TWO-STEP APPROACH IN SOIL-STRUCTURE INTERACTION: HOW GOOD IS IT?

by

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Introduction

The difficulty in performing soil-structure interaction analyses for complicated structures exhibiting three-dimensional geometry has motivated the frequent use of an approximate procedure that separates the computation into two steps (Fig. 1). In the first step, the three-dimensionality and structural detail is ignored, concentrating efforts instead on computing the response of an "equivalent" two-dimensional (coarse) model of the structure to the seismic environment, typically with finite elements. The resulting motion of the foundation (consisting of both translations and rotations) are then used in the second step as input to a refined three-dimensional model of the structure, assuming the support to be rigid (i.e., no soil-structure interaction). While this technique appears to be attractive for the potential savings in the analyses, it often produces erroneous results because of the inconsistencies in the dynamic properties of the two models used. In particular, spurious amplification peaks may be observed at the natural frequencies of the (detailed) structure that would not have developed if the analysis had been done in a single step. This phenomenon is explored below for the simple but illustrative case of shear beam modeled with springs and lumped masses.

Direct method vs. two-step approach:

Consider a cantilever shear beam (Fig. 2) supported on a single spring-dashpot system representing the flexibility of the soil. A unit harmonic excitation is prescribed at the support of the "springs", i.e., under the soil spring-dashpot system. The motion amplitude observed at a structural location, and in particular at the top of the structure,
Step 1
Coarse structural model
Detailed soil model

Seismic input

Step 2
Detailed structural model

Foundation motion from step 1 as input

Fig. 1

Discrete shear beam

Fig. 2

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is referred to as the transfer function (or frequency response function) for that location. If \( u_{gf} \) and \( u_{gs} \) are the transfer functions from the ground \((g)\) to the foundation \((f)\) and to the top of structure \((s)\) respectively, then it can be shown that

\[
    u_{gs} = u_{gf} \cdot u_{fs}
\]

where \( u_{fs} \) is the transfer function from the foundation, assumed fixed, to the top of the structure (Fig. 3). In essence, the justification for the two-step approach is embodied in equation (1); the determination of \( u_{gf} \) is obtained in the first step of analysis, while \( u_{gs} \) is computed in the second step, using as input a seismic excitation with amplitude \( u_{gf} \).

It can also be shown that the response of the foundation in this example is given by

\[
    u_{gf} = [k_f - \omega^2(m_f + \sum_j \gamma_j^2 a_j)]^{-1} \cdot k_f \cdot u_g
\]

where \( k_f = k + i\omega c \) is the complex soil stiffness, \( m_f \) = mass of foundation, \( \gamma_j \) = \( j \)th modal participation factor of the structure on fixed base for horizontal base motion \( u_q = u_1 \) (i.e., in the 1st or horizontal coordinate direction), \( \omega \) = excitation frequency, and

\[
    a_j = \frac{\omega_1^2 + 2i\beta_j \omega_1 \omega}{\omega_1^2 - \omega^2 + 2i\beta_j \omega_1 \omega}
\]

are the amplification functions for the structural modes (i.e., on fixed base) having natural frequency \( \omega_j \) and fraction of critical damping \( \beta_j \). More generally, the equation corresponding to equation (2) for systems of higher complexity involving multiple seismic inputs and/or degrees of freedom per nodal point is of the form

\[
    U_f = [K_f - \omega^2(M_f + \Gamma^T A \Gamma)]^{-1} K_f \cdot U_g
\]

with matrices \( K_f, M_f, \Gamma, A = \text{diag} \{ a_j \} \) and vectors \( U_f, U_g \) substituting for the scalars \( k_f, m_f, \gamma_j, a_j, u_{gf}, u_g \). The participation factors

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Fig. 3

\[ \frac{\\omega_1}{\\omega_x} = 1 \]
\[ \beta = 0.05 \]

\[ u_{qs} \]
\[ u_{fs} \]
\[ u_{gf} \]

Fig. 4

\[ \frac{\\omega_1}{\\omega_x} = 0.5 \]
\[ \beta = 0.05 \]

\[ n_1 = 5, n_2 = 20 \]
\[ n_1 = 5, n_2 = 5 \]
\[ n_1 = 20, n_2 = 20 \]

