Volume I

NUMERICAL METHODS IN GEOMECHANICS

Edited by C. S. Desal

Papers presented and discussed at the Second
International Conference on Numerical Methods
in Geomechanics held at Virginia Polytechnic
Institute and State University, Blacksburg,

Sponsor: Engineering Foundation Conferences
Partial Support: National Science Foundation
Technical Co-sponsors:
1. Geotechnical Engineering Division,
   American Society of Civil Engineers
2. German Geotechnical Society
3. Department of Civil Engineering, Virginia
   Polytechnic Institute and State University

Published by
American Society of Civil Engineers
345 East 47th Street, New York, NY 10017
DYNAMIC SOIL STRUCTURE INTERACTION

by

J. M. Roesset, ** A.M. ASCE and E. Kausel, * A.M. ASCE

INTRODUCTION

The effect of local conditions on the dynamic response of structures, particularly to seismic excitations, has been a subject of considerable interest in recent years. A number of sophisticated mathematical techniques, elaborate computer codes and simpler engineering approximations have been developed to solve the problem in a single step, considering the combined soil structure system, or in three separate steps (determination of the motion at the base, computation of the dynamic stiffnesses of the foundation, and soil structure interaction analysis). The advantages and disadvantages of each one of these two approaches and the possibility of obtaining with either sensible results were discussed in a previous paper (14). A controversy seems to persist, however, on the validity or adequacy of each method. Comparative studies, intended in general to show the superiority of any one approach, have indicated in the past a lack of clear understanding as to the correct way to apply the other. This situation is probably aggravated by the larger emphasis placed lately on the development of computer codes over the derivation of simplified procedures, comprehensive studies which would shed more light on the importance of various approximations or adequate justifications of the numerical procedures used.

The purpose of this paper is to discuss further the available solutions and their applicability in practical situations, as well as some of the main limitations existing in both methods. It is important to keep in mind that numerous uncertainties are present in various phases of the analysis and that no single procedure or computer program will be able to generate deterministically exact answers to the true physical problem. A considerable amount of engineering judgment will always be necessary to estimate key parameters and the most one can reasonably expect is to obtain results which are physically logical. Explicit recognition of the fact that no solution can be actually labeled as the "exact" solution would probably help in the above-mentioned controversy. It is clear, on the other hand, that several aspects of the solution are now well understood and that any method should be able to reproduce them, at least approximately. The existence of uncertainties should not be extrapolated to the point of negating this knowledge, the need for further research or the use of improved methodologies.

** Professor of Civil Engineering, M.I.T., Cambridge, Mass.
FOUNDATION MOTION

The determination of the motion at the base of the foundation is normally associated with the problem of soil amplification.

It is common today to specify the motion directly at the free surface. This is a reasonable approach if the soil can be classified as "firm ground," since most of the statistical data used to derive design spectra is based on motions recorded on this type of soil. The records will include thus already a certain measure of amplification. When dealing with soft, deep soil deposits, the motion at the free surface will be, however, substantially different in frequency content. To consider then the same earthquake characteristics as for an average firm ground would not be logical. To derive spectra which would be valid for all possible soil conditions, including extreme cases, would also be meaningless. A more sensible approach, if the motion is to be specified at the surface, is to provide a number of response spectra for several typical soil conditions.

Surface Foundations

For a surface foundation, or a foundation with very small embedment, the motion specified at the surface can be used directly as input for the soil structure interaction analysis in the three step approach. The direct approach would require on the other hand determination of a motion at the bottom boundary which will reproduce the specified input at the surface. This may be construed as an obvious advantage of the three step approach for this situation. A computational advantage may indeed exist. However, in order to use in the second step soil properties consistent with the expected levels of strain, some amplification theory or approximate formulae based on this theory must be applied. The theoretical limitations of this theory would thus still be present.

