently rammed by an 80 ton car at varying speeds (Figure 4). The speed was adjusted in steps so as to achieve prescribed levels of acceleration of up to four times the acceleration of gravity at the anchoring points of the container.

Acceleration signatures were recorded by sensing elements at various points in the container (Figure 5), and extensive measurements were made of the motions experienced by the drums. The container withstood these tests without external damage, no drums were broken, and no fill material was spilled. Following these
tests, the container received certification by the Deutsche Bahn, the Germanischer Lloyd, and the Bundesanstalt für Materialprüfung as being fit for the transport of dangerous goods.

Six months later, on 6 December 1994, a fully loaded container was subjected to a drop test at the port facilities of Blohm + Voss in Hamburg. The container was hung by one edge 30 cm above the ground and dropped with some inclination onto a big concrete pad covered by a thick steel plate. The container crashed first onto a corner, then pivoted about this point and slammed onto the surface with great impact. The motion signatures were recorded at various points with triaxial devices; additionally, the test was captured on video. While the container suffered some external damage, no fill material was spilled to the exterior.

MATHEMATICAL MODELS

Clearly, full scale dynamic tests are very expensive and time consuming. Even though a number of essential tests are prescribed by the licensing agencies and must unavoidably be carried out, it is also true that these tests may often not be exhaustive enough. One can easily imagine — or postulate — scenarios not covered by the tests. Thus, it is desirable to supplement the physical tests with mathematical models that can be used to make predictions about hypothesised accident conditions. Such tools can be invaluable for the prediction of the behaviour of the container during such conditions, and can also be used to interpret any measurements made during actual tests. These considerations moved the authors to develop a discrete numerical model for the analysis of the container when subjected to dynamic conditions. In essence, the authors implemented a finite element model with discrete elements that allow for large motions of the components.

Consider a container loaded onto a flat car (Figure 3), which is in turn impacted from the rear end by another heavy car (Figure 4). The container is loaded with two layers of drums on trays, and there are eight rows with three drums in each (see Figure 8 below). Thus, the container contains a total of $2 \times 3 \times 8 = 48$ drums. To a first approximation, such a collision elicits mostly longitudinal horizontal forces within the container, so it suffices to represent this system in terms of a plane (two-dimensional) geometry. In such model, we have made the following assumptions:

(i) Both the ramming and flat cars as well as the container itself — but not its contents — are infinitely rigid separate bodies of known masses.

(ii) The shock absorbers between the cars are modelled as a linear spring-damper system, which may neither exceed a maximum elongation, nor be subjected to tension. Upon reaching its maximum elongation, the ensuing collision is assumed to be of negligible duration (the time required for a shock wave to travel through the flat car is between one and two orders of magnitude smaller than the response time of the container with the drums inside).

(iii) The drums in the container are rigid bodies connected by elastic springs (the trays and the restraining devices); each drum can both translate as well as rotate about a transversal axis.

(iv) The longitudinal motion of the bottom edge of the lower row of drums is impeded by friction against the bottom of the container. Upon exceeding its maximum frictional value, each drum may slide independently.

(v) Inelastic damping forces are neglected.

Using these assumptions, motion predictions were then made for tested configurations considering three mechanical models of increasing sophistication:

(1) First, the two colliding cars were assumed to be perfectly rigid and obeying the physical laws governing an elastic–plastic collision (conservation of momentum and energy balance). This is the very simplest model that was used to make predictions about the abrupt change in velocity that takes place when the shock absorbers between the cars have reached their maximum deformation (21 cm). Since the equations for this model are well known, they need not be presented here. However, full details can be found elsewhere.

(2) Next, a second model was considered involving the collision of two rigid bodies separated by a spring. As shown later on, this model can be evaluated in closed form, supplements the previous model, and allows one to make simple predictions about the orders of magnitude of the response (accelerations, forces, etc.).

(3) Finally, a more elaborate third model was developed in which the contents of the container are discrete masses separated by springs that can move, rotate, or slide relative to each other. The resulting system of non-linear equations was then solved numerically by means of a special computer code written in the FORTRAN language. Motion signatures were then evaluated at various points and compared with those recorded at the Minden facilities.

Model 1: Elastic–plastic collision of two rigid bodies

This model involves ten equations which are presented elsewhere.

