ON THE USE OF A SOIL BOX FOR LARGE SCALE TESTING OF SOIL-STRUCTURE INTERACTION EFFECTS

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ABSTRACT
This paper examines the feasibility of, and the scaling requirements for, the use of a soil box in a large-scale shaking table. Such a box would be intended for experimental studies on ground-structure interaction effects. There are two separate aspects to consider in the design of a soil box for testing soil-structure interaction effects with a shaking table. The first aspect is the seismic response of the soil alone, in the absence of any structure, which is generally known as the free-field problem. The goal consists in estimating or predicting the seismic environment at a given site in the absence of the structure to be erected, i.e. the intensity and spatial variation of the seismic motions at and near the surface. On the other hand, once this seismic environment is known (or presumed to be known), one can proceed to the second aspect of the seismic phenomenon, which is the analysis of the structure for soil-structure interaction effects, a phenomenon which is controlled—in addition to the dynamic rigidity of the soil-structure system—only by the seismic motions in the near-field i.e. the neighborhood of the interface between the soil and the structure before the latter is erected. The soil-structure interaction problem can in turn be described by two distinct phenomena: on the one hand is the kinematic interaction or wave passage, which results from the spatial variability of the seismic motions across the foundation interface; on the other is the inertial interaction, which results from the dynamic response of the superstructure above the ground and the inertial forces that it feeds back into the compliant soil. Both the free-field problem and the soil-structure interaction problem are important in seismic testing, although different scaling strategies may apply when attempting to study them in the laboratory. Clearly, the free field problem can be analyzed only with small-scale models; less obvious, but equally true is the fact that the soil-structure interaction problem also requires scaled models, not so much because of the structure, but because of the soil surrounding the structure. This practical restriction introduces a host of difficulties for testing, because the phenomenon is non-linear, and not all parameters can be scaled as desired or required. In this paper, both aspects of the seismic interaction problem are considered in turn.
INTRODUCTION
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of the soil alone, in the absence of any structure, which is generally known as the free-
field problem. This part of the phenomenon may require simulating in the laboratory a
complex seismic environment, such as an alluvial basin consisting of layered soils,
nearby rock outcroppings, and perhaps even topographic irregularities such as hills or
depressions. The goal is an estimation or prediction of the seismic environment at a given
site in the absence of the structure to be erected, i.e. the intensity and spatial variation of
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It is now well known that soil-structure interaction expresses itself through two
distinct phenomena: on the one hand is the kinematic interaction or wave passage, which
results from the spatial variability of the seismic motions across the foundation interface;
on the other is the inertial interaction, which results from the dynamic response of the
superstructure above the ground and the inertial forces that it feeds back into the
compliant soil. Both the free-field problem and the soil-structure interaction problem are
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attempting to study them in the laboratory, as will be seen. In this paper, both aspects will
be considered in turn.

MODELING THE FREE-FIELD
Model size
If the soil deposit —say an alluvial valley— has a "simple" geometry that consists of
horizontal layers underlain by crystalline rock, if there are no nearby rock outcroppings,
and if the motions are not severe enough to cause strongly inelastic deformations in the
ground, then it is possible to use analytical/numerical models to describe the motion in
the soil at arbitrary points. These ideal conditions only rarely apply: in most cases, one
must deal with dipping layers, the presence of nearby topographic irregularities, and the
spatial variation of material properties. Among the most difficult problems to analyze
(and much less predict) is the seismic behavior of alluvial basins of intermediate lateral
extent or depth, and surrounded by hills or material discontinuities along an irregular
contour. While numerical models (such as finite elements or finite differences) could be
used —at least in principle— one soon discovers that trustworthy models are
overwhelmed by the computational demands and complexities imposed by the three-
dimensional, time varying nature of the problem. Indeed, most useful three-dimensional
discrete models require inordinately large computational resources. Thus, this is an area
where testing in the laboratory could provide significant benefits in advancing the state of
knowledge.