Embedded Foundations

For a foundation with substantial embedment, some analysis must be performed to determine a motion at the base consistent with the specified input at the surface. Analytical or semi-analytical solutions based on a continuous formulation have not yet been developed. The normal procedure is thus to find first a compatible motion at the bottom boundary of a discrete finite element model (for a one-dimensional soil profile without excavation) and to solve next the two- or three-dimensional (cylindrical) model with the excavation (fig. 1). This analysis is similar to that performed in the direct, or single step, approach, except for the fact that no structure would be present (only a massless rigid foundation). The effect of the structure on the vertical stresses in the soil should be taken, however, into account for the selection of appropriate curves of modulus and damping versus strain if a cyclic linear analysis is used to account for nonlinear soil behavior. It would seem that in this case the direct approach has a definite advantage, since the same model of the soil would be used anyway. This is only true if the model of the structure is very simple. Otherwise the increase in the number of degrees of freedom and in the cost of the analysis when the structure is included may be important. Some simplified procedures have been derived in addition (20) to estimate the motions at the foundation level from one-dimensional analyses. While these procedures are only approximate, they seem to provide reasonable answers, slightly on the conserva-
tive side. Morray's study (20) indicates that a consistent motion at the foundation should have both a translational and a rotational component and that the latter may increase the structural response by 15 to 20% in some range of frequencies. Its magnitude is, however, strongly dependent on the stiffness of the foundation sidewalls and on the condition of the backfill (the translational component is on the other hand insensitive to those conditions).

The main point of controversy in relation to this phase of the analysis lies still in the fact that the translational component of the motion at the base of an embedded foundation will show, in general, a deamplification with respect to the surface motion, which will lead to a decrease in the structural response, particularly if the rotational component is neglected. It has been argued that if the procedure described above is to be applied, the specified motion should first be amplified and that the resulting motion at the surface should then have unusually large accelerations. The first argument ignores the fact that recorded motions do already include some amplification for average firm ground conditions, although it is true that design spectra specified at the surface of other typical soil profiles should include amplification effects (and will normally do so). The second argument would seem to be based on the assumption that recorded motions are specified at bedrock, and on the results of early amplification studies with linear elastic behavior and no radiation (bedrock motion). These results showed an extreme frequency sensitivity of the soil deposits, with very large amplifications, especially at the fundamental frequency including radiation and simulating nonlinear soil behavior. These peaks are, however, substantially reduced, and the motions obtained at the free surface of deep soil deposits may be in fact smaller than the motions on rock.

Various alternatives have been suggested to avoid the deamplification effect, such as assuming that the specified motion occurs at the foundation level but in the far field (in a one-dimensional geometry) or that it is directly the motion at the base of the foundation (ignoring then any rotational component). The first alternative is not logical, since it would still require an analysis and the resulting motion at the foundation would still show amplifications and deamplifications in various frequency ranges. The second alternative is more practical, although its physical significance is highly questionable. It is probably conservative, and it avoids entirely the first step, making thus the three step approach computationally advantageous. It leads, however, to inconsistencies if various buildings are founded at different levels, and as pointed out before for the case of a surface foundation, some amplification theory must still be used to determine strain levels for the determination of the foundation stiffnesses.

One-Dimensional vs. Two-Dimensional Amplification

The root of the controversy is clearly a lack of confidence in one-dimensional amplification theory (shear waves propagating vertically), which is the basis for most amplification studies (even if a two-dimensional disturbance is introduced by the excavation). Too much emphasis has been placed in the past in the use of this theory, and not enough work has been done with other types of waves. Some studies for SV and P waves with arbitrary angles of incidence in the rock (11) show clearly that the high frequency selectivity exhibited by one-dimensional results, particularly when radiation damping is neglected, is substantially reduced in most other cases. This point is illustrated in fig. 2, where
amplification functions are shown for shear waves travelling vertically, with and without radiation (rigid rock or elastic rock), for SV waves with an angle of incidence α at bedrock (almost normal incidence at the free surface in all cases), and for a combination of SV and P waves with the amplitude of the shear wave approximately 2.5 times that of the P wave (the angle α is in this case that of the dilatational wave). These results are not inconsistent with studies which have shown the qualitative validity of the one-dimensional amplification theory (as those conducted in relation to the Caracas earthquake of 1967), but which have not really investigated in depth its quantitative accuracy. On the other hand, when the nonlinear soil behavior is taken into account, either approximately through an iterative linear analysis, or more rigorously through a true nonlinear analysis in the time domain (3,7), a reduction in the amplification peaks and a decrease in the frequency selectivity are also obtained (even moreso for the true nonlinear analysis).