Model 2: Collision between two rigid bodies separated by springs

The parameters are displayed in Figure 6. The dynamic equation for this model is:
\begin{equation}
\begin{bmatrix} m_p \\ m_f \end{bmatrix} \begin{bmatrix} \dot{u}_p \\ \dot{u}_f \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} u_p \\ u_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{equation}

(1)

where \( k \) is the spring constant of the shock absorber. In matrix form, this equation can be written as:

\[
\mathbf{MU} + \mathbf{KU} = \mathbf{0}
\]

(2)

which is subjected to the initial conditions:

\[
\mathbf{U}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \dot{\mathbf{U}}_0 = \begin{bmatrix} v \\ 0 \end{bmatrix}
\]

with \( v \) being the impact velocity. Using modal superposition, the solution can be obtained as:

\[
\mathbf{U} = \Phi \mathbf{Q} = \sum_{j=1}^{2} \left( a_j \cos \omega_j t + \frac{b_j}{\omega_j} \sin \omega_j t \right) \phi_j
\]

(4)

where \( \omega_j, \phi_j \) are the eigenvalues and eigenvectors of the system, and \( a_j, b_j \) are integration constants, which can be obtained from the initial conditions.

The former follow from the eigenvalue problem

\[
\mathbf{K} \phi_j = \omega_j^2 \mathbf{M} \phi_j
\]

(5)

whose solution is

\[
\omega_1 = 0, \quad \phi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \omega_2 = \sqrt{(1 + \mu) \sqrt{k/m_f}}, \quad \phi_2 = \begin{bmatrix} \mu \\ -1 \end{bmatrix}
\]

(6a, 6b)

with auxiliary parameter \( \mu = m_f/m_p \).

The integration constants \( a_j, b_j \) are given by

\[
\begin{align*}
a_1 &= \frac{\phi_j^T \mathbf{M} \dot{\mathbf{U}}_0}{\phi_j^T \mathbf{M} \phi_j} \\
b_1 &= \frac{\phi_j^T \mathbf{M} \mathbf{U}_0}{\phi_j^T \mathbf{M} \phi_j}
\end{align*}
\]

(7)

so that

\[
a_1 = a_2 = 0, \quad b_1 = b_2 = \frac{v}{1 + \mu}
\]

(8)

After insertion of these constants in Equation 4, we obtain

\[
\mathbf{U} = \frac{v}{1 + \mu} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \omega_2 \begin{bmatrix} \frac{\mu}{(1 + \mu) \sqrt{k/m_f}} \\ -1 \end{bmatrix}
\]

\[
\dot{\mathbf{U}} = \frac{v}{1 + \mu} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \cos \omega_2 t \begin{bmatrix} \frac{\mu}{(1 + \mu) \sqrt{k/m_f}} \\ -1 \end{bmatrix}
\]

\[
\ddot{\mathbf{U}} = \frac{v}{1 + \mu} \cos \omega_2 t \begin{bmatrix} -\frac{\mu}{(1 + \mu) \sqrt{k/m_f}} \\ 1 \end{bmatrix}
\]

(9a, 9b, 9c)

From these equations, we can derive the following relationships:

(a) Compression of the shock absorber:

\[
e = u_p, \quad u_t = \frac{v}{\omega_2} \sin \omega_2 t
\]

(10)

(b) Duration (\( t = T \)) of collision: it is a function of the deformation of the shock absorbers, and follows from the condition \( e = 0 \).

\[
T = \frac{\pi}{\omega_2} = \sqrt{(1 + \mu)} \sqrt{k/m_f} = \pi \sqrt{(1 + \mu)} \sqrt{k/m_f}
\]

(11)

(c) Impact velocity eliciting the maximum allowable deformation of the shock absorber:

\[
v_m = \omega_2 e_{\text{max}} = \frac{\pi e_{\text{max}}}{T}
\]

(12)

(d) Maximum acceleration of the flat car with mass \( m_f \):

\[
\ddot{u}_t_{\text{max}} = \frac{\sqrt{v \omega_2}}{\sqrt{(1 + \mu)} \sqrt{k/m_f}} = \frac{v}{\sqrt{(1 + \mu)} \sqrt{k/m_f}} = \frac{\pi}{\sqrt{(1 + \mu)} \sqrt{k/m_f}} T
\]

(13)

(e) Velocity after collision:

Setting \( t = T = \pi/\omega_2 \), we obtain from Equation 9b:

\[
a_p = \frac{1 - \mu}{1 + \mu} v, \quad u_t = \frac{2}{1 + \mu} v
\]

(14)

These equations agree with those obtained for model 1 (not presented here).

