IMPACT OF WEIGHT FALLING ONTO THE GROUND

By Jose M. Roesset, Member, ASCE, Eduardo Kausel, Member, ASCE, Vicente Cuellar, Jose L. Monte, and Julian Valerio

ABSTRACT: The application of some nondestructive testing procedures for soils and pavements, such as the spectral analysis of surface waves method, requires the use of dynamic sources at some point on the surface of the ground to create vibrations that can be recorded and related to the material characteristics of the medium. These sources are typically accomplished by means of falling weights of various sizes. This paper applies a simple mass-spring-dashpot model to assess the characteristics of the forces transmitted to the ground by the falling weight so as to be able to rationally decide on its size, mass, and dropping height. It is found that if inelastic effects are disregarded, a falling weight always rebounds; that the drop height affects only the impact velocity and amplitude of the contact force—not the duration of contact; and that increases in the drop weight lead to reductions in damping, increases in contact time, increases in contact force, and enhancement of the low-frequency components in the Fourier spectrum of the contact force.

INTRODUCTION

In the application of nondestructive testing techniques—such as the Spectral Analysis of Surface Waves (SASW) method—to measure elastic moduli in soils or pavements, it is often necessary to use dynamic sources having broad frequency spectra and adequate levels of energy. Such characteristics are typically achieved by means of percussive tools (e.g., hand-held hammers of different sizes), or by dropping a weight onto the ground. In either case, the spectral characteristics and highest frequency present in the excitation depend on the duration of impact and on the variation in time of the force between the impacting body and the ground during contact. The amplitude of that force and the ensuing deformation of the ground, on the other hand, determine the levels of wave energy transmitted to the soil. This energy is generally less than the kinetic energy on impact because some energy remains in the weight as it rebounds, and because some energy is lost as a result of nonlinear processes in the neighborhood of the point of collision. In general, it is particularly difficult to achieve impact forces of long duration and high energy able to elicit deeply penetrating low-frequency waves with amplitudes exceeding the background noise level; indeed, the depth to which properties can be obtained is limited primarily by the availability of powerful enough—yet nondestructive—sources in the long period range.

A simple procedure to estimate the values of the mass and drop height required to produce the desired amplitudes of motion over a specified frequency range was recently suggested by Gucunski (1991). This procedure

1Prof., Dept. of Civ. Engrg., Univ. of Texas, Austin, TX 78712.
2Prof., Dept. of Civ. Engrg., Massachusetts Inst. of Technol., Cambridge, MA 01239.
4Head, Special Testing Program, CEDEX, Alfonso XII-3, Madrid, Spain.
5Head, Dynamic Methods Program, CEDEX, Alfonso XII-3, Madrid, Spain.

Note. Discussion open until January 1, 1995. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on April 23, 1993. This paper is part of the Journal of Geotechnical Engineering, Vol. 120, No. 8, August, 1994. ©ASCE, ISSN 0733-9410/94/0008-1394/$2.00 + $.25 per page. Paper No. 5957.

1394
is based on modeling the impact problem by means of a simple mass-damper system, with the dashpot representing either the soil, a specially designed impact box, or a combination of both. The force in such a system decays exponentially with time. Although this model is very simple and may provide sensible results, it is clearly an approximation and furthermore physically impossible, because a constant load implies an infinite displacement. The objective of the present work is to extend Gucunski's studies on amplitude and time variation of the impact force produced by a falling weight, modeling both the stiffness and the damping of the ground.

FALLING WEIGHT

A rigorous analysis of the collision of a falling body with the ground, even if inelastic effects are disregarded and ideal geometries and materials are considered—such as a sphere or cylinder impinging on an elastic, homogeneous, isotropic half-space—calls for the rather formidable mathematical and numerical tools of elastodynamics. Moreover, Hertz's classical formulation for the collision of two spheres—with the half-space modeled as a sphere of infinite radius—does not apply to the falling-weight problem, because the contact phenomenon with the half-space is dominated by wave propagation and radiation effects. Indeed, the duration of contact for two colliding spheres is much longer than the longest period of vibration of the spheres, so the collision is quasistatic; by contrast, the infinitely large half-space lacks any characteristic frequencies, and dynamic interaction effects are important. Fortunately, a rigorous solution is not necessary for the engineering problem being considered, because only order-of-magnitude estimates are needed. Thus, only an approximate model for the falling-weight problem is considered in the present paper.

