Modelling hysteretic damping

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ABSTRACT: To simulate the loss of energy through hysteresis due to nonlinear soil behavior it has become accepted practice to use linear hysteretic damping, which is frequency independent, adjusting its value through a series of linear iterative analyses, on the basis of some characteristic strain. While true hysteretic damping is indeed frequency independent for the same level of steady state response in each frequency, the response of a system does not have the same amplitude at all frequencies. As a result the linear hysteretic model commonly used tends to filter excessively high frequency components in convolution (soil amplification) analyses. For deconvolution analyses, to obtain compatible motions at some depth which will produce a desired result at the surface, this leads to instabilities as the depth increases. The nature of this problem is investigated in this paper comparing the results of true nonlinear analyses in the time domain with those of the linear iterative analyses.

1 INTRODUCTION

Damping has been traditionally represented by viscous dashpots, with a resistance which is linearly proportional to the rate of deformation (relative velocity between the nodes they connect), because this model leads to sets of linear differential equations (ordinary or partial differential equations) which can be easily solved in closed form for simple analytical forms of the dynamic excitation. This type of damping dissipates a fraction of the total strain energy of the system in each cycle of steady state vibration which is linearly proportional to the frequency. It would be characteristic of linear viscoelastic materials with a constant viscosity coefficient. Material damping in soils and structures is associated however with nonlinear hysteretic behavior and the resulting loss of energy, through hysteresis loops, is a function of the amplitude of the steady state vibration but not of the velocity with which the loop is traversed. The fraction of the strain energy which is dissipated per cycle is thus independent of frequency for a fixed amplitude of vibration. It is not, however, independent of the strain rate which involves both amplitude and frequency. To simulate this type of energy losses it has become customary to use linear hysteretic damping, which is only properly defined for steady state vibration (thus in the frequency domain) and which is independent of both frequency and amplitude. To account for the amplitude dependence in the real system iterative analyses must be conducted computing the amplitude of vibration (actually the strain amplitude) and adjusting the value of the damping accordingly for the next iteration. For a single frequency (monochromatic) excitation and steady state response the maximum strain amplitude is used to define the level of nonlinear behavior and thus the appropriate values of the equivalent stiffness (shear modulus) and damping. For transient excitations, where multiple frequencies are involved, a characteristic strain, which is typically taken as two thirds of the maximum strain, is used.

While this approach is reasonable when dealing with a steady state response of a single frequency, the problem becomes more complicated when studying the response at a multidegree of freedom system with several frequencies involved. This is the situation encountered for instance in soil amplification studies to obtain site specific design motions at the free surface of a soil deposit which are consistent with some specified earthquake input at rock outcropping or on a different type of soil (convolution analyses), or when it is desired to obtain compatible motions at some depth of the soil profile, consistent with a desired earthquake input at the free surface, to perform soil structure interaction studies (deconvolution process).

The use of constant, frequency independent, linear hysteretic damping in relation to soil amplification, or convolution, analyses tends to filter excessively the high frequency components of motion which have smaller amplitudes but are assigned the same amount of damping as the fundamental frequency of the soil deposit. Starting with a realistic seismic input at the outcropping of rock one would end up with a motion at the free surface of a soil deposit,
particularly soft and deep soil deposits, which may not have any energy above 8 or 10 Hz. These are clearly unrealistic motions which defy actual observations. When performing, on the other hand, deconvolution, if one starts with a realistic earthquake record at the free surface and proceeds gradually down the soil profile the high frequency components are steadily amplified and at a certain depth (function of the soil properties) the solution becomes unstable. To avoid this problem in practical applications the frequency components of the surface motion above 8 or 10 Hz are sometimes arbitrarily suppressed.

The objective of this paper is to illustrate some of the limitations involved in dynamic analyses with linear hysteretic damping, particularly in the context of soil amplification studies, where this has led in the past to questionable assumptions and simplifications. Since hysteretic damping is intended to simulate nonlinear material behavior, the exact solution (within the accuracy allowed by truncation and round-off errors) would be obtained performing nonlinear analyses in the time domain. To illustrate the problem and the approximation introduced by the use of linear hysteretic damping, a close-coupled multidegree of freedom system simulating a shear beam or a soil deposit subjected to vertically propagating shear (SH) waves will be considered. Analyses will be carried out both in time domain with nonlinear springs and in the frequency domain using the iterative linear approach. The same discrete model will be used for both sets of analyses for consistency and the curves relating the variation of the stiffness and damping to the level of shear strain for the iterative analyses will be those corresponding to the nonlinear springs used in the time domain solution.

