
Athens, Greece, 1993

Determination of the Elastic Properties of a Miocene Marl by the Spectral Analysis of Surface Waves

by

Vicente Cuéllar and José Luis Monte
Centro de Estudios y Experimentación de Obras Públicas, Madrid, Spain

Eduardo Kausel
Massachusetts Institute of Technology, U.S.A.

José Manuel Roësset
The University of Texas at Austin, U.S.A.
Determination of the Elastic Properties of a Miocene Marl by the Spectral Analysis of Surface Waves

Cuéllar, V. and Monte, J.L.
Centro de Estudios y Experimentación de Obras Públicas, Madrid, Spain

Kaussel, E.
Massachusetts Institute of Technology, Cambridge, U.S.A.

Roesset, J.M.
The University of Texas at Austin, U.S.A.

ABSTRACT: In this paper a discussion is made about the use of the Spectral Analysis of Surface Waves for defining the elastic modulus of a soft rock around the concrete lining of a hydraulic tunnel. After a brief geological description of the rock mass, a summary is given of the tests performed in the laboratory to characterise the material and the technique used to collect data from inside the tunnel is presented. Following this, an explanation is provided of the procedure used for defining shear wave velocities around the tunnel from those data. Finally and on the basis of a comparison between 3-D solutions for the propagation of surface waves in horizontally layered systems and in tunnels, considerations are made on the validity of the interpretation method used.

1. INTRODUCTION

The Alcubierre Tunnel, constructed 30 years ago and located in North-East Spain supplies the whole southern area of the Alcubierre Sierra with water.

The 5,169 m. long tunnel runs through a Miocene formation of gypseiferous marl with a maximum rock cover of 90 m. The tunnel is oval, having arches with three different radii: 4.00 m. at the roof, 11.50 m. at the bottom and 2.00 m. at the side walls, there being a height and a free span of 5.50 and 7.75 m., respectively. The concrete lining has an average thickness of 0.60 m. (see Fig. 1).

The lining has deteriorated in certain places, as a result of being attacked by the water, high in sulphate content, that flows in the rock mass. With a view to repairing the structure, a zonification study of the tunnel was undertaken in which ten different sections were checked by drilling 2 and 3 m. deep and 100 mm. in diameter boreholes into the tunnel walls, so that the state of the concrete and the rock mass around the tunnel could be analyzed. In addition to laboratory tests, con-
2. GEOLOGY

The Alcubierre Sierra lies in the Ebro Basin, which was formed by tectonic subsidence during the Lower Tertiary Period.

The lacustrine valley is covered with both detrital and evaporitic sediments which, in the tunnel area, constitute a fine-grained gypiferous marl from the Miocene Epoch.

The depression is generally flat, but fluvial erosion has been extremely active on these materials which, being unconsolidated, were easily eroded, giving rise to the formation of the gullies and deep cut banks that characterize the Alcubierre landform.

3. LABORATORY TESTS

The tests, conducted with rock samples extracted "in situ" were mainly devised to determine their density, strength and aggressivity to concrete. The cores which were fine-grained, had a clayey matrix of low plasticity, with a Liquid Limit ranging from 24 to 44 and a Plasticity Index lying between 20 and 22 (see Fig. 2).

The water content of the samples ranged from 3% to 15% and the dry density values varied from 20 to 23 KN/m³.

Table 1 shows the results of the unconfined compression tests conducted on rock samples obtained from inside the tunnel at depths of between 1.5 and 2.5 m. The Young moduli indicated in the Table, were estimated for a stress level below 2 MN/m². As can be seen from the Table, it is a soft rock whose strength and elastic modulus are highly variable, depending on the degree of moisture.

Table 1. Geotechnical parameters of the Alcubierre marl.

<table>
<thead>
<tr>
<th>Water Content (%)</th>
<th>Dry Unit Weight (MN/m²)</th>
<th>E (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>23</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
<td>5</td>
</tr>
</tbody>
</table>

The concrete cores analyzed, showed an average apparent density value of 2,340 Kg/m³ without there being appreciable differences between the determinations carried out on the samples obtained from the roof and from the bottom. The average unconfined compressive strength obtained in cylindrical samples has been 25 MN/m².

4. SPECTRAL ANALYSIS OF SURFACE WAVES

In the last decade, Stokoe and his co-workers at Texas University, Austin, introduced the Spectral Analysis of Surface Waves (S.A.S.W.), into the Geotechnical Engineering field, see Heisey, Stokoe, Hudson and Meyer (1982), Stokoc and Nasarian (1984) and Sanchez Salinero (1987). This technique has been used in Spain by Cuéllar (1988, 1992), Valerio (1990) and Otoe (1992).