Following Lysmer (1965) and Verbic and Veletsos (1972), the vertical dynamic impedance functions for a rigid circular plate welded to an elastic half-space can be approximated closely by the stiffness functions of a mass-spring-dashpot system in parallel, with the mass representing the participating mass of soil; if the soil is not nearly incompressible, then the soil mass can be neglected. This suggests that one can assess the effects for the problem described by means of an ideally rigid mass impinging on a (perhaps inelastic) spring-dashpot system. Whereas such an idealization constitutes a considerable simplification of the problem of a weight falling onto the ground, its solution does, nonetheless, provide valuable insight into the far more complex contact problem. As it turns out, even this truly elementary model is complicated enough to require numerical evaluations and it yields results that are useful, if not always intuitive. Such a model is considered in the following pages.

IMPACT OF RIGID MASS ON ELASTIC SPRING-DASHPOT SYSTEM

Consider a rigid mass impinging vertically on a spring-dashpot system with velocity $v$, and moving thereafter as one system under the influence of the gravitational force. Clearly, this is a classical one-degree-of-freedom system governed by the linear equation

$$m\ddot{u} + c\dot{u} + ku = mg$$

(1)

whose characteristic parameters, the undamped frequency, the undamped period, and the fraction of damping, can be denoted with the usual notation
ω, T, and β, respectively. Dividing by mg and defining the dimensionless time τ = ωt = 2π(t/T) as well as the parameters

\[ \delta = \frac{u}{u_s} = \frac{uk}{mg} \quad \text{(dimensionless displacement)} \]  
\[ \nu = \frac{\dot{\delta}}{\dot{\delta}_t} = \frac{\dot{u}w}{g} \quad \text{(dimensionless velocity)} \]  
\[ \alpha = \frac{\ddot{\delta}}{d\delta \dot{\delta}^2} = \frac{\ddot{u}}{g} \quad \text{(dimensionless acceleration)} \]

one obtains the dimensionless equation

\[ \ddot{\delta} + 2\beta \dot{\delta} + \delta = 1 \quad \text{(3)} \]

whose solution is

\[ \delta = 1 + (\delta_0 - 1)C + [\nu_0 + \beta(\delta_0 - 1)]S \quad \text{(4a)} \]
\[ \nu = \nu_0C + (1 - \delta_0 - \beta\nu_0)S \quad \text{(4b)} \]
\[ \alpha = (1 - \delta_0 - 2\beta\nu_0)C + [\beta(\delta_0 - 1) + \nu_0(2\beta^2 - 1)]S \quad \text{(4c)} \]

in which δ₀ and ν₀ = initial conditions; and C and S are given for (4a), an underdamped system, by

\[ C = e^{-\beta t} \cos(\xi t) \quad \text{(5a)} \]
\[ S = e^{-\beta t} \frac{\sin(\xi t)}{\xi} \quad \text{(5b)} \]
\[ \xi = \sqrt{1 - \beta^2} \quad \text{(5c)} \]

for (4b), a critically damped system, by

\[ C = e^{-\tau} \quad \text{(5d)} \]
\[ S = \tau e^{-\tau} \quad \text{(5e)} \]

and for (4c), an overdamped system, by

\[ C = e^{-\beta t} \cosh(\xi t) \quad \text{(5f)} \]
\[ S = e^{-\beta t} \frac{\sinh(\xi t)}{\xi} \quad \text{(5g)} \]
\[ \xi = \sqrt{\beta^2 - 1} \quad \text{(5h)} \]

**REBOUND AND FREE FLIGHT**

The normalized contact force during contact is given by

\[ \varphi = \frac{ku + cu}{cu_0} = \frac{m(g - \ddot{u})}{cu_0} = \frac{1 - \alpha}{2\beta\nu_0} \quad \text{(6)} \]

that is

\[ \varphi = 1 + (\delta_0 - 1 + 2\beta\nu_0)C + [\beta(1 - \delta_0) + \nu_0(1 - 2\beta^2)]S \quad \text{(7)} \]