For engineered structures to be adequately supported, the ground must have
sufficient stiffness and strength. With a few notable exceptions —such as the soils
underneath Mexico City—most soils propagate seismic disturbances and waves at speed of no less than 150 m/s, and this speed generally increases with depth. Earthquakes, on the other hand, are energy-rich only in the band between 1 and 5 Hz. We observe also that dominant frequencies go hand in hand with the stiffness of the soil, with soft soils propagating preferentially the low frequency components while rapidly attenuating the higher ones. For a typical earthquake with a dominant frequency of about 3Hz, the wavelengths in soft ground are on the order of 50m or greater. An intermediate size alluvial basin of this material that is 200m in depth and 1000m in width and length, when measured in wavelengths, has dimensions of 4x20x20. In a 1:100 model for this basin, the typical wavelength would scale down to about ½ meter in length. To achieve this goal, one could increase the frequency of the excitation applied to the model, change its material parameters (i.e. mass density and modulus of elasticity), or both. Assuming that one chooses not to significantly change the mass density—a reduction in wave speed would require a denser material—it follows that the soil box would have a size of 2x10x10m, and the model would weigh some 500t (=2.5x2x10x10). While this quantity is somewhat on the heavy side, it is of the same order of magnitude as the mass contemplated for large scale structures. On the other hand, the seismic waves in bedrock travel much faster than those in the soil, perhaps at no less than 3km/s. Hence, the wavelengths there are perhaps 20 times greater than those in the soil. For a 10m wide model, this would represent a wavelength of 10m, which could crudely be simulated by means of four independently moving platforms of 2.5m each (or perhaps neglected altogether and modeled with a single platform). While only a rough sketch, this analysis appears to indicate that with appropriate scaling of parameters, it may be possible to construct useful models of soil basins.

**Soil stiffness**

One of the most difficult aspects in the construction of soil and alluvial basin models is the proper representation of the variation of soil stiffness with depth. For cohesionless granular materials such as sands, the shear modulus changes roughly in proportion to the square root of the effective vertical stress (basically, the weight of the soil above the elevation considered minus the buoyancy caused by interstitial water, if any). In other words, the moduli are controlled by gravity. This is one of the reasons why centrifuges have gained a strong foothold in geotechnical engineering: they allow to increase gravity hundertfold, so a 1:100 model preserves the stress gradient (and thus the moduli) of the prototype. Clearly, this is not possible in a large shaking table, so other strategies must be used. While there are no current established procedures or guidelines on how to simulate gravity in a soil model other than with a centrifuge, some alternatives might deserve further exploration.

A first alternative could consist in forcing water to flow in a vertically downward direction, and draining this water at the bottom of the model. In principle, the drag forces induced by the flow should increase the effective stress in the soil, accumulate with depth, and partially simulate gravity. However, the water also has inertial characteristics which can interfere with the motion and behavior of the soil particles and with the propagation of waves, particularly near the surface. In addition, the buoyancy effect of the water will partly offset the gains obtained from the drag forces.
A second strategy could employ a thin gas-impermeable membrane over the surface of the model, together with the application of a partial vacuum at the bottom. Among the disadvantages of such an approach would be the potential interference of the membrane with motion measurements and surface waves, and more importantly, the independence of the applied pressure differential from the soil depth. Thus, there would be no stress gradient (or moduli gradient) in the soil other than that produced by normal gravity.

Yet another possibility could consist in mixing into the soil some heavy metal particles so as to increase substantially the mass density of the material. In a 1:3 scale model, a 3:1 increase in mass density (for example, using steel "sand") would preserve gravity effects (see Appendix, eq. 17). Another alternative would be adding ferromagnetic particles to the soil and applying a magnetic field so as to attract the particles from the bottom. Among the disadvantages: the added particles could constitute environmental hazards, they could change the behavior of the soil fabric at small scale (friction between particles, etc.), any steel particles could simply rust away with time, or they could be susceptible (mechanically and electrically) to the magnetic fields induced by the testing devices and thus interfere with waves in the model, particularly if the shaking table is driven by electromagnets instead of conventional hydraulic actuators. (It should be added that the behavior of clays is also governed in part by electrochemical forces between the clay particles, which could conceivably be disrupted by the electromagnetic fields set up by such a shaking table; if so, this could lead to possible changes in material behavior).

