DYNAMIC SOIL-STRUCTURE INTERATION OF NUCLEAR POWER PLANT STRUCTURES

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Acknowledgment
This document presents a faithful reproduction of the original type-written lecture notes used in Santa Margeritha, for which only a poor copy remained available. It differs from the original solely in the use of modern formatting and in the removal of a handful of typographical errors.
Dynamic Soil-Structure Interaction of Nuclear Power Plant Structures

By E. Kausel

The layout of nuclear power plant sites is characterized by a complicated array of structures, erected at relatively close distances. This cluster of structures is unusual in the sense that they are very massive and heavy when compared to standard buildings typically found in the construction industry. A typical containment structure has a relative solid concrete density between 25 percent and 30 percent in contrast to conventional structures having a relative density of about 10 percent. As a consequence, interaction effects between the soil and the structure during seismic excitations can no longer be disregarded. The following pages present a review of the procedures available for the dynamic analysis of nuclear power plant sites accounting for soil-structure interaction effects.

The development of a suitable mathematical model for the seismic analysis of nuclear power plant sites follows in various steps. Each of these steps involves important decisions on the part of the analyst, and the results obtained in the final analysis can only be as good as the weakest link in this chain of decisions.

The principal branching points in the decision tree leading to the seismic model can be described by the following questions:

1. What is the geometrical (physical) model?
2. What is the excitation?
3. How do we solve the dynamic equations?

These questions will be discussed in more detail in the following.

1. Geometrical Model (Physical Idealization)

1.1 Rigid vs. flexible subgrade:

It is generally accepted that the soil-structure interaction effects are negligible for very firm or rigid subgrade materials. Each structure at the site can then be analyzed individually for a prescribed earthquake motion at its base, in the way it has been traditionally done with conventional structures.

On the other hand, for soft to intermediately firm soils (having a shear wave velocity of less than, say 700 m/sec), these interaction effects can no longer be ignored. A proper model should then account for the subgrade flexibility and the coupled interaction between the soil and the structures.
In the more general case, the array of structures and the soil should be regarded as a single continuum, with dynamic interactions and feedback among the various structures through the soil. These interactions arise as a result of perturbations and waves generated by the vibrating structures, which in turn modify locally the seismic excitation. However, the problems involved in modeling simultaneously more than one structure are formidable. Even if considerations about feasibility, cost and computer storage are not a serious constraint, it is simply not practical to try to analyze the whole system as one single unit. Considering the numerous uncertainties implicit in the final dynamic model, particularly in the soil properties and the excitation, the results obtained using currently available methodologies with such an enlarged model are not necessarily better or more exact than those obtained analyzing just one or at the most two structures at a time. Nevertheless future developments and advances in the state-of-the-art may alter these considerations.

1.2 Two-dimensional vs. three-dimensional idealization:
Real structures or soil deposits rarely possess symmetry w/r to a vertical axis or w/r to vertical planes. While nuclear containment structures and power plant buildings may have a near axisymmetric or rectangular configuration, there are many buildings for which these conditions do not hold. Soils, on the other hand, are not homogeneous, and their properties vary generally in the horizontal direction as well as with depth. In most cases, however, the material subgrade properties vary more rapidly in the vertical than in the horizontal direction, and the idealization of the founding soil as a set of horizontal layers may be adequate. It is therefore of interest to investigate the possibility as well as the accuracy of reproducing a three dimensional configuration with a two-dimensional model, as often done in practice. A full three-dimensional analysis is in most cases prohibitively expensive if an adequate representation of the physical system is to be provided. For structures with cylindrical symmetry, an analysis in cylindrical coordinates with an appropriate Fourier expansion in the circular direction can provide the true spatial behavior with considerable savings in storage requirements and time of computation. A further reduction in cost is achieved by using an “equivalent” two dimensional model, particularly for rectangular foundations. Preliminary analyses show reasonable agreement between two-dimensional and three-dimensional solutions (9). The main advantage of the plane model is that it allows to consider several adjacent structures, whereas in the cylindrical three-dimensional solution one is limited to concentric structures if more than one building is present.