When the contact force vanishes and the system velocity is negative (i.e.,
upwards), the mass rebounds and initiates free flight, while the spring-
dashpot system creeps toward the rest position. The two systems are then
characterized by the dimensionless equations. For mass
\[ \delta = \frac{1}{2} \tau^2 + \nu \tau + \delta_r \]  
(8a)
\[ \nu = \tau + \nu_r \]  
(8b)
And for the spring dashpot
\[ \delta = \delta_r e^{-\gamma/2\ell} \]  
(9a)
\[ \nu = \nu_r e^{-\alpha/2\ell} = -\frac{\delta_r}{2\ell} e^{-\gamma/2\ell} \]  
(9b)
where the subscript \( r \) refers to the conditions at rebound.

Figs. 1 and 2 illustrate application of (4) and (7)-(9) to systems with 75%
and 500% damping, with the indicated values for the dimensionless impact
velocity parameter \( \dot{u}_0/gT = v_0/2\pi \).

**IMPACT VELOCITY VERSUS REBOUND TIME (DURATION
OF CONTACT)**

As stated earlier, rebound occurs when the contact force drops to zero. Solving from (7) for the impact velocity, when the numerator vanishes, one
obtains
\[ \nu_0 = \frac{C - \beta S - 1}{2\beta C + (1 - 2\beta^2)S} \]  
(10)
which constitutes an implicit equation for the duration of contact in terms
of the impact velocity and damping. Numerical evaluation of this equation
gives the results shown in Fig. 3. Noteworthy in this figure is that even
systems with greater than critical damping (\( \beta > 1 \)) may rebound if the impact
velocity is large enough, as is demonstrated in Fig. 2, which shows the
rebound of a system with 500% damping. Another interesting observation
in Fig. 3 is the presence of two bounds for the velocity and contact time,
both of which depend on the fraction of damping: below a certain velocity,
the system no longer rebounds, whereas at very high impact velocity, the
contact time approaches a finite asymptotic value. This limit is reached
when the denominator in (10) is zero; setting this term to zero and solving
for the minimum duration of contact possible, one obtains
\[ \tau_{\min} = \frac{1}{\sqrt{1 - \beta^2}} \arccos(2\beta^2 - 1); \quad \beta < 1 \]  
(11a)
\[ \tau_{\min} = 2; \quad \beta = 1 \]  
(11b)
\[ \tau_{\min} = \frac{1}{\sqrt{\beta^2 - 1}} \arccosh(2\beta^2 - 1); \quad \beta > 1 \]  
(11c)

Eq. (11c) indicates that the shortest contact time is never zero, no matter
how high the damping value. On the other hand, Fig. 3 also shows that the
maximum duration of contact—if rebound occurs—is always less than the
resonant period, in fact less than half the period when the damping exceeds
FIG. 1. Displacement Time History; $\beta = 0.75$, Impact Velocity $\dot{u}_0/gT = 5$

FIG. 2. Rebound of Highly Damped System; $\beta = 5.00$, Impact Velocity $\dot{u}_0/gT = 300$

... the critical value. The shortest possible contact time for a specific impact velocity and any arbitrary value of damping is given by the envelope to the curves in Fig. 3 (dashed lines); such envelope can be approximated closely by a curve of the form $t_d/T = 0.52/\dot{u}_0/gT^{0.2}$. Although this envelope constitutes only a low bound for the contact time, it has the advantage of being simple to compute. It should be noted that Hertz's model for the collision of two spheres also predicts contact times that vary as the inverse...
fifth root of the impact velocity, although the proportionality constant changes with the physical parameters.