**SOIL BOX VS. STRUCTURAL MODEL**

*Soil around a structure*

It is well known that the dynamic pressure bulb — i.e. that part of the soil affected by soil-structure interaction— is comparable in size to the foundation dimensions. Considering typical soils with mass density 2500 kg/m³ and a building with an average density of about 250 kg/m³ (i.e. "10% solid") with a nearly compensated square foundation (i.e. one in which the weight of the soil removed is similar to the weight of the building), such a building will have one basement floor for every ten stories above ground. Assuming a pressure bulb with the same characteristic width and depth as the foundation, the soil mass for that bulb will occupy a volume some 20 times the size of the foundation (namely a soil ring or collar around the foundation, and a basement-deep soil layer underneath it). Thus, the soil box around and underneath the structure will weigh about twenty times more than the building itself. In general, such a soil mass is too large to be included in a natural scale model of the structure. Even for a small building weighing some 500 tons, the soil box around it would weigh about 10,000 tons! This inescapable fact remains true even if one were to model the soil in only one horizontal direction (i.e. "plane strain" conditions): the soil box would still be some eight to tens times heavier than the building. Thus, a soil box to natural scale cannot realistically be achieved. To bring down the weight by a factor of at least 20 without changing the material density, the physical dimensions of the model would have to be reduced by a factor $(20)^{1/3}$, which is somewhat less than 3. It can be concluded then that an experiment on soil-structure interaction will involve models at a scale of at most 1:3, possibly much smaller than that. Analyses for other structural types (e.g. bridges, tunnels,
dams, retaining walls etc.) would also call for reduced-scale models, perhaps down to rather small scales.

**Boundaries of soil box**

One of the recurring difficulties in both experimental as well as numerical analyses for soil-structure interaction effects is the appropriate modeling of the boundaries of the soil box (or as it is sometimes called, the soil island). A common strategy in experiments for soil amplification and soil liquefaction is to assume that in soft soil deposits, the seismic waves propagate vertically, or nearly so. This pragmatic assumption is based in part on the fact that if a large impedance contrast (stiffness mismatch) exists between the soil and the underlying rock, it will cause the waves to refract to a near vertical path. If so, then shear waves (S-waves) will induce little, if any, vertical motion in the soil particles, and a horizontal roller boundary at the lateral edges of the model (numerical or experimental) will suffice to accurately guide the shear waves. In the laboratory, this ideal boundary condition is accomplished by means of the laminar box: the lateral walls of such a box consist on many thin steel plates, each of which has a rectangular opening that forms the interior of the box when all plates are stacked up. These plates can slide smoothly on top of each other, and they provide the appropriate boundary condition for shear waves propagating vertically in the box. However, this mechanical boundary is not adequate for other types of waves (P waves, surface waves, etc.), or even for S waves at oblique angles, which could in fact be generated by material discontinuities, dipping layers, topographic irregularities, or structural models in the box. Thus, such features will always produce reflections and echoes to some degree. In some cases, it may be possible to design experiments so that any reflections of the waves scattered by the structure (or inclusion) will have dissipated before they return to the model. The larger the structural model in relation to the size of soil box, however, the more difficult it is to avoid such echoes.

Another strategy consists in building a wave-absorbing layer along the walls of the soil box, similar in concept to those used in shipping packages to prevent damage to the contents, or to the soft walls in an anechoic recording studio. Materials used for such devices include foams, sponges, corrugated cardboard, or even saw dust, all of which exhibit a highly non-linear dynamic behavior. However, their efficacy in absorbing elastic waves of long and intermediate wavelength, such as those possible in a soil box, has not yet been fully demonstrated (an much less understood).

More problematic still is the modeling of an appropriate transmitting-absorbing boundary for the bottom of the soil box. In principle, this boundary would have to be able to transmit the incident waves induced by the shaking table without impediment, yet absorb (perhaps not fully!) any reflections returning from the soil box. While elaborate boundaries based on control theory could be devised, no such boundaries have been used or attempted to date in seismic or geotechnical testing.

**Non-linear effects in the prototype**

Perhaps the most difficult aspect to represent faithfully in a smaller than natural scale model of a structural system is the material and geometric non-linear behavior of the system. As mentioned previously, for small seismic excitations, these effects may not be important, but damaging earthquakes invariably take structures well into the inelastic
range, not to mention soils, which exhibit inelastic, irreversible behavior even at small strains. In addition, one must often contend in seismic engineering with geometric non-linearities, such as separation of the foundation walls from the soil, lift-off of unanchored fluid storage tanks, pounding of buildings, sliding of earth slopes and retaining walls, and so forth. The common denominator in non-linear processes such as these is their failure to obey the principle of superposition.