**Results obtained with model 2**

The physical constants are:

\[
m_p = 80 \text{ ton} = \text{mass of ram car}, \quad m_f = 50 \text{ ton} = \text{mass of flat car + container + cargo}, \quad k = 5619 \text{ kN.m}^{-1} = \text{stiffness of shock absorber} (= 590/0.105) \text{ total stiffness of two shock absorbers in series} = k/2, \text{then two parallel sets left and right gives 2(k/2) = k}, \quad e_{\text{max}} = 0.21 \text{ m} = \text{maximum clearance of shock absorbers} (= 2 \times 0.105)
\]
Using these values as well as Equations 10 to 14, we obtain

\[ \mu = \frac{58}{8} = 0.625 \text{ mass ratio} \]

\[ \omega_2 = 13.51 \text{ rad.s}^{-1} \text{ eigenfrequency (} = 2.15 \text{ Hz)} \]

\[ T = 0.232 \text{ s duration of impact} \]

\[ v_m = 10.22 \text{ km.h}^{-1} \text{ speed eliciting maximum spring travel of shock absorbers} \]

\[ \ddot{u}_{\text{max}} = 0.30v \quad \text{g maximum acceleration (v in km.h}^{-1} \text{)} \]

It follows that an impact velocity \( v = 10 \text{ km.h}^{-1} \) will produce a maximum acceleration of 3g in the flat car (or container). Since this value is close to that necessary to elicit full stroke of the shock absorbers, it follows that beyond this impact velocity, the acceleration of the container will grow very rapidly.

Figure 7 depicts the maximum acceleration of one of the anchor points of the container as computed with the aid of model 2 considered in this section. This figure also shows the values recorded during the actual tests at the DB Minden facilities (where the actual mass ratio was 0.625). As can be seen, the simple model provides excellent estimates of the maximum acceleration, even though the actual system exhibits a more complex (non-linear) behaviour due to the changing participation of the contents of the container with the impact velocity.

**Model 3: Finite element model**

In this alternative, the container and the drums are modelled as discrete masses with translational and rotational inertias joined by springs representing the elastomeric trays supporting the drums. As shown in Figures 8 and 9, this model contains two layers of spring-connected drums. Along the bottom of the container, the springs can develop in either translational direction only the maximum force allowed by friction. After this limit is reached each stack of drums may begin to slide (independently of each other) with respect to the container. The details of this model are developed in the sections that follow.

After the two cars make contact, momentum begins being transferred through the shock absorber to the flat car with the container, which in turn transfers some of its kinetic energy to the drums in the form of a wave propagating through them in both a horizontal as well as vertical direction. If and when the maximum stroke of the shock absorbers is reached, an elastic–plastic collision ensues, which changes the momentum abruptly, and thus the velocity of the flat car. This collision is associated with very large accelerations, which do not immediately affect the drums, since the compliance of the trays and the sliding capability of the drums on the bottom of the container limit the forces that can be transferred to them. A shock wave then develops which again propagates through the system eliciting translations, rotations, and sliding. Each drum then experiences different motions, and must be analysed in turn.

**Stiffness of supporting trays**

(i) **Longitudinal stiffness.** With reference to Figure 10, we consider two neighbouring drums at a certain elevation (i.e. upper, middle and bottom level), and idealise the tray connecting the drums as a system of parallel

![Figure 8. Container with finite masses.](image)

![Figure 9. Cross section of drums with longitudinal and rotational springs.](image)

Figure 7. Maximum acceleration of container plotted against impact velocity.
springs. If $E$ is the modulus of elasticity and $c$ the thickness of the trays, then the total centre-to-centre longitudinal stiffness of the tray material between the drum is

$$k = \frac{1}{2} Ec \left[ \frac{b - 2r}{a} + 2 \int_0^\alpha \left( \frac{dy}{a - r \sin \theta} \right) \right]$$

(15)

with $y = r (1 - \cos \theta)$, $dy = r \sin \theta \, d\theta$.