A numerical search, using (10), of the minimum velocity for which rebound occurs and the contact time associated with that velocity for a given value of damping, and an evaluation of (11) for the minimum contact time for infinite impact velocity yields the results shown in Figs. 4 and 5. On the other hand, the duration of contact for zero damping can be found explicitly from (10), namely

\[
\frac{\tau_d}{2\pi} = \frac{t_d}{T} = \frac{1}{2} + \frac{1}{\pi} \arctan \left( \frac{1}{v_0} \right), \quad \beta = 0
\]  

(12)

CONTACT FORCE

During first impact, the initial displacement is zero, so that the normalized contact force has an initial value equal to 1. Whether thereafter this force first rises or falls depends on the value of the derivative at time zero; this derivative is obtained from (7), and is given by

\[
\phi_0 = \frac{2\beta + v_0(1 - 4\beta^2)}{2\beta v_0}
\]  

(13)

which is nonpositive only if
Thus, if damping is less than 0.5, the force on impact can never be the maximum force, but must necessarily rise before decaying. If damping is greater than this value—even if greater than critical—and the impact velocity is not large enough, the force will also rise. Only for sufficiently large damping and impact velocity does the contact force decrease with time from

\[ v_0 \geq \frac{2\beta}{4\beta^2 - 1} \] (14)
the start. This is illustrated in Fig. 6, which shows the contact force for a system with 50% damping and for various impact velocities (the force time histories for systems with larger values of damping resemble the two lower curves in Fig. 6).

On the other hand, computations for impact velocities exceeding the largest value indicated in Fig. 6 result in force time histories and contact times not very different from those shown. This appears to suggest that the force is controlled by the dashpot, and that the system is behaving mostly like Gucunski's mass-damper system. To explore this issue, consider the equations of a mass-damper system in dimensionless form (obtained by deleting the third term in (3) and integrating):

\[ v = \left( v_0 - \frac{1}{2\beta} \right) e^{-2\beta v} + \frac{1}{2\beta} \]  

so that the normalized force is

\[ \phi = \left( 1 - \frac{1}{2\beta v_0} \right) e^{-2\beta v} + \frac{1}{2\beta v_0} \]  

Clearly, (15) and (16) correspond to a system that never rebounds, no matter how high the impact velocity, and the largest force occurs on impact. An evaluation of the contact force by both (7) and (16) when the system is subjected to a very high impact velocity yields the results shown in Fig. 7. The results are nearly identical for 500% damping, they are similar for 200%, and there are clear differences for damping values equal or less than critical; the discrepancy between the two solutions would increase for lower values of damping or impact velocity. Also, whereas the force for the mass-spring-damper system ceases at a finite time (the mass rebounds), the force in the case of a mass-damper system tends asymptotically to the (normalized) weight of the mass, and the system creeps at constant velocity.
Fig. 7. Contact Force: Full Solution versus Mass-Damper Solution; Impact Velocity $\dot{u}_0/gT = 100$

At the opposite extreme is the case of a lightly damped system; in this situation, the maximum force occurs (approximately) at maximum deformation of the spring, and can be estimated by equating the kinetic energy on impact with the strain energy in the spring. When this is done, one obtains

$$F_{\text{max}} = \frac{\dot{u}_0}{g} = \frac{\sqrt{km}}{c} = \frac{1}{2\beta}$$

Fig. 8 shows the maximum contact force as a function of damping; each point in this figure is obtained by searching (7) in time, and choosing an impact velocity such that the ratio of dimensionless impact velocity to damping remains constant (this strategy is equivalent to changing the mass of the system while keeping all other parameters constant). Also shown as a dashed line in this figure is (17). When the velocity-damping ratio exceeds a value of about 10 (which is a likely situation), the maximum force becomes insensitive to this parameter and exhibits a bimodal behavior: for damping larger than 0.5, the maximum force is simply the initial force in the dashpot, and for values of damping less than 0.5, it is described closely by (17). On the other hand, larger values of the velocity-damping ratio (e.g., 1,000), lead to results that are virtually identical to those for 100.