More research work is necessary to determine the effect of other types of waves on the motion at the base of an embedded foundation, and on the resulting levels of strain in all cases. A better knowledge of the potential wave content of earthquakes, as a function of magnitude and distance, would also allow to improve present solutions. In the meantime, with today’s state of knowledge, the limitations of the one-dimensional theory must be recognized, but negating entirely its validity, when used judiciously, does not seem reasonable (particularly if it is negated for some phases of the analysis but accepted, explicitly or implicitly, for others).

Additional Considerations

Several additional aspects of this type of study deserve also consideration. Such, for instance, as the generation of artificial motions which will envelop or average the design spectra, the boundary conditions and element size for finite element studies (embedded foundation) and the use of the iterative linear analysis to simulate nonlinear soil behavior. Variations in the results using different samples of artificial earthquakes are smaller than for real earthquakes (which may be interpreted as an advantage or a disadvantage), since these motions are intended to have smooth spectra. Even so these variations exist, and they are a function of the number of corrections or iterations performed in the generation process to ensure the smoothness of the spectra. The use of a single artificial motion would not seem to be an ideal solution.

The use of appropriate boundary conditions and the restrictions on the size of the finite elements necessary to obtain accurate results are common to the investigation of the foundation motion and to the determination of the foundation stiffnesses. They are discussed in more detail in the next section. The validity of the iterative linear analysis concerns also various phases of the analysis and deserves a section of its own.

FOUNDATION STIFFNESSES

A significant debate has been going on in the last few years over the convenience and accuracy of using so-called "analytical" solutions for the foundation stiffnesses versus their determination in each specific case using finite elements. Since both approaches require an analytical formulation and the use of a numerical solution algorithm, a more
appropriate distinction could be made between continuous and discrete solutions.

**Continuous Solutions**

Solutions are already available for the case of a strip footing (12) and a circular foundation (24) resting on the surface of an elastic, homogeneous halfspace. Of these solutions, those provided by Veletsos (24) for the circular foundation are particularly useful. Algorithms for the determination of the dynamic stiffnesses of strip and rectangular footings (8) and circular foundations (17) resting on the surface of a horizontally layered stratum are also available. The use of these algorithms is particularly convenient when there is a small number of layers (one or two) over a rigid rock or an elastic halfspace. It should be noticed, however, that they become computationally more expensive than a finite element solution, with consistent lateral boundaries at the edge of the foundation, if a large number of layers (of the order of 10 to 20) is desired to reproduce properly the variation of soil properties with depth.

Use of the available halfspace solutions, properly modified to account for other effects, is particularly attractive. These solutions require not only a surface foundation (or very small embedment) but also homogeneous properties with depth. A procedure has been suggested (9) to account for the variation of properties due to nonlinear soil behavior by applying the iterative linear analysis to the one-dimensional problem and averaging in some way the resulting equivalent values of modulus and damping. The basic idea is reasonable and would produce good results (particularly for moderate and large excitations) when applied in combination with the algorithms mentioned above for layered systems, without averaging the properties. The rules suggested to obtain the average properties require, however, further studies to confirm their validity. Several approximate procedures have also been suggested to simulate the effect of embedment (4, 21, 22). Elsabee's rules (4) are particularly simple and seem to provide reasonable results. The frequency dependence is assumed to be the same as for the surface foundation and the static stiffnesses are obtained from the formulae:

\[
k_{x0} = \frac{8GR}{2-v}(1 + \frac{1}{2} \frac{R}{H})(1 + \frac{2}{3} \frac{E}{R})(1 + \frac{5}{4} \frac{E}{H})(1 + 2iD)
\]

\[
k_{\phi0} = \frac{8GR^3}{3(1-v)}(1 + \frac{1}{6} \frac{R}{H})(1 + \frac{2}{E} \frac{E}{R})(1 + 0.7 \frac{E}{H})(1 + 2iD)
\]

\[
k_{x\phi} = \frac{1}{3} E k_x
\]

for \( \frac{E}{H} \leq \frac{1}{2} \) and \( \frac{H}{R} > 2 \).

In these expressions \( G \) is the shear modulus of the soil and \( v \) its Poisson ratio. \( H \) is the depth of the stratum, \( E \) the depth of embedment, \( R \) the radius of the foundation and \( D \) the amount of internal damping in the soil (of a hysteretic nature).