Equation 15 can be expressed as

$$k = \frac{Ec}{2a} \left[ \frac{b - 2r}{1 - \alpha \sin \theta} - \frac{\pi}{2} \int_0^{\pi/2} \left( \frac{d\theta}{1 - \alpha \sin \theta} \right) \right]$$

(16)

in which $\alpha = r/a < 1$. To evaluate this integral we expand the integrand in series:

$$I = \int_0^{\pi/2} \left( \frac{d\theta}{1 - \alpha \sin \theta} \right) \frac{\pi}{2}$$

$$= \int_0^{\pi/2} (1 + \alpha \sin \theta + \alpha^2 \sin^2 \theta + \ldots) d\theta - \frac{\pi}{2}$$

$$= \frac{\pi}{2} \left[ 1 + \frac{1}{2} \alpha^2 + \frac{1.3}{2 \cdot 4} \alpha^4 + \frac{1.3.5}{2.4.6} \alpha^6 + \ldots \right]$$

$$+ \left[ \alpha + \frac{2}{3} \alpha^3 + \frac{2.4}{3.5} \alpha^5 + \frac{2.4.6}{3.5.7} \alpha^7 + \ldots \right]$$

(17)

which can be evaluated recursively with the integrals

$$I_n = \int_0^{\pi/2} \sin^n \theta \, d\theta = \frac{n - 1}{n} I_{n-2}, \quad I_0 = \pi/2, \quad I_1 = 1$$

(18)

Thus, it follows that the longitudinal stiffness is

$$k = Ec \left[ \frac{b}{2a} + 1 \right]$$

(19)

which can be evaluated numerically, as shown in Figure 11. The dimensions in our case are: $r = 0.652/2 = 0.326 \text{ m}$, $a = 0.724/2 = 0.362 \text{ m}$, and $b = 0.72 \text{ m}$, so that $b/2a = 0.99$ and $\alpha = r/a = 0.90$. For this value of $\alpha$, the integral $I$ is approximately 10. The longitudinal stiffness of the trays for each pair of drums is then $k = Ec(0.99 + 10) = 11Ec$. Since there are three parallel stacks of drums, their total longitudinal stiffness must then be the triple of this value, that is

$$k = 33Ec$$

(20)

For example: in our case, the trays have thickness $c = 0.15 \text{ m}$ and modulus of elasticity $E = 5000 \text{ kN.m}^{-2}$, the longitudinal stiffness is then $k = 24,750 \text{ kN.m}^{-1}$.

It follows that the longitudinal stiffness of trays has an order of magnitude $10^4 \text{ kN.m}^{-1}$; however, the exact value is uncertain, because the trays are neither welded to each other nor are they monolithically connected to the drums. In other words, these stiffnesses have probabilistic values, so that parametric studies are necessary.

(ii) Rotational stiffness of trays. When the drums rotate, they press against the trays (Figure 12). To model this problem, we can idealise the edge of the drums as a ring footing on elastic (Winkler) foundation.

The effective width $w$ of the elastic foundation is comparable to the thickness $c$ of the trays. This gives a Winkler constant $q = Ew/c$. Thus, the rotational stiffness is

$$k_0 = \int_0^{2\pi} qx^3 dx = \int_0^{2\pi} qr^3 \cos^3 \theta \, d\theta = \pi qr^3$$

(21)

Figure 11. Evaluation of longitudinal stiffness.

Figure 12. Parameters for rotational stiffness.
that is
\[ k_0 = \frac{\pi E w r^3}{c} \sim \pi E r^3 \quad (22) \]

Later on we will also need the rotational stiffness associated with three parallel stacks of drums and normalised by the total height of the drums, namely
\[ \gamma = \frac{3 k_0}{h^2} = 3 \pi E r \left( \frac{r}{h} \right)^2 \quad (23) \]

Using values \( r = 0.326 \text{ m}, h = 1.048 \text{ m} \) and \( E = 5000 \text{ kN.m}^{-2} \), Equation 23 yields a normalised rotational stiffness of \( \gamma = 1487 \text{ kN.m}^{-1} \). This value is one order of magnitude smaller than the longitudinal stiffness of the trays (about 20 times smaller).

Of the three tray elevations in our model, only the bottom and middle elevations have any significant rotational stiffness, because the top elevation is not in contact with the roof.