The tests are conducted by placing two receivers with preselected spacing, on the ground surface. A vertical impulse is then applied to the surface and this generates a transient signal F(t)
containing a certain range of frequencies. The wave group is monitored by the receivers and captured in the time domain as $X_1(t)$ and $X_2(t)$ signals, (see Fig. 3). The coherence function $\gamma^2(f)$ and the phase $\phi(f)$ of the cross spectrum $G_{x_1x_2}$ of both signals are then calculated, using a spectra analyzer (see Fig. 4).

![Fig. 3. Input and receivers signals in SASW tests](image)

The coherence function $\gamma^2(f)$ and the phase $\phi(f)$ of the cross spectrum $G_{x_1x_2}$ of both signals are then calculated, using a spectra analyzer (see Fig. 4).

$$\gamma^2(f) = \frac{G_{x_1x_2}(f)G_{x_1x_1}(f)}{G_{x_1x_1}(f)G_{x_2x_2}(f)}$$  \hspace{1cm} (1)

$$G_{x_1x_2}(f) = X_1(f)X_2^*(f)$$  \hspace{1cm} (2)

where:

$$G_{x_1x_1}(f) = X_1(f)X_1^*(f)$$  \hspace{1cm} (3)

$$G_{x_2x_2}(f) = X_2(f)X_2^*(f)$$  \hspace{1cm} (4)

and $*$ stands for the complex conjugated.

For each frequency $f$, $\phi(f)$ represents the phase difference between signals $X_1(t)$ and $X_2(t)$.

Accordingly, the transmission time $t$, between receivers for a wave of frequency $f$, will be:

$$t = \frac{\phi(f)}{360f}$$  \hspace{1cm} (5)

where $\phi(f)$ is given in degrees and $t$, in seconds.

By knowing the distance "d" between the receivers, the velocity $c$, with which each frequency is transmitted from one to the other, can be calculated straightforwardly:

$$c = \frac{d}{t}$$  \hspace{1cm} (6)

That velocity corresponds to a wavelength $\lambda$ such that

$$\lambda = \frac{c}{f}$$  \hspace{1cm} (7)

Then, a curve relating $c$ with $f$ or $c$ with $\lambda$, the so called dispersion curve, is obtained by using Eqs. (5), (6) and (7) with different frequencies.

The coherence function, $\gamma^2(f)$, is used for selecting the frequency range applicable to the calculation procedure represented in the equations above. This is possible because in the frequency domain such a function plays the same role as the squared value of the correlation coefficient of two variables in the time domain. So one gets:

$$0 < \gamma^2(f) < 1$$  \hspace{1cm} (8)

In Eq. (8) $\gamma^2(f)$ will be 0 when signals $X_1(t)$ and $X_2(t)$ are not correlated, and it will take a
value 1, when they are linearly dependent. Usually, a value of $\gamma'(f)$ equal to or greater than 0.9 is required.

5. INTERPRETATION PROCEDURES

To backcalculate the material properties from the experimental dispersion curves obtained with the Spectral Analysis of Surface Waves method, one assumes a soil profile, conducts an analytical study to obtain the theoretical dispersion curve corresponding to that profile, compares this theoretical curve to the experimental results, introduces appropriate modifications to the profile changing the properties and thicknesses of the different layers, and repeats the process iteratively until satisfactory agreement is reached.

As described by Roesset, Chang and Stokoe (1991) there are two alternative ways to determine the theoretical dispersion curve for a given soil profile, one based on the assumption of two-dimensional plane Rayleigh waves, considering only one mode of propagation, which is normally the fundamental one, and the second based on a three-dimensional solution computing the displacements caused by a dynamic load acting on the surface of a layered half-space using the Green's functions derived by Kausel (1981). The first approach is clearly more economical and faster while the second is more accurate and reproduces better the actual field conditions.

6. RESULTS OBTAINED

Figs. 5, 6 and 7 show the dispersion curves obtained from different parts of the tunnel by repeatedly striking on the lining with a hand vibrator. Thus, Fig. 5, for instance, represents the dispersion curve obtained from the bottom of the tunnel at section PK-5900. This section is in a zone where the groundwater table, due to the lack of a suitable drainage, remains slightly above the lining at the bottom of the tunnel. By contrast, Figs. 6 and 7 show other tunnel zones where no groundwater level was detected from the inside.
By comparing Figs. 6 and 7, it can be seen that both may be roughly represented by the same smoothened dispersion curve indicated by the continuous line in Fig. 8. The curve with the dotted line in that figure, represents the 3-D solution corresponding to the layer system indicated in Fig. 9.