On the other hand, it is important to give due consideration to the coupling of the motion components when the structure is not plane-symmetric or axisymmetric. Furthermore, wave passage effects may induce torsional motions in otherwise ideally axisymmetric or plane-symmetric structures.

At present time, most analysis using finite element methods idealize the structure either in an axisymmetric or in an equivalent two-dimensional configuration. The aim is to minimize the total number of equations needed to solve a given system. An excellent approximation in axisymmetric systems subjected to vertically propagating shear waves is to equate radial and tangential components in the axisymmetric expansion in the
circular direction, that is, \( u_r = u_{i\theta} \) (the index 1 refers to the component with Fourier number one). In this way, the number of degrees of freedom is reduced from three to two in each node, without discernible loss of accuracy. This approximation holds because the loading (the seismic motion) satisfies this condition exactly, while the diaphragm action of the mat prevents an ovoidal distortion of the structure. For other loading conditions, this approximation may not be applicable.

1.3 Continuum solutions vs. finite elements:
Basically, there are two alternatives to incorporate the effects of the subgrade flexibility:

In the first approach, the mathematical model of the soil and the structures is based on finite elements and regular linear members, or finite difference schemes. This method is usually referred to as the direct, complete or one-pass approach.

Alternatively, it is possible to model the subgrade by stiffness functions which can be interpreted as a set of springs, dashpots and masses. The structure is again modeled with finite elements or regular linear members. This approach is normally referred to as the spring method, substructure method or three-step solution.

The results obtained with these two approaches may differ in general by a lesser or greater extent, depending on the assumptions made. Each method has its own advantages and limitations from a practical standpoint, but many of the problems encountered in the solution are common to both. In particular, one of the most important points in either case is the definition of the design earthquake and the free field ground motion, that is, the motion that would occur in the soil deposit if no structure were present. As will be shown later on, these two methods are mathematically equivalent; if often they are classified as different, it is only because of inconsistencies in their implementation.

1.4 Finite element discretization:
Finite element (and finite difference) techniques have some well established restrictions in order to provide acceptable accuracy. In order to reproduce adequately the propagation of waves through the continuum, the size of the elements should not be larger than about 1/6 to 1/8 of the smallest wave length of interest. This wave length is simply given by the relation

\[
\lambda_{\text{min}} = C_s T_{\text{min}} = C_s / f_{\text{max}}
\]

where \( T_{\text{min}} \) represents the smallest period of interest and \( f_{\text{max}} \), conversely, the largest frequency; \( C_s \) is the shear wave velocity of the soil. It is possible to use, of course, larger elements far away from the region of interest, which in this case is the neighborhood of the foundation. The use of elongated elements in the region of interest and near the free surface of the soil will not permit the propagation of waves in their long direction, except for very long periods, and as a result, they will mask the radiation damping at intermediate and high frequencies. Also, the element size under the footing is controlled by consideration about stress gradients. The number of elements must be
enough to achieve acceptable resolution in those regions where the stresses are expected to change rapidly in magnitude with distance.

For solution procedures in the time domain, it is usually advantageous to work with diagonal (lumped) mass matrices. Frequently domain solutions, on the other hand, can be performed either with lumped or consistent matrices. A combination of both, typically 2/3 consistent and 1/3 lumped, seems to improve the dynamic responses significantly.

1.5 Boundary Conditions:
Another important consideration is the selection of the appropriate boundary conditions in the finite model to simulate the semi-infinite subgrade continuum. There is a tendency to ignore this factor by asserting that it may be neglected if the boundaries are far away from the structure. It is clear that the effect of the structure will die out with distance if there is internal damping in the soil. However, the necessary distance to guarantee good results is typically greater than the distance that can comfortably be handled in practical situations. While this distance would decrease for large values of material damping, as might be caused by a very strong earthquake, the number of degrees of freedom to reproduce with accuracy the total domain will generally be very large.