These formulæ assume that the sidewalls of the foundation are rigid and welded to an undisturbed backfill. If there are no sidewalls or if the backfill is greatly disturbed, the horizontal stiffness is only slightly affected. The rotational stiffness decreases, however, approaching the value for a surface foundation. The coupling stiffness is also strongly affected and can even change sign in the limiting case of no
sidewalls. A consistent assumption should be used for this step and for the determination of the foundation motion discussed in the previous section.

**Discrete Solutions**

Discrete formulations based on finite differences or finite elements can be used for the determination of the foundation stiffnesses. Finite element models have become, however, particularly popular because of the ease with which they can reproduce arbitrary geometries and variations in material properties. As with any discrete formulation care must be, however, exercised to ensure that the solution reproduces well the true physical behavior.

The effect of the mesh size on the stiffnesses of a circular foundation was investigated by Kausel (13), who found that for a given frequency \(\omega\) good solutions are obtained as long as the size of the elements is kept smaller than 1/4 to 1/6 of the corresponding wavelength. A solution often applied in practice is to use small elements in the immediate neighborhood of the foundation and to increase gradually their size with distance. This approach implies the existence of an effective dynamic pressure bulb under the foundation representing the zone which must be reproduced with accuracy. The existence of this effective zone of influence depends strongly on the amount of internal soil damping as is more clear for rocking than for horizontal or vertical motions. So, while the procedure makes sense and it can provide better results than a regular mesh with the same total number of degrees of freedom, the transition in size should be gradual. Using for instance horizontally elongated elements immediately to the side of the foundation will greatly distort the rocking stiffness.

Of special interest for this problem is the effect of the bottom and lateral boundaries. Most finite element programs assume the bottom boundary to be rigid, neglecting thus the effect of vertical radiation. The importance of this effect will be a function of the relative stiffness of the underlying rock with respect to the soil, the depth of the deposit and the amount of internal damping. If there is a soil deposit over much stiffer rock, assuming a rigid bottom will not introduce a significant error. If on the other hand this abrupt transition in properties does not exist physically, the bottom boundary should be placed at a depth of at least 8R for horizontal excitation and 2 to 4R for rocking. Placing the bottom boundary at a sufficient distance from the foundation may be required anyway to reproduce properly the variation in soil properties with depth.

For a layer of finite depth on rigid rock, lateral radiation cannot take place below the fundamental shear frequency of the stratum (and is very small in rocking below the fundamental dilatational frequency). Above this frequency, or over the complete range for a half space, the dissipation of energy through lateral radiation becomes very important. This effect is accurately reproduced in a finite element model through the use of a consistent boundary matrix (2, 13, 25) which allows us to place the boundary directly at the edge of the foundation for linear analyses. While the computation of the consistent boundary matrix requires the solution of a quadratic eigenvalue problem, the reduction in the number of degrees of freedom necessary for the analysis in comparison with other boundaries more than offsets this extra computation. It was argued for some time that the type of lateral boundary used made very
little difference in the results. This conclusion was based on the comparison of elementary and viscous boundaries. A comprehensive comparative study by Etouney (5) indicated that while there is in fact little difference below the fundamental frequencies of the stratum (shear frequency for horizontal excitation, dilatational frequency for rocking), the differences are significant above these frequencies. To obtain reasonable results in this higher frequency range with elementary or viscous boundaries, they should be placed at a distance of 10 to 20R for 5% internal damping and 5 to 10R for 20% damping. Computer programs widely used such as LUSH (19) did not include originally any special boundary, but they have incorporated the consistent boundary matrix in the more recent versions.

Some comparative studies (18) have implied that finite element solutions were unable to reproduce properly the foundation stiffnesses for a halfspace. The studies did not make use, however, of the consistent boundary; they were applied to the case of a strip footing (where the static horizontal stiffness is zero for a halfspace but finite for a layer), and no internal damping was assumed in the soil, although the possibility of reproducing the radiation effects through an equivalent hysteretic internal damping was investigated (a poor solution, since radiation damping is of a viscous nature, proportional to frequency). Kaiser (13) and Chang Liang (4) had shown on the other hand that finite element models, with the appropriate precautions, can indeed reproduce well halfspace continuous solutions for circular foundations and even for strip footings (except at very low frequencies). This is particularly so if there is some internal damping, as can be normally expected. For zero damping the solution for a layer of finite depth will show important oscillations and will in fact be zero at the fundamental frequency of the stratum, but a small amount of damping smoothes considerably these oscillations. Equally important is the fact that a perfectly homogeneous halfspace is not the situation most commonly encountered in practice.