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matrix $\Gamma$ has up to six columns, depending on how many seismic components are prescribed under the springs. This matrix can also be written as

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \Gamma_j \end{bmatrix}$$ (5)

with $n$ being the number of degrees of freedom of the structure (i.e., the number of modes on fixed base), and the modal row-vector $\Gamma_j$ being given by

$$\Gamma_j = \left\{ y_{j1}, y_{j2}, \cdots, y_{j6} \right\}$$ (6)

Defining the $(6 \times 6)$ virtual modal mass matrices

$$\Delta M_j = \Gamma_j^T \Gamma_j$$

it follows that equation (4) is of the form

$$U_f = \left[ K_f - \omega^2 (M_f + \sum \Delta M_j a_j) \right]^{-1} K_f U_g$$ (7)

These virtual mass matrices can be shown to satisfy the relationship

$$\sum \Delta M_j = M_s$$ (8)

with $M_s$ being the $(6 \times 6)$ mass matrix of the structure, assumed rigid, relative to the coordinate point at which the impedances $K_f$ are attached. Thus, the complete mass matrix for the structure and the foundation would be $M_s + M_f$.

Equation (7) (with equation (2) being a particular case) describes the motion of the foundation for harmonic seismic inputs with amplitude $U_g$. It can be observed that the effective inertia of the system,

$$M_{\text{eff}} = M_f + \sum \Delta M_j a_j$$ (9)

depends on the amplification factors $a_j$; thus, it is a function of the
excitation frequency of the seismic motion, and the natural frequencies and modal damping values of the structure on fixed base. If the superstructure did not have any damping at all, then at each natural frequency the effective mass would become infinitely large. As a result, the foundation would remain exactly motionless at these frequencies; at other structural locations, the motion would have a finite amplitude which could be interpreted as resulting from the indeterminate product 0 · ∞ (motion of foundation × structural amplification). On the other hand, most real structures have some amount of structural damping of the order of 2% to 7% of critical in each mode. As a result, the effective mass is not infinitely large, although it is still substantial when compared with the actual mass of the structure. Hence, the transfer functions for the motion of the foundation will not display a zero value but a small value at each of the resonant frequencies of the structure. When this "small" value is multiplied times the "large" amplification function for the structure, one obtains the finite response of the structural masses, accounting for interaction, at the natural frequencies of the structure (Fig. 3). If, on the other hand, the structural frequencies are changed in the second step, as they must be if the structural model is changed, then the large amplification peaks for the altered structural model will not coincide with the valleys in the foundation response obtained with the original structural model. As a result, large spurious amplifications will be observed in the response of the hybrid system. Peaks will then be observed not only at the coupled interaction frequencies, but at the frequencies of the structure on fixed base as well. It should be observed that the amplitude of these spurious peaks is controlled by the structural damping only, and not by the effective soil-structure interaction damping (i.e., accounting for radiation). Further, they will develop even for biases in the structural frequencies as small as 5%, as will be shown. The actual bias in the frequencies of a solid block modeled, for example, with finite elements as opposed to those of a refined model with lumped masses and linear members, is likely to be much higher.

Example

To illustrate the effect of changing the structural model in the second step of analysis, consider the lumped mass shear beam, supported
on a single translational spring at the base, shown in Fig. 2. For a structure discretized into n masses (n+1 when counting the foundation), the inertia properties are

\[ M = mn \quad \text{(total mass)} \]

\[ S = \frac{mhn^2}{2} = \frac{MH}{2} \quad \text{(static moment)} \]

\[ J = \frac{mh^2n^3}{3} \left(1 + \frac{1}{2n^2}\right) = \frac{MH^2}{3} \left(1 + \frac{1}{2n^2}\right) \quad \text{(moment of inertia w.r. to base)} \]

The above formulae indicate that when changing the number of masses in the structural model it is not possible to match simultaneously the static moment (center of mass) and the moment of inertia; nevertheless, the error is small, since \(1/2 \ n^2 \ll 1\). Also, since this example does not involve rotation, this error is irrelevant.