Consider, for example, a structural model for the inelastic behavior of a building subjected to an earthquake strong enough to produce horizontal and vertical accelerations close to 1g. In such a model, it would not suffice to simply use materials that scale the elastic moduli and the yielding behavior in beams and columns; one would have to scale gravity as well, since the stresses due to weight interact importantly with the dynamic stresses due to the seismic motion, and will most certainly affect the inelastic behavior of load-bearing members. While in principle one could increase the density of the material in inverse proportion to the length scale and in direct proportion to the elastic moduli scale (see Appendix, eq. 17), it is very difficult to accomplish precisely that with conventional, off-the-shelf materials. For example, switching from concrete to steel may triple the density, but it also increases Young's modulus by an order of magnitude; in addition, the inelastic behavior of steel is very different from that of concrete.

Initial in-situ stresses can also bias the non-linear behavior of a system. Examples are the residual stresses in welded connections, the horizontal components of stress in compacted fills around the foundation of a building, or the axial forces in pre-stressed and post-tensioned concrete members. These are difficult or impossible to reproduce in scaled models.

CHOOSING THE SCALING PARAMETERS
As shown in the Appendix, problems in structural dynamics are controlled by several dimensionless parameters that cannot all be chosen independently. In essence, one has reasonable freedom in choosing the size of the model, that is the length scaling. One can also choose among several off-the-shelf materials to build these models, which fixes simultaneously (at least) four parameters, namely the mass density, the elastic moduli, the viscosity, and the yield stresses (and inelastic behavior). With these choices, all other physical parameters become dependent, and cannot be freely chosen: time and frequency scaling, wave velocities and wavelengths, mass, stiffness and damping, and last but not least, gravity.

Clearly, all of the previously cited difficulties with scaled models are minimized or disappear altogether when testing structures at natural scale. One could perhaps also develop special testing materials, but those are likely to be rather expensive. Thus, models should be as large as practically realizable.

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APPENDIX

Modeling and scaling of a one degree-of-freedom system

In this section, we consider briefly the most important scaling parameters necessary for a one degree of freedom dynamic system. This will give us an overview of the independent parameters available for consideration in testing, even for a structure consisting of many degrees of freedom.

The equation of motion of a 1-dof system subjected to a dynamic force is

\[ m\ddot{u} + c\dot{u} + ku = f g(t) \]  

in which \( m, c, k, f \) are, respectively, the mass, damping constant, stiffness and force amplitude, \( g(t) \) is a dimensionless function of time \( t \), and the dots indicate derivatives with respect to time. Dividing by the mass, this equation can written in the more convenient form

\[ \ddot{u} + 2\beta\omega u + \omega^2 u = \omega^2 u_s g(t) \]  

in which

\[ \beta = \frac{c}{2\sqrt{km}} = \text{fraction of critical damping (a number)} \]  \hspace{1cm} (3a)
\[ \omega = \frac{k}{\sqrt{m}} = \text{the natural frequency (in rad/s) of the system, and} \]  \hspace{1cm} (3b)
\[ u_s = \frac{f}{k} = \text{the static deflection} \]  \hspace{1cm} (3c)

In the following, we shall denote the scaled variables (i.e. those pertaining to the model) by means of added single quotes.

a) Length (l) scaling:

\[ \frac{l}{l'} = \lambda \Rightarrow u / u' = \lambda \quad \text{or} \quad u = \lambda u' \]  \hspace{1cm} (4)

b) Time (t) scaling:

\[ \frac{t}{t'} = \tau \Rightarrow t = \tau t' \quad \dot{u} / \dot{u}' = \lambda / \tau \quad \ddot{u} / \ddot{u}' = \lambda / \tau^2 \]  \hspace{1cm} (5)

Introducing these scaling relations into the differential equation, we obtain

\[ \frac{\lambda}{\tau^2} \frac{d^2 u'}{dt'^2} + \frac{\beta}{\tau} \frac{d u'}{dt'} + \omega^2 \lambda u' = \omega^2 \lambda u_s g(t) \]  \hspace{1cm} (6)

which yields
\[
\frac{d^2 u'}{dt'^2} + 2\beta(\omega\tau)\frac{du'}{dt'} + (\omega\tau)^2 u' = (\omega\tau)^2 u'_{\tau} g(t)
\] (7)