COEFFICIENT OF RESTITUTION AND REBOUND ENERGY

The (absolute) ratio of the rebound velocity and the impact velocity is normally referred to as the coefficient of restitution. A numerical evaluation
of this number is shown in Fig. 9 for a range of damping values. It can be observed that this coefficient is not very sensitive to the impact velocity, except in the immediate neighborhood of the rebound limit at low velocities. Thus, a good estimate for the coefficient of restitution can be obtained by solving for $C$ and $S$ from (5) and (11), substituting the result obtained into (4b), dividing by the initial velocity, taking the limit when this velocity is infinitely large, and changing the sign to account for the fact that the rebound velocity is negative. When this is done, the following expression is obtained:
For example, when damping is critical, the minimum contact time is $2 \frac{\pi}{2} \beta [(11b)]$. It follows that $R = e^{-2} = 0.135$. More generally, a numerical evaluation of (11) and (18) for a range of damping values yields the results shown in Fig. 10.

The fraction of energy actually retained by the mass is not quite equal to the square of the coefficient of restitution because the mass rebounds from a different position on impact; thus, its potential energy is different. The actual energy ratio is as follows:

$$
\epsilon = \frac{\frac{1}{2} v_r^2 - \delta_r}{\frac{1}{2} v_0^2 - \delta_0}
$$

(19)

in which the terms representing the potential energy have negative signs because the displacements are positive downwards. Introducing the condition of zero initial displacement and expressing the displacement at rebound in terms of the velocity at rebound [see (9b)], one obtains

$$
\epsilon = \frac{v_r^2 + 4\beta v_r}{v_0^2} = R^2 - \frac{4\beta R}{v_0}
$$

(20)

which is somewhat less than $R^2$. Thus, when damping is near critical or greater, the fraction of energy transmitted from the mass to the ground (i.e., $1 - \epsilon > 1 - 0.135^2$) is greater than 98%.

Clearly, these computations of the coefficient of restitution are based on a purely elastic solution. Real materials, on the other hand, always experience inelastic effects, a situation that becomes more pronounced as the
impact velocity increases. Hence, rebound velocities during actual testing can be expected to be less—perhaps much less—than those predicted by Figs. 9 and 10.

**IMPULSE APPLIED TO THE GROUND**

The equation of impulse (i.e., the principle of conservation of momentum) for this mechanical system—assuming that it rebounds—is

$$\int_0^{t_d} F(t) \, dt = \int_0^{t_d} (F - mg) \, dt + mg t_d = m(\dot{u}_0 + \dot{u}_r) + mg t_d \quad (21)$$

in which the rebound velocity is taken positive upwards; thus, the first term on the right-hand side represents the total change in momentum. Notice that the left-hand side of this equation is the same as the static (i.e., zero frequency) component of the Fourier transform of the force time history. Normalization of this equation by the appropriate parameters yields the following result:

$$\int_0^{t_d} \varphi(\tau) \, d\tau = \frac{1}{2\beta} \left( 1 + R + \frac{\tau_d}{v_0} \right) \quad (22)$$

which establishes a relationship between the normalized impulse, the coefficient of restitution, the fraction of damping, the duration of contact, and the impact velocity. A numerical integration of (7) and computation of $R$ via (22) yields results that are virtually indistinguishable from those in Fig. 9.

Fig. 11 shows the normalized impulse [i.e., (22)] as a function of the damping ratio, choosing as before the independent parameter as the ratio of the dimensionless velocity to the damping; as can be seen, all curves tend

![Diagram showing the relationship between force impulse and damping ratio](image-url)
to a common asymptote—shown as a dashed line—which corresponds to the first term in (22). The impulse appears to be insensitive to the velocity-damping parameter when the latter exceeds a value of about 50. The normalized impulse for high velocity and low damping is essentially a line parallel to the asymptote at twice the value, since \( R \) approaches unity when damping is decreased. Curves for values of the velocity-damping ratio less than 2 (approximately) are not shown because the system no longer rebounds and the integral in (22) does not exist (the force approaches the weight of the mass, and the integral diverges).