**Influence of gravitation on stiffness ('Pi-Delta effects')**

As the drums oscillate back and forth and rotate about a horizontal axis, their centre of gravity moves with respect to the edge of the drums, which can lead to inverted pendulum effects. This phenomenon can be captured in the stiffness matrix by means of a correction term. The derivation of this term can be effected by means of energy principles.

We consider a stack of two drums in their displaced configuration as shown in Figure 13. We define

- \( Q_1 \) and \( Q_2 \) = weight of the drums
- \( P_1, P_2, P_3 \) = lateral forces (transferred by the trays)
- \( u_1, u_2, u_3 \) = horizontal displacements of the trays
- \( v_1, v_2 \) = vertical displacements of the centres of gravity (positive downward)
- \( \theta_1, \theta_2 \) = rotation of the drums around the horizontal axis

The potential energy of the drums is

\[ \Pi = -(Q_1 v_1 + Q_2 v_2) \quad (24) \]

We next express the vertical displacements of the centres of the drums in terms of the rotations as
\[ v_1 = 2v_2 + \frac{h}{2} (1 - \cos \theta_2) \approx 2v_2 + \frac{h}{4} \theta_2^2 \quad (25a) \]
\[ v_2 = \frac{h}{2} (1 - \cos \theta_2) \approx \frac{h}{4} \theta_2^2 \quad (25b) \]

On the other hand, the rotations are also functions of the lateral displacements:
\[ \theta_1 = \frac{u_1 - u_2}{h}, \quad \theta_2 = \frac{u_2 - u_3}{h} \quad (26) \]

Hence, the potential energy is
\[ \Pi = -\frac{1}{4h} [Q_1(u_1 - u_2)^2 + (2Q_1 + Q_2)(u_2 - u_3)^2] \quad (27) \]

In matrix form this expression may be written as
\[ \Pi = -\frac{1}{2} U^T B U \quad (28) \]

in which
\[ U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad B = \frac{1}{2h} \begin{bmatrix} Q_1 & -Q_1 \\ -Q_1 & 3Q_1 + Q_2 & -Q_1 \\ -Q_1 & -Q_1 & 2Q_1 + Q_2 \end{bmatrix} \quad (29) \]

and since \( Q_1 = Q_2 = mg \), it follows that \( B \) can be reduced to
\[ B = \frac{mg}{2h} \begin{bmatrix} 1 & -1 \\ -1 & 4 & -3 \\ -3 & -3 & 3 \end{bmatrix} \quad (30) \]

The total potential energy of this system is the sum of the deformation energy (the elastic energy stored in the trays), the potential energy of the gravitational forces, as well as the potential energy of the external forces \( P \), i.e.
\[ E = \frac{1}{2} U^T K U - \frac{1}{2} U^T B U - U^T P \quad (31) \]

where \( K \) is the stiffness matrix of the tray. The condition of equilibrium is simply \( \delta E = 0 \), that is
\[ \delta E = \delta U^T (KU - BU - P) = 0 \quad (32) \]

from which we obtain
\[ (K - B)U = P \quad (33) \]

Thus, it suffices to work with a modified stiffness matrix \( \tilde{K} = K - B \), to consider the effect of the gravitational forces.
Mass matrix (stack of 2 drums)

We consider the initial forces in each drum, which must be in equilibrium with the internal forces acting at its top and bottom surface (Figure 14).

Denoting with \( m \) and \( J \) the mass and mass moment of inertia of the drum we obtain the dynamic equilibrium equation:

\[
F_1 + F_2 = m\ddot{u}
\]

\[
(F_1 - F_2) \frac{h}{2} = J\ddot{\theta}
\]

(34a)
(34b)

On the other hand, the displacement \( u \) and rotation \( \theta \) of the centroid of the drum can be expressed in terms of the translations \( u_1, u_2 \) of the lid and bottom:

\[
u = \frac{(u_1 + u_2)}{2}
\]

\[
\theta = \frac{(u_1 - u_2)}{h}
\]

(35a)
(35b)

Inserting these relationships into Equations 34a and 34b we obtain

\[
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} = \frac{1}{4} \begin{bmatrix}
m + n & -m - n \\
m - n & m + n
\end{bmatrix} \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

(36)

where

\[
n = \frac{4J}{h^2} = m \left[ \frac{1}{3} + \left( \frac{r}{h} \right)^2 \right]
\]

(37)

is the normalised moment of inertia of the cylindrical drum. The mass matrix for a stack of two drums is obtained by overlapping the mass matrices (Equation 36) for each drum, that is

\[
M = \frac{1}{4} \begin{bmatrix}
m + n & -m - n \\
m - n & 2(m + n) & m - n \\
m - n & m + n
\end{bmatrix}
\]

(38)

This expression constitutes the desired mass matrix.