2 FORMULATION

A soil profile with a depth of approximately 30 m (100 ft), a shear wave velocity of 240 m/sec (800 ft/sec) and a mass density of 1900 kg/m$^3$ (a unit weight of 120 lb/ft$^3$ ft) was assumed for all the studies. The soil layer was subdivided into ten sublayers and each one was represented by a shear spring. The corresponding multidegree of freedom system thus had ten masses and ten springs. The top mass was equal to 2850 kg and the other nine were 5700 kg. Each spring had an initial stiffness (in the linear elastic range) of 36.5 MPa. Initial analyses conducted with a single degree of freedom model increased the shear wave velocity by 11% to ensure the same fundamental natural frequency of 2Hz for low levels of strain. In this case, the mass was 28,500 kg and the initial spring constant 4.5 MPa.

The variation of the shear modulus with level of strain for the selected material was given by the values of $G/G_{\text{max}}$ versus $\gamma$ listed in table 1 ($G_{\text{max}}$ is the initial shear modulus) which correspond to an actual soil tested by K.H. Stokoe at the

<table>
<thead>
<tr>
<th>$\gamma \times 10^6$</th>
<th>$G/G_{\text{max}}$</th>
<th>D</th>
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<tbody>
<tr>
<td>5.35</td>
<td>1.000</td>
<td>0.0</td>
</tr>
<tr>
<td>10.35</td>
<td>0.991</td>
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<td>19.42</td>
<td>0.973</td>
<td>0.007</td>
</tr>
<tr>
<td>40.88</td>
<td>0.925</td>
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<tr>
<td>99.33</td>
<td>0.824</td>
<td>0.035</td>
</tr>
<tr>
<td>170.70</td>
<td>0.727</td>
<td>0.063</td>
</tr>
<tr>
<td>304.20</td>
<td>0.623</td>
<td>0.083</td>
</tr>
<tr>
<td>569.20</td>
<td>0.505</td>
<td>0.109</td>
</tr>
<tr>
<td>1201.00</td>
<td>0.371</td>
<td>0.142</td>
</tr>
</tbody>
</table>

Each nonlinear spring was modelled in the time domain analyses by nine elastoplastic springs in parallel, selected so as to provide the same variation of the stiffness with level of deformation. The values of damping corresponding to the set of springs are given in table 1. They were slightly different from those determined experimentally, a situation often encountered in practice (a multilinear or even a curvilinear spring which reproduces exactly the variation of the shear modulus measured in the laboratory, will not exactly match the measured values of damping).

Equivalent linear analyses are normally performed using a continuous formulation as presented for instance by Roeset and Whitman (1969) and implemented with the iterative scheme in program SHAKE (Schnabel et al 1972). To ensure complete consistency between the models used in the nonlinear and the iterative linearized solutions the same discrete model was used here in both cases. For the iterative analyses the values of the relative stiffnesses and the corresponding damping were the ones consistent with table 1 (they were actually computed for each value of the spring distortion using the set of 9 elastoplastic springs).

The solution in the time domain was carried out through a step by step integration of the equations of motion using the central difference formula. The iterative analyses were carried out in the frequency domain. Starting in each case with the initial material properties corresponding to very low levels of strain a complete solution was obtained. The time histories of the deformations of each spring were computed using the Fast Fourier transform. For harmonic steady state excitations the amplitude of the deformation, once a steady state response had been reached, was used as characteristic strain. For transient analyses using an earthquake record, the characteristic strain was selected as two thirds of the maximum computed value. The spring stiffnesses and the damping were then adjusted.
correspondingly for the next iteration. The damping was introduced through the use of a complex modulus of the form G(1+2id). The iterations continued until the maximum strains in all the springs differed by less than 5% in two consecutive cycles.