**EXAMPLE OF APPLICATION—CYLINDRICAL MASS**

Consider a cylindrical mass impinging on an elastic half-space. Following Lysmer (1965) and Verbie and Veletsos (1972), the half-space can be modeled in the low-frequency range by means of a spring, a mass, and a dashpot. The presence of a participating mass of soil in this model introduces a minor difficulty in that the mass has inertial properties only, but is not subjected to gravity. This means that (1) should be replaced by

\[
(m + m_s)\ddot{u} + c\dot{u} + ku = mg
\]

At first glance, it would seem that the solution to this equation could simply be obtained from the equations previously derived by writing it in the form of (1), which is achieved by multiplying and dividing the right-hand side by the total mass, and redefining the acceleration of gravity as \( g' = gm/(m + m_s) \). In addition, the initial velocity of this system would have to be reduced by this same mass ratio, on account of the principle of conservation of momentum. With such changes, the dimensionless solution (4) would indeed be preserved. This isomorphism, however, does not carry over to the contact force because the latter is the force between the drop mass and the participating mass of soil, not between the total mass and the spring-dashpot system. It follows that neither (6) nor (7), not giving the contact force, nor the equations and conclusions based on them (contact time, coefficient of restitution, impulse), apply in this case.

A more careful examination of this problem reveals, however, that the participating mass of soil cannot instantaneously acquire the initial velocity implied by the principle of conservation of momentum because the travel times of waves within the active soil bulb are comparable to the contact time of the weight and the soil (i.e., the participating mass of soil is not ideally rigid). Hence, the very concept of a participating mass of soil in the context of a falling weight problem is dubious. For this reason, and because of the many uncertainties inherent in modeling the actual physical phenomenon, the participating mass of soil is best left out.

Omitting the participating mass of soil, the relevant parameters for the problem considered are as follows:

\[
k = \frac{4Gr}{1 - v} = \frac{4\rho_s C_s^2 r}{1 - v}; \quad \text{(half-space spring)} \tag{24a}
\]

\[
c = \frac{k r}{C_s}; \quad \text{(half-space dashpot)} \tag{24b}
\]

\[
m = \pi r^2 h p; \quad \text{(mass of cylinder)} \tag{24c}
\]

where \( G, C_s, \rho_s, v, r, h, \) and \( \rho \) = the shear modulus, shear wave velocity,
soil mass density, Poisson’s ratio of the soil; and the radius, depth, and density of the cylinder, respectively; also, \( \vartheta = a \) a numerical coefficient that depends on Poisson’s ratio for the soil (generally it has a value in the vicinity of 0.75). From these parameters, it follows that

\[
\beta = \vartheta \frac{1}{\pi (1 - v)} \frac{\rho_s}{\rho} \frac{r}{h}
\]

This formula demonstrates that damping depends only on the aspect ratio and mass-density ratio of the falling weight; hence, an increase in the size of the cylinder does not lead to changes in damping. Since, in most practical cases, the dropped weight has a diameter comparable to the depth, it follows that the aspect ratio \( r:h \) is of order 0.5; then using reasonable values for the other parameters in this formula and considering only solid cylinders, it becomes clear that damping cannot be too large (say, less than 50% of critical). It can be expected that this finding holds true also for other geometric shapes of the dropped weight.

If the cylinder drops from a height \( H \) above the ground, then the impact velocity is \( u_0 = \sqrt{2gh} \); it follows that

\[
\frac{u_0}{gT} = \frac{v_0}{2\pi} = \frac{1}{\pi} \sqrt{\frac{2}{\pi (1 - v)}} \frac{C_s}{\sqrt{gr}} \sqrt{\frac{\rho_s}{\rho}} \sqrt{\frac{H}{h}}
\]

The factors other than the shear wave velocity in this formula typically evaluate (in metric units) to a number of order unity (in s/m), so that the dimensionless velocity is of the order of the shear wave velocity. This means that the dimensionless velocity is typically of order 100 or greater, that is, it is large. Again, this situation can be expected to remain valid for other shapes of the falling weight.