Two-Dimensional vs. Three-Dimensional Solutions

While finite element formulations are available for the case of circular foundations (13), and they are not much more expensive than for strip footings (introducing small simplifications) particular emphasis has been placed on the study of the latter. Some of the best-known computer programs, such as LUSH, are in fact for two-dimensional analyses (plane strain problem).

A comparison between two-dimensional and three-dimensional solutions (16) showed differences of up to 50% in the structural response (amplified response spectra) with the plane strain solution being unconservative. It should be noticed, however, that the comparison was performed for a halfspace condition (where the strip footing has zero static stiffness) and for a soil without any internal damping. Comparisons by Kaiser (13) for an actual soil profile also indicated a difference, but only of the order of 15%.

A more recent study (1) shows also differences between the structural response from two-dimensional and three-dimensional analyses. It concludes, however, that these differences are due to the structural model and that the horizontal component of motion at the base of the structure from the two cases is very similar (the rotational component seems to be ignored). The study recommends the use of a complete two-dimensional soil structure model just to obtain the base motion and then
the use of a structural analysis program, with a three-dimensional model of the structure and the computed motion applied at the base, to determine the structural response. A proper justification, beyond the comparison of the horizontal component of the base motion is, however, lacking. The advantage of this procedure over the regular three step approach is also debatable.

It is clear that a plane strain model cannot reproduce exactly a three-dimensional situation and that solutions obtained with two-dimensional computer programs will only be approximate. A simplified procedure to reproduce three-dimensional effects by adding a viscous boundary on the sides of a two-dimensional slice has been presented (10). It is intended to reproduce the case of a square or rectangular foundation, but the results have only been compared to those of a circular foundation (assuming the approximate procedure to be "exact") and not to better solutions for the same situation.

More extensive studies are necessary to estimate better the magnitude of the potential errors introduced by two-dimensional solutions (or the modified versions) for typical situations. It is not likely that they be as large as those for the case considered by Luco (16). The fact that they exist and that they may be of the same order as those introduced by the use of available continuous solutions, modified for embedment and strain level (or variable soil properties), should be, however, recognized. For circular foundations the use of formulations in cylindrical coordinates (13) would eliminate these errors, but uncertainties will still persist, as for continuous solutions, in relation to the conditions of the soil in the backfill.

NONLINEAR SOIL BEHAVIOR

It has been long recognized that one of the key factors in the application of these theories to practical situations is the consideration, at least approximately, of nonlinear soil behavior.

From laboratory tests curves relating shear modulus and damping to maximum level of strain are obtained for a cyclic, steady state condition. Families of these curves are usually provided for different values of the vertical stress. Nonlinear soil behavior can then be simulated in the analyses with various levels of sophistication:

a) by performing a single linear analysis estimating values of moduli and damping for the soil on the basis of approximate formulae or simpler analyses.

b) by performing a series of linear analyses, using for each one soil properties based on the levels of strain obtained from the previous analysis. The strain used to determine values of moduli and damping from the experimental curves cannot be the maximum strain which may only occur once. Typically a value of 2/3 of the maximum has been used as characteristic strain. A more economical and equally reasonable solution is to use the root mean square strain multiplied by 2/3 and by the ratio of the maximum input acceleration to the root mean square acceleration (16).

c) by using a discrete model and assuming nonlinear constitutive relations for the material, performing the analysis in the time domain. For a one-dimensional condition this requires the definition of a shear stress - shear strain law which would yield a variation of
secant modulus and damping similar to that of the experimental curves. Various mathematical models have been suggested, with a Ramberg-Osgood type model being frequently used.