It follows that frequencies scale in inverse proportion to the time scaling factor:

\[
\omega' = \omega\tau \quad \frac{\omega}{\omega'} = 1 / \tau
\] (8)

c) **Mass density** \((\rho)\) **scaling**

\[
\frac{m}{m'} = \frac{\rho}{\rho'} \frac{l^3}{l'^3} = \mu \lambda^3 \quad \text{in which} \quad \mu = \frac{\rho}{\rho'}
\] (9)

d) **Stiffness** \((E\text{-moduli})\) **scaling**

As a specific example, consider either a bending beam or a shear beam. The stiffness of these structural elements is proportional to

\[
k \propto \frac{EI}{L^3} \rightarrow \frac{E'I^4}{l'^3} = El \quad \text{and} \quad k \propto \frac{GA}{L} \rightarrow \frac{E'I^2}{l} = El
\]

Defining the elastic moduli scaling factor as

\[
\frac{E}{E'} = \frac{G}{G'} = \epsilon
\] (10)

we obtain

\[
\frac{k}{k'} = \frac{EI}{E'I'} = \lambda \epsilon
\] (11)

On the other hand, wave velocities ("celerities") are related to moduli and mass density by an equation of the form \(C = \sqrt{E / \rho}\). It follows that wave velocities scale as

\[
\frac{C}{C'} = \sqrt{\epsilon / \mu}
\] (12)

Also,

\[
\frac{\omega}{\omega'} = \frac{1}{\tau} = \sqrt{\frac{k'm'}{k'm}} = \sqrt{\frac{\lambda \epsilon}{\mu \lambda^3}} = \frac{1}{\lambda} \sqrt{\frac{\epsilon}{\mu}}
\]

It follows that

\[
\tau = \lambda \sqrt{\frac{\mu}{\epsilon}}
\] (13)
that is, the time, length, mass density and moduli scaling factors are related, and cannot all be independently chosen.

e) Stress ($\sigma$)
Since the strain in the model must be the same as in the prototype, it follows that

$$\frac{\sigma}{E} = \frac{\sigma'}{E'} \Rightarrow \frac{\sigma}{\sigma'} = \varepsilon$$

(15)

It follows that for inelastic materials, yield stresses must also obey this ratio.

f) Damping (viscosity $\eta$)

$$2\beta = \frac{c}{\sqrt{km}} \Rightarrow \frac{c}{c'} = \sqrt{\frac{k'm'}{km}} = \sqrt{\frac{1}{\lambda \varepsilon \mu \lambda^2}} = \frac{1}{\lambda^2 \sqrt{\mu \varepsilon}} = \frac{\eta}{\eta'}$$

Because of scaling difficulties, the material viscosity of a model will not, in general, satisfy this equation. It follows that the fraction of critical damping in the model will be given by

$$\frac{\beta'}{\beta} = \lambda^2 \sqrt{\mu \varepsilon} \frac{\eta'}{\eta}$$

(14)

in which the viscosity ratio may stay close to 1 in value.

g) Gravity ($g$)

$$\gamma = \frac{g}{g'} = \frac{\lambda}{\tau^2} = \frac{\varepsilon}{\lambda \mu}$$

(16)

Unless a centrifuge is being used, this number will almost certainly not be scaled appropriately. Hence, gravity-related phenomena, such as the reduction of structural frequencies induced by the so-called $\Pi-\Delta$ effects (and the previously cited in situ moduli of granular materials) will not be scaled correctly.

h) Stresses due to gravity
These stresses are proportional to weight and inversely proportional to the cross-sections. Hence
To maintain the same stress ratio as for dynamic stresses given by eq. 15, we must have

\[ \frac{\sigma_g}{\sigma'_g} = \frac{\rho L^3 g / A}{\rho' L'^3 g' / A'} = \frac{\mu \lambda' \gamma}{\lambda^2} = \mu \lambda \gamma \]  

(17)

which agrees with equation 16. If gravity cannot be scaled down in a model (i.e. \( \gamma = 1 \)), its effects can still be preserved by choosing \( \mu \lambda = \epsilon \). For example, one could increase the mass density by the same factor used to reduce the size of the model while preserving the same moduli (not easy to accomplish).