For example, consider a soil with a shear wave velocity of 200 m/s, a Poisson’s ratio equal to 0.33, \( \vartheta = 0.75 \), and a cylinder with radius \( r = 0.20 \) m and depth \( h = 0.40 \) m that is dropped from a height \( H = 5 \) m, and assume that the mass densities of the cylinder and soil are 2,500 kg/m\(^3\) and 2,200 kg/m\(^3\), respectively. Thus, the drop weight has a mass of 125.7 kg, the impact velocity is 9.9 m/s, the undamped period is \( T = 0.007 \) s, the fraction of critical damping is 34%, the coefficient of restitution is \( R = 0.42 \) (i.e., 82% of the impact energy is transferred), and the dimensionless impact velocity is \( v/gT = 147 \). Since the latter number is high, it follows that the contact time can be obtained from Fig. 5, which yields \( t_d = 0.4T = 0.003 \) s. Figs. 12 and 13, on the other hand, show for this case the time history of the contact force and the amplitude of its Fourier spectrum [which is of primary interest when evaluating numerically the displacements on the ground’s surface by means of Green’s functions such as those given by Kausel (1981)]; also in these figures, are the computations for a simple mass-damper model given by (16), which leads to noticeably different results (although the order of magnitudes are similar). As can be seen, the maximum force does not occur on impact but some time later, and is about 21% higher than the initial force.

Clearly, the computed numbers are only ideal reference values, not only because of the simple half-space model used and the assumption of linearity, but also because a falling weight is rarely of cylindrical shape; indeed, even if the shape were cylindrical, it would almost never hit the ground with its flat side perfectly parallel to the ground’s surface.
SELECTION OF DROP MASS AND DROP HEIGHT

The two parameters that can be selected, within reasonable limits, when dropping a weight onto the soil is the drop height and the drop mass. The height affects only the initial velocity, which is proportional to the square root of the height. Since in most practical cases, the dimensionless velocity is large and damping is moderate, it follows that the mass rebounds and the contact time is nearly independent of the drop height. Thus, the
initial force, the maximum force, and the amplitude of the Fourier spectrum of the force are proportional to the square root of the falling height—provided the system remains linear. Also, since displacements (i.e., ground vibrations) at some distance to the source—the impact point—depend linearly on the contact force, it follows that they grow with the square root of the falling height.

On the other hand, the size of the drop mass affects not only the inertia of the dynamic system, but also the stiffness and damping constants as well,
since a larger mass is generally associated with a larger contact area with the ground. These changes lead in turn to increases in the natural period and reductions in the fraction of critical damping. If the participating soil mass is zero and if the mass density and geometric shape of the falling weight remain constant, it follows that the system period is directly proportional, and the fraction of damping inversely proportional, to the cubic root of drop mass. In the range of large dimensionless impact velocities, both of these effects lead to increases in the contact time, which do, therefore, grow somewhat faster than the cubic root of the mass. If, however, it is the contact area that is preserved rather than the aspect ratio or density of the falling weight, then the cubic root proportionality changes into a square root proportionality. Either way, the initial force remains constant, but the reduction in damping is likely to increase—certainly increase, if damping is less than 50%—the maximum force. Thus, the increases in both contact time and the values of the maximum force contribute to an increase in the amplitude of the Fourier spectrum, and, ultimately, to increases in the amplitude of vibrations elicited in the ground at some distance from the source. Unlike the dropping height, however, which affects mostly the amplitude of the displacements and not their variation in time, the size of the mass has an effect on the spectral contents of the force and, therefore, on the vibration signatures. An indication of the growth with the drop mass of the low-frequency part of the force spectrum (of interest with regards to deeply penetrating waves) can be obtained by observing the static component of the Fourier spectrum, that is, the impulse.

Figs. 14, 15, and 16 illustrate these points for the example presented previously; they show the contact time, the maximum force, and the maximum Fourier amplitude, respectively, as the mass is increased by one order of magnitude while the contact area is preserved. As can be seen in Fig. 14, the contact time approaches one half of the system period, which in turn grows in proportion to the square root of the mass; also, the time at

\[ \text{Amplitude (N-sec)} \]

\[ \text{Mass (kg)} \]

**FIG. 16. Example of Application: Zero-Frequency Component of Fourier Transform versus Drop Mass**
which the maximum force is attained approaches one half of the contact
time, that is, one quarter of the period. Fig. 15 reveals that the maximum
force grows slightly faster than the square root of the mass. Fig. 16, on
the other hand, demonstrates an essentially linear dependence of the Fourier
amplitude of the contact force with the mass. Notice that when the mass is
increased over the range shown, the fraction of critical damping drops from
0.34 to 0.07, the coefficient of restitution grows from 0.42 to 0.8, and the
energy ratio decreases from 0.82 to 0.36. Thus, even though the kinetic
energy on impact increases by a factor of 20, the energy actually transferred
to the ground increases only by a factor of 8.8 (\(= 20 \times 0.36/0.82\)).