For a two-dimensional situation it is necessary to define not only the variation of the shear modulus, but also an additional parameter. In some cases the value of Poisson's ratio is assumed to remain constant, while in others the bulk modulus or the constrained modulus are maintained independent of strain. A proper study of the effect of these different assumptions is still lacking. These limitations apply equally to the linear iterative analysis and to the time domain analysis. They are complicated further for three-dimensional studies. Before embarking on the extension of present procedures to more complex situations, research seems necessary to determine on the basis of experimental data appropriate constitutive relations for soils under two-dimensional and three-dimensional states of stress and dynamic loading.

**Internal Soil Damping**

Internal soil damping in the iterative analysis is assumed to be of a hysteretic nature, through the use of complex moduli. This selection is based on experimental results which indicate that the damping is mainly due to nonlinear behavior and that the energy dissipated per cycle is independent of frequency. Loco, Hadjian and Dos stated in a recent paper (18) that, "From the results obtained, it appears that the commonly used frequency-independent damping is not appropriate to represent the internal energy dissipation of soils at low frequencies. The use of this type of damping may lead to significant underestimation of structural response." This conclusion is surprising, since their study did not contain any comparison with nonlinear analyses in the time domain, but only an attempt to reproduce radiation through internal damping. The fact that hysteretic damping will not reproduce well radiation effects is evident, since radiation damping increases almost linearly with frequency. The reference to internal energy dissipation is, however, misleading. Studies by Constantinopulos (3) for the one-dimensional case have shown that reasonable results can be obtained with the iterative procedure only if the damping in each layer is assumed to be hysteretic, as could be logically expected.

**Iterative Linear Analyses**

The iterative linear procedure is the one most commonly used in practice. A more comprehensive set of studies (3) using a Ramberg-Osgood model for the time history analysis and the corresponding variations of modulus and damping for the iterative procedure, showed that while the latter reproduces well nonlinear effects from a qualitative point of view, it overestimates slightly the accelerations and it may underestimate substantially the strains. The discrepancy in the numerical results increases with the level of excitation. A recent paper by Finn, Lee and Martin (7) confirms these conclusions and finds even larger discrepancies between both approaches, with a constitutive law for the material which is also of a Ramberg-Osgood type, but somewhat different (loops do not close at the same points in each cycle).

Comparative studies between nonlinear and iterative linear solutions for the two-dimensional soil structure interaction problem have been performed by Ettouney (6), using constitutive laws for the material
also of the Ramberg-Osgood type, but closer to Finn's than to Constantopoulos' in their behavior. The general trends were the same, but the discrepancies in the solution larger than those for a one-dimensional case (particularly those of Constantopoulos' study). Differences were very small for such measures of the response as maximum accelerations, but significant for levels of strain and amplified response spectra. In all cases nonlinear analyses show a larger reduction in the frequency selectivity of the soil deposit (broader spectra with less pronounced peaks) than the iterative analyses. These tend to be therefore conservative in some ranges of frequencies but unconservative in others. Ettouney studied also the effect of performing a simple linear analysis with values of moduli and damping resulting from an iterative one-dimensional analysis (a much more economical solution) as suggested in (9, 15). Variation in the results with those of a two-dimensional iterative analysis is very small, particularly for moderate to large excitations. Results for a particular case are shown in fig. 3. They are typical of those obtained for a variety of situations.

All these studies cast some doubt on the validity of the iterative procedure. It should be noticed, however, that the results are sensitive to the assumed constitutive laws and that apparently small changes in these seem to introduce important variations in the response. Pending more studies of the kind mentioned above, an indictment of present procedures should not be made. Considering the uncertainties in soil properties and behavior, the approximation furnished by the iterative linear procedure may be acceptable, except for the determination of maximum strains. The use of cyclic analyses for the complete two-dimensional or three-dimensional model, which is expensive, would appear, however, to be unwarranted, and the savings achieved by applying the iteration only to the one-dimensional case could be better applied to the consideration of different inputs and different soil properties.

It is important to notice in any event that the sensitivity of the results to the assumed soil behavior and to the procedure used to simulate it may be larger than to other approximations which are the source of so much controversy.

The Inverse Problem

As mentioned in a previous section, when the input is specified at the surface of the soil, the inverse amplification problem is normally used to determine a compatible motion at the bottom boundary of a discrete model. The iterative linear scheme is applied in this case to determine soil properties consistent with the levels of strain.