![Dispersion curve](image)

**Fig. 8.** Dispersion curves from sections in dry ground

![Horizontally layered system](image)

**Fig. 9.** Horizontally layered system used in 3-D solution

As can be seen in Fig. 8, the theoretical solution thus obtained fits very well to the experimental curve, proving the existence of a 2 m. thick deteriorated layer of rock around the tunnel. Likewise, it can be seen that, using the geotechnical parameters given in Table 1 for the rock in a dry state, the value $v_s = 900$ m/sec. associated to that layer corresponds to a Young modulus of 4350 MN/m², 1.45 times greater than the one obtained in the laboratory with samples extracted from bore holes within that thickness range. That value is consistent with the one suggested by Deere, Hendron, Patton and Cording (1966), for rocks with RQD close to 100%. Lower values would have been obtained behind the lining when applying the same interpretation procedure to the dispersion curve in Fig. 5. This is also consistent with the fact that, as indicated in Table 1, the wetter the material the less the Young's modulus.

7. DISCUSSION

When applying the SASW technique in a tunnel one would use either of the two interpretation procedures previously discussed assuming a horizontally layered system. A question must be raised then as to the validity of this formulation given the tunnel geometry. One would expect that for very small wavelengths relative to the radius (or an equivalent radius) of the tunnel, the assumption of horizontal layers would still be valid. As the wavelengths approach or exceed the tunnel diameter the actual geometry of the problem should have, however, an effect on the results which is not well known or understood yet.

To assess the importance of this effect and the validity of the usual interpretation (or inversion) procedures when dealing with a tunnel a computer program was developed to simulate the actual experimental setup. The tunnel is modelled as a circular cavity in an infinite visco-elastic medium consisting of annular rings with constant properties in the circumferential (azimuthal) direction (see Fig. 10). The medium is discretized using finite elements in cylindrical coordinates with linear displacement expansions in the longitudinal and circumferential directions and a quadratic expansion in the radial direction. Following a formulation developed
by Kang (1990, 1991) the loads and displacements are expanded in a Fourier series in the longitudinal direction (actually a discrete Fourier transform from the space to the wavenumber domain) while a cloning procedure is used in the circumferential direction to achieve a very large number of elements with a relatively small computational effort. The solution provides for a given frequency the steady state displacements at equally spaced points along lines parallel to the axis of the tunnel. From these results one can then obtain the phase differences between the positions where the receivers would be located, the corresponding wave propagation velocity (phase velocity), and the wavelength. Each analysis for a given frequency provides thus one point of the dispersion curve.

A number of parametric studies were conducted to verify the accuracy of the solution and to provide guidance in the selection of parameters (mesh size, etc). In particular a similar solution was implemented in cartesian coordinates for horizontally layered systems and the results compared to those provided by the Green's functions (Sykora and Roesset, 1992).

In Fig. 11 a comparison has been made between the 3-D layer solution for the situation sketched in Fig. 9 and the 3-D tunnel solution for the case in Fig. 10. It can be seen that for wavelengths equal or less than the radius of the tunnel both solutions are practically the same.

8. SUMMARY AND CONCLUSIONS

In this work a gypsiferous marl that makes up the deteriorated zone around the Alcubierre tunnel has been investigated.

The material is a soft rock from the Miocene Epoch highly susceptible to moisture changes with Young's modulus determined in the laboratory ranging from 1800 MN/m² at a water content of 10% to 3000 MN/m² at a moisture of 3%.

The mechanical behavior of the rock in situ has been assessed by generating surface waves on the lining and finding the dispersion curves for both wet and dry zones in the tunnel.

From the interpretation of the dispersion curves, using a 3-D solution for horizontally layered
systems based on the Green's functions, a 2 m thick altered zone around the tunnel has been detected where the rock presents an elastic modulus 1.45 times greater than the one determined in the laboratory. The results obtained in that way have been compared with those provided by a 3-D solution for a tunnel with a circular cross section and they have been found to be equivalent for wavelengths equal or less than the radius of the tunnel.

The determination of the theoretical dispersion curve using the tunnel solution is much more expensive computationally than using the Green's function. One would expect therefore that the bulk of the analyses would be carried out in practice using the standard techniques and that the more accurate formulation would be used only to check the validity of the results at a relatively small number of frequencies.

ACKNOWLEDGEMENTS

This research was carried out during the stay of Professors E. Kausel and J. M. Roesset at the Public Works Research and Studies Center of the Spanish Department of the Environment and Transport in 1992. The authors would like to express their thanks to the Directorate General of Scientific and Technical Research of the Spanish Department of Education and Science for his contribution to support that stay.

REFERENCES