On the other hand, the frequencies, modal shapes and participation factors for the superstructure on fixed support are

\[ \omega_j = 2 \frac{\sqrt{k}}{\sqrt{m}} \sin \theta_j \quad \text{, frequencies, rad/sec} \]

with \( \theta_j = \frac{\pi}{4n} (2j - 1) \quad \text{, } j = 1, 2, \ldots, n \)

\[ \phi_j = \frac{\sqrt{2}}{\sqrt{mn}} \begin{bmatrix} 1 \\ \cos 2\theta_j \\ \cos 4\theta_j \\ \vdots \\ \cos 2(n-1)\theta_j \end{bmatrix} \quad \text{modal shape} \]

\[ \gamma_{j1} = -(-1)^j \frac{m}{\sqrt{2n}} \cot \theta_j \quad \text{p. factor for base translation} \]

Also, the swaying frequency associated with a rigid structure is
$$\omega_x = \sqrt{\frac{k_x}{M}}$$

It is convenient at this point to introduce the following notation:

- $n_1$ = number of masses in structural model used in the first step to determine the motion of the foundation.
- $n_2$ = number of masses in structural model used in the second step to determine the motion at the top.

Thus, if $n_1 = n_2$, the model is consistent, and the motions computed with the two-step approach coincide with those of a direct evaluation. In general, when the model is "refined" in the second step, the number of masses increases, i.e., $n_2 \geq n_1$.

Fig. 4 shows the motion at the top of the structure for $\omega_1/\omega_x = 0.5$, $\beta = 0.05$ (structural damping), and three structural configurations: $(n1 = 5, n2 = 5)$, $(n1 = 20, n2 = 20)$, and $(n1 = 5, n2 = 20)$. Thus, the first two are consistent models, while the last is inconsistent. The stiffness of the "refined" model (i.e., $n2 = 20$) was adjusted so as to match the fundamental frequency $\omega_1$ of the "coarse" (i.e., $n1 = 5$) model. It can be observed that the results of the consistent models are in good agreement, while the inconsistent model shows discrepancies at higher frequencies. Thus, the refinement (change!) of the structure in the second step deteriorates rather than improves the computation.

The situation described above is the most favorable possible for the two-step approach with dissimilar structures, since the fundamental frequencies were matched exactly. In actual computations, no such exact match is accomplished, so that even the fundamental frequency may differ between steps. In fact, in some situations it may not even be possible to attempt a match, as when the nature of the structure is altered entirely. An example is the case of a coarse structure modeled first with finite element in two dimensions, and then refined to a structure with lateral and torsional coupling in three dimensions. The effect of a bias in the frequencies is illustrated in Fig. 5. Here, the motion was computed first with a fully consistent model ($n1 = 20, n2 = 20$);
Peak \{ - - - 14 \ (n1 = n2 = 20) \\
--- 25!! \ (n2 = n2^* = 20) \}

*Structural frequencies biased 5%.

\[ \frac{\omega_1}{\omega_X} = 0.5 \]

\[ \beta = 0.05 \]

Fig. 5
the computation was then repeated with a model having again 20 masses, but changing the stiffness of the structure in the second step so as to bias the frequencies by 5% (all frequencies are multiplied by the factor 0.95). It may be observed that the amplitude of the amplification peaks nearly doubles as a result of the bias. Even worse results are obtained when using models with a different number of masses and frequencies that do not match exactly.

Conclusions:

The results shown here, as well as further numerical experiments for other soil configurations (changing stiffness of soil spring, adding rocking spring, etc.), lead to the following conclusions:

a) The technique of refining the structure in the second step of the two-step approach deteriorates rather than improves the computations. It is then preferable to adhere to the coarse model throughout the analysis. The requirement on the consistency of the structural models may in some cases be as important or even more important than the need to consider soil-structure interaction, unless the structure has considerable damping.

b) For structures that must clearly be modeled in three dimensions, it may be necessary to use the impedance (soil springs) approach in connection with equation (4), or generalizations of two-dimensional finite element codes that are compatible with three-dimensional superstructures (in essence, an implicit impedance approach).

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