3 SINGLE DEGREE OF FREEDOM SYSTEM

To better visualize some of the characteristics of the nonlinear response, a single degree of freedom model of the complete soil deposit was studied first. As stated earlier, the natural frequency of the system for very low levels of strain was 2Hz. The system was subjected to a harmonic, single frequency base motion of the form

\[ \ddot{u}_0 = 0.3 A \sin 2\pi ft \text{ m/sec}\]

where \( f \) is the frequency of the excitation in Hz and \( A \) is a scale factor.

Figure 1 shows the time variation of the mass acceleration (the acceleration at the free surface of the soil deposit) resulting from the nonlinear analysis and Figure 2 the corresponding results for the iterative approach with linear hysteretic damping for a frequency \( f = 1 \) Hz and a scale factor of 5. The response predicted by the linearized solution is slightly higher (by about 20%) than the nonlinear results for this particular case. Figures 3 and 4 show the corresponding Fourier amplitude spectra computed using the part of the response records between 5 and 15 seconds to guarantee that a steady state condition had been reached. Both spectra show a similar peak at the excitation frequency of 1 Hz. This is the only nonzero component of the response for the equivalent linear system. The nonlinear system exhibits on the other hand a second smaller peak (about 10% of the other one) at a frequency of 3 Hz and even smaller peaks at 5, 7, 9 Hz, etc. Because the nonlinearity is associated with the material properties, one would expect to see components at the third harmonic (three times the excitation frequency) and other odd numbered multiples of the input frequency. While the contribution of these higher order harmonics to the total response (the maximum ground acceleration) is small, they could affect more significantly the response of structure with a natural frequency of 3f resting on top of the soil. Figure 5 shows the final amplification function used in the iterative solution. It can be seen that the effective natural frequency of the system has been reduced to 1 Hz. The maximum amplification at this frequency is about 1.71, which would imply an effective damping of 29% (for this level of excitation all the elasto-plastic springs have reached their yield level; the maximum acceleration is simply the yield force divided by the mass). Figure 6 shows finally the variation of the maximum response acceleration with the scale factor \( A \) for the two models.

4 MULTIDEGREE OF FREEDOM MODEL

The multidegree of freedom model of the soil layer with ten masses and springs was subjected to the same harmonic excitation of the previous case. Figures 7 and 8 show the time history of the absolute acceleration of the top mass (motion at the free surface of the soil) for the nonlinear and the iterative linear analyses. Figures 9 and 10 show the corresponding Fourier amplitude spectra. It can be seen that the maximum acceleration from the true nonlinear analysis is now somewhat lower (30 to 50%) than that of the linearized solution. More importantly, the nonlinear response contains a substantial amount of high frequency energy (at three times the frequency of the excitation).

Figures 11 and 12 show the absolute acceleration of the top mass (the seismic motion at the free surface of the soil deposit) due to an earthquake at the base simulating a white noise multiplied by a trapezoidal envelope with a linear portion increasing from 0 to 1 over the first 5 seconds, a constant value of 1 from 5 to 15 seconds and a descending ramp (from 1 to 0) from 15 to 20 seconds. The maximum base acceleration is approximately 0.1 g. Figures 13 and 14 again show the corresponding Fourier amplitude spectra using the segment of the response from 5 to 15 seconds. The maximum acceleration is again higher for the nonlinear solution. It can be clearly seen that the iterative solution reduces excessively the high frequency components of the response.

REFERENCES

Figure 1: Nonlinear time history response (SDOF, 1Hz)

Figure 2: Iterative linear history response (SDOF, 1Hz)

Figure 3: FFT for nonlinear time history response (SDOF, 1Hz)

Figure 4: FFT for iterative linear history response (SDOF, 1Hz)

Figure 5: Transfer function for iterative linear response (SDOF, 1Hz)

Figure 6: Peak value response versus Scale factor (SDOF, 1Hz)
Figure 7: Nonlinear time history response (MDOF, 1 Hz)

Figure 8: Iterative linear history response (MDOF, 1 Hz)

Figure 9: FFT for nonlinear time history response (MDOF, 1 Hz)

Figure 10: FFT for iterative linear history response (MDOF, 1 Hz)
Figure 11: Nonlinear time history response (MDOF, EQINP)

Figure 12: Iterative linear history response (MDOF, EQINP)

Figure 13: FFT for nonlinear time history response (MDOF, EQINP)

Figure 14: FFT for iterative linear history response (MDOF, EQINP)