The uniqueness of a solution and the convergence of the procedure for this inverse problem are not guaranteed. Final values of modulus and damping are dependent on the initial estimates and different patterns can be obtained varying these. A reasonable and consistent approach is to start with the small strain values.

Convergence of the scheme encounters often numerical difficulties for deep deposits, particularly the way these analyses are performed in practice today. More studies are necessary on these points and on the sensitivity of the results to the types of waves considered (rather than only vertically propagating shear waves).
METHODS OF SOLUTION

Further approximations and sources of discrepancies are inherent often in the methods of solution used.

A solution in the frequency domain has become particularly popular in recent years because of the frequency dependence of soil amplification effects and of the foundation stiffnesses. The method is only valid for linear problems and requires therefore the use of the iterative procedure to simulate nonlinear soil behavior. It can be applied to the direct solution and to the three step approach, but for the latter it requires some further simplification (applying the iterative scheme to only one phase of the analysis). An important parameter, in all cases, is the frequency increment, or number of points, used in the determination of the Fourier transform of the input motion and for the transfer functions (amplification functions, dynamic foundation stiffnesses structural response). It is common to use a smaller number of points for the transfer functions and to determine intermediate values by interpolation. Interpolation on the real and imaginary parts seems more appropriate than on the amplitude and phase. Studies by Scarlatti (23) have shown that a reduction in the frequency interval over values normally used in practice, particularly in the vicinity of the fundamental frequencies in the system, can produce variations in the results of 10 or 15% and in some cases, when soil structure interaction effects are very small, a complete reversal on the nature of these effects (from a small increase in the magnitude of a response parameter to a slight decrease or vice versa).

A solution in the time domain for linear behavior requires the definition of frequency independent internal damping. The use of damping matrices obtained as a combination of the mass and the stiffness matrices (in particular the so-called Rayleigh damping) is common. Some care must be, however, exercised with this solution since low and high frequency components are liable to be filtered out entirely. This problem does not exist if nonlinear soil behavior is considered. This is clearly the case where the time domain approach is most appropriate. In rigor the nonlinear time solution should be applied to the total problem (combined soil structure system). The same approximations discussed for the frequency solution are, however, applicable. The consistent lateral boundaries can no longer be applied in these cases, since they are only defined in the frequency domain, and, therefore, the soil model should be extended to the sides at a sufficient distance from the foundation. Special attention must be paid in these analyses to the time step of integration and to the numerical procedure used. Methods which introduce fictitious damping into the system (in all modes) in order to guarantee numerical stability, may produce well-behaved but unreliable results.

There has been a tendency lately to favor the direct approach over the three step solution. From a rigorous point of view the former has clear advantages for a nonlinear solution in the time domain (if accurate constitutive laws for the soil were available) or when the flexibility of the foundation had a significant effect in the response (which is not normally the case). Modeling of the structure in this approach is, however, often very crude and unacceptable for structural design. The three step approach lends itself better to the use of various approximations, to sensitivity studies on the effect of various assumptions and to the determination of significant parameters. It is thus hard and un-
realistic to assert dogmatically the superiority of any one approach over the other, when they are both applied judiciously. A combination of both and various approximations may be desirable in many practical situations.

SUMMARY

It was the purpose of this paper to review critically the uncertainties existing in the different phases of soil structure interaction analysis and the approximations introduced by various procedures. A considerable amount of research is still necessary to determine the sensitivity of results to usual approximations, to the assumed soil behavior and to variations in the specified motion (all these factors should be considered in toto). In the meantime it should be remembered that no single method will provide a deterministically exact answer to the true physical problem, and that variations in key parameters may be more advisable than the use of sophisticated computer codes, when this sophistication introduces only minor refinements.

ACKNOWLEDGMENT

This paper is based on basic research conducted at the Civil Engineering Department of M.I.T. under a grant from the National Science Foundation, RANN Division, and on comparative studies for actual containment structures carried out at the Structural Division of Stone & Webster Engineering Corporation.

APPENDIX I. REFERENCES


Motion specified at surface

Determine compatible motion at bedrock

Determine motions at foundation

Input at bedrock compatible motion

FIG. 1
FIG. 2 - SV & P WAVES ($A_s = 2.5 A_p$)
FIG. 3 - ARS at Top of Containment