In the numerical solution by time step integration, we need also the inverse of this matrix, which can be shown to be given by:

\[
M^{-1} = \frac{1}{cd} \begin{bmatrix}
c & -b & b^2/a \\
-b & a & -b \\
\frac{b^2}{a} & -b & c
\end{bmatrix}
\]

(39a)

\[
a = \frac{(m + n)}{4}, \quad b = \frac{(m - n)}{4}, \quad c = \frac{2a}{b^2/a}
\]

(39b)

Elastic equilibrium (stack of 2 drums, see Figure 9)

We consider the forces exerted by the trays and their relationship with the nodal displacements and external forces. For this purpose, we number the stacks from left to right with indices \( j = 1, 2 \ldots 8 \), and consider an arbitrary stack, in which we define:

\[
l = \text{left}
\]

\[
r = \text{right}
\]

\[
a = \text{bracing bars (between nodes and walls)}
\]

\[
f = \text{flat car}
\]

\[
p = \text{ram car}
\]

\[
i = 1, 2, 3 = \text{upper, middle, and lower edge (of the tray)}
\]

\[
u_{ij} = \text{displacement of node in elevation } i, \text{ column } j
\]

\[
u_f = \text{displacement of flat car}
\]

\[
\theta_i = \frac{(u_{i,j} - u_{i,j+1})}{h}
\]

\[
theta = \text{rotation of drums}
\]

\[
f_i = \text{external forces at elevation } i, \text{ column } j
\]

\[
M_i = \text{internal forces}
\]

\[
k_{ii} = \text{longitudinal stiffness of trays to left and right}
\]

\[
k_{iu} = \text{rotational stiffness of trays}
\]

\[
k_{ia} = \text{stiffness of bracing bars}
\]

\[
k_{iu} = \text{normalised rotational stiffness}
\]

\[
k_{ii} = \text{deformation of tray to the left}
\]

\[
k_{ii} = \text{deformation of tray to the right}
\]

\[
k_{ia} = \text{deformation of bracing bars}
\]

Conditions of equilibrium:

upper elevation or node:

\[
P_{ij} = F_i + k_{ij}\Delta_{ii} + k_{ir}\Delta_{ir} + k_{ia}\Delta_{ia}
\]

(40a)

\[
M_i = k_{ia}\theta_i
\]

(40b)

upper drum:

\[
F_1 + F'_2 = 0
\]

(41a)

middle elevation (primes: 'from above', double primes: 'from below')

\[
P_{2j} = F''_2 + F''_j + k_{2i}\Delta_{2i} + k_{2r}\Delta_{2r} + k_{2a}\Delta_{2a}
\]

(42a)

\[
M'_2 = -M''_2 = k_{2b}(\theta_i - \theta_2)
\]

(42b)
lower drum:
\[ F'' + F_3 = 0 \]  (43a)
\[ M'' + M_3 = (F'' - F_3) \]  (43b)
bottom elevation or nodes:
\[ P_{j3} - \Gamma_3 + k_{31} \Delta \theta_1 + k_{33} \Delta \theta_3 \]  (54a)
\[ M_3 = k_{33} \theta_2 \]  (54b)
Eliminating the internal forces and moments as well as the rotations, we obtain after brief algebra the expression:
\[ P_j = -K_j U_{j-1} + (K_0 + K_1 + K_2)
+ K_3 U_j - K_j U_{j-1} - K_j E u_t \]  (45)
where
\[ P_j = \begin{pmatrix} P_{1j} \\ P_{2j} \\ P_{3j} \end{pmatrix}, \quad U_j = \begin{pmatrix} u_{1j} \\ u_{2j} \\ u_{3j} \end{pmatrix}, \quad E = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \]  (46a)
\[ K_0 = \text{diag} \{k_{00}\}, \quad K_j = \text{diag} \{k_{jj}\}, \quad K_i = \text{diag} \{k_{ii}\}, \quad U_0 = E u_t \]  (46b)
\[ K_0 = \begin{pmatrix} \gamma_1 + \gamma_2 & -(\gamma_1 + 2\gamma_2) & \gamma_1 \\ -(\gamma_1 + 2\gamma_2) & \gamma_1 + 4\gamma_2 + \gamma_3 & -(2\gamma_3 + \gamma_1) \\ \gamma_2 & -(2\gamma_3 + \gamma_1) & \gamma_2 + \gamma_3 \end{pmatrix} \]  (46c)
On the other hand, \( P_{j1} = P_{3j} = 0 \). Furthermore the external force \( P_{3j} \) acting on the bottom is determined by the friction and sliding condition along this surface.