These results are consistent with the data presented previously in Figs. 8
and 9. Indeed, the dimensionless velocity-damping ratio for this case is 147/
0.34 = 432, a (rather high) number that remains constant as the mass is
increased. Also, damping is less than 0.34. It follows that the force is given
by the dashed line in Fig. 8 [i.e., by (17)], which implies \(F_{\text{max}} = u_0 \sqrt{km}\).
Also, the normalized impulse (Fig. 11) is essentially the reciprocal of damp-
ing (twice the asymptote), so the actual impulse is \(I = (1/\beta)(cu_0/\omega) = 2mu_0,\)
which does indeed grow linearly with the mass.

CONCLUSIONS

A simple mass-spring-dashpot model is used in the present paper to sim-
ulate the impact of a weight falling onto an elastic half-space, and to assess
the characteristics of the forces transmitted to the ground during a non-
destructive testing procedure, such as the SASW method. The results ob-
tained lead to the following general conclusions.

For most reasonable configurations in a falling-weight test, the dimen-
sionless impact velocity is high (say, above 100), and damping is moderate
to light (on the order of 50% or less). This implies in turn the following:
(1) A simple mass-dashpot model is not satisfactory; (2) the falling weight
always rebounds (may not be true if inelastic collision takes place); (3) the
contact time is of the order of one half the interaction period; it is certainly
greater than one half the fifth root of the dimensionless
impact velocity; and (4) the coefficient of restitution has a probable value
of about 1/3 (less if inelastic effects are considered).

The drop height affects only the impact velocity and the amplitude of the
contact force, both in proportion to the square root of the drop height. It
does not affect the contact time, the damping or the variation in time of
the response. Hence, the amplitude of vibrations elicited (and perhaps re-
corded) in the ground also increase with the square root of the drop height.

Increases in the drop weight reduce the damping and increase the coef-
icient of restitution; thus, the energy actually transferred to the ground
grows at a rate slower than the mass. While inelastic processes at the point
of impact would increase the fraction of energy released by the mass, that
additional energy would not necessarily be converted into wave energy, and
would thus not be available for testing.

The system period and the contact time are directly proportional, and
the fraction of damping inversely proportional, to the cube root of the mass
(assuming constant aspect ratio and mass density).

The maximum contact force can be estimated as the initial force (i.e.,
the force in the dashpot) times 1 or the reciprocal of twice the damping,
whichever is greater. This means that for damping ratios less than 0.5, the
maximum force grows as the 2/3 power of the mass; the initial impact force,
however, grows only as the cubic root of the mass.
The impulse and low-frequency components of the Fourier spectrum of the force grow essentially linearly with the mass (the force amplitude contributes a 2/3 power dependence, and the contact time the remaining 1/3). This implies that amplitudes of displacements and vibrations induced on the ground’s surface at some distance from the source point grow roughly in proportion to the drop weight.

At high dimensionless impact velocities, the time variation of the contact force is very nearly triangular in shape (Fig. 6). It follows that, to a first approximation, the amplitude of the Fourier transform of the contact force varies as the momentum, that is, linearly with the mass; also, the frequency axis contracts in proportion with the reciprocal of the contact period, in other words, it contracts with the cube root of the mass.

ACKNOWLEDGMENT

This research was carried out during the sabbatical stay of the first two writers in the fall of 1992 at CEDEX, the Center for Experimental Research and Studies, Department of Public Works, Madrid, Spain. The writers would like to thank the Directorate General of Scientific and Technical Research of the Spanish Department of Education and Science, and the senior administration at CEDEX for their support.

APPENDIX. REFERENCES