**Dynamic equilibrium equations**

From Equations 33, 38, and 45 we obtain:

(a) drums (\( 1 \leq j \leq 8 \)):
\[ P_j = M \dot{U}_j - K_j U_{j-1} + (K_0 + K_1 + K_2)
+ K_i + K_n - B) U_j - K_j U_{j+1} - K_j E u_t \]  (47)
(b) flat car with container:
\[ m_n u_r + k_n (u_r - u_n) + \sum_{i=1}^{3} k_{11} (u_r - u_{1i})
+ \sum_{j=1}^{8} \sum_{i=1}^{3} k_{22} (u_r - u_{2j}) = -\sum_{j=1}^{8} P_{3j} \]  (48)
(c) ram car:
\[ m_n u_p + k_n (u_p - u_n) = 0 \]  (49)
When the drums are not sliding, then \( \dot{u}_{3j} = \dot{u}_r \) (kinematic condition). In such a case, we compute the force \( P_{3j} \) as the sum of the internal forces on the drums (from Equation 47). If the result exceeds the allowable friction \( R \), then we replace \( P_{3j} \) with \( R \) sign (\( P_{3j} \)), and the drum begins to slide (\( \dot{u}_{3j} \neq \dot{u}_r \)). This sliding continues until the speed difference \( \dot{u}_{3j} - \dot{u}_r \) changes sign. When this happens we must also test whether or not the drum begins immediately to slide in the opposite direction, or if it sticks to the bottom.

**Numerical integration**

We integrate Equations 47–49 by means of the method of finite differences:
\[ \dot{U}_{j+1} = U_j + \Delta t \dot{U}_j \]  (50a)
\[ U_{j+1} = U_j + \Delta t \dot{U}_{j+1} \]  (50b)
While selecting the time step \( \Delta t \leq T_{min}/\pi \) we must be careful not to exceed the shortest period of the system divided by \( \pi \) to ensure numerical stability.

**Summary of results**

Figures 15 and 16 show a comparison of the motions in the front left row of barrels in the container, as computed with the discrete model and as recorded at the Minden test facilities, for a collision velocity of 10 km h\(^{-1}\). Figure 15 gives the motion at the bottom of the lower layer of barrels (similar to the container’s motion), while Figure 16 provides the acceleration at the top of the upper layer of barrels. In both cases, the synthetic signatures have been low-pass filtered to match the filtering used during the tests (although this made little difference in the computed motions). While the computed and recorded motions are clearly not the same, they exhibit similar overall trends, the orders of magnitude of the acceleration are similar, and there is general concordance in the duration of the process and in the frequencies of oscillation. Indeed, the differences
in these figures are no worse than the variations that were observed in actual motions recorded at the left, centre, and right barrel columns (not shown), which on account of symmetry could have been expected to be very similar, or even the same. This experimental obser-

vation indicates that the trays supporting the barrels may not have had symmetric stiffness/geometric properties, and/or that non-linear processes (e.g. local yielding of materials, etc.) played an important role. Nonetheless, the numerical model provided useful results concerning overall physical behaviour.

CONCLUSION

Discrete numerical models for the dynamic analysis of box containers intended for the transport of drums filled with radioactive materials, such as that briefly described herein, can provide valuable insight into the behaviour of such systems during hypothesised accident conditions.

More importantly, they can be used to supplement and expand the results obtained in actual testing, to help in their interpretation, and above all, to test for design changes in the computer without the need for expensive experiments. The model considered in the context of the new 20' box container designed by CORROBESCH/STM has yielded prediction numbers that were in reasonable agreement with the signatures recorded during actual tests.

The computer model is not limited to the shown symmetrical configuration of loaded drums in the container. It can also be applied for any other configuration with minor changes.

REFERENCES