A finite element idealization of a semi-infinite soil continuum is limited by two boundaries: a bottom boundary, and lateral boundaries.

For shallow strata over hard rock, the bottom boundary is usually defined as a rigid interface at bedrock, at which the displacements are specified. For deep soil strata over rock, or when there is a deep layer of soil without a clear, sharp change in elastic properties, it becomes necessary to define the bottom boundary at some arbitrary depth. This depth has to be at least two foundation diameters to ensure that the waves generated by the vibration of the foundation are significantly attenuated before being reflected at this boundary. An improvement can be achieved using a set of springs and dashpots as suggested by Kuhlemeyer (11); however, this constitutes only an approximate technique, because the values of these elements are based on the assumption of normal wave front incidence. Nevertheless, they would accurately model a one-dimensional wave propagation situation.

The lateral boundaries, on the other hand, can be modeled in at least three ways:

a) Elementary boundaries
No attempt is made in this case to account for wave permeability, relying on the internal damping of the soil and the distance to the boundary to avoid the effect of the reflected waves (box effect).

Three cases are often used:
- fixed boundaries, in which the displacements are specified
- free boundaries with specified stresses
- mixed boundaries, where both stresses and displacements are specified
The specified stresses or displacements are usually chosen so as to satisfy the conditions prevailing in the free-field. If no structure were present, the procedure would yield the exact free-field solution. The structure will generate, however, a two-dimensional wave propagation problem with a complicated combination of waves hitting the boundaries at varying angles. These waves will then be reflected in different forms depending on the precise free-field conditions.

b. Viscous boundaries
An attempt is made in this case to absorb the waves radiating away from the structure, modeling the dynamic behavior of the far field by combinations of springs and dashpots, or by procedures which can be interpreted in terms of these elements (1, 11, 14). These procedures are based again on the one-dimensional wave propagation theory, and assume basically a pattern of waves and their angle of incidence on the boundary. Therefore, they are only approximate in a two or three-dimensional situation.

c. Consistent Boundaries
Waas (23), developed a transmitting boundary which reproduces the far field in a way consistent with the finite element expansion used to model the core region. The method is based on the exact solution to the general two-dimensional wave propagation problem in a layered stratum and it can be shown mathematically to correspond precisely to the solution on an infinite number of columns of finite elements all of the same vertical size as the last column of the core region.

Kausel (6, 7), generalized the concept to the (axisymmetric) three-dimensional case and arbitrary expansion order in the finite elements, obtaining results which are in excellent agreement with known analytical solutions for circular plates. Perhaps the most important point is that this boundary not only accounts consistently and accurately for the true physical behavior, but is also easy to implement and not very time consuming. It allows, furthermore, placing the lateral boundary directly at the edge of the foundation, with a considerable reduction in the number of degrees of freedom and corresponding savings in time of computation.

To visualize better the effects of the lateral boundaries, it is necessary to consider the physical behavior of waves radiating from the foundation. For a half-space, this radiation takes place in all directions. For a layered stratum resting on rigid rock, radiation can only take place laterally and for frequencies higher than the first natural frequency of the stratum. Thus, below this frequency, all boundaries should produce essentially the same results. If this is the range of frequencies of interest, the lateral boundary conditions are of little concern. At the resonant frequency of the stratum and above it, a considerable amount of radiation takes place in a true two-or three-dimensional situation. Notice that this lateral radiation would not occur in a one-dimensional situation, and is therefore masked entirely when using finite elements very elongated horizontally. The need to reproduce properly this effect depends thus on the frequency range of interest, the depth and properties of the soil stratum, its internal damping, and the type of excitation.
1.6 Material Properties

It has been generally recognized that the nonlinear characteristics of the subgrade material are among the most important factors controlling the magnitude of soil effects. Therefore, dynamic analyses for soil amplification and soil structure interaction problems must account, in at least an approximate way, for the dependence of the soil properties on the levels of strain.

While it is theoretically possible to perform true incremental analyses in which the subgrade properties are adjusted according to the instantaneous load path and levels of strain, such alternative is seldom used in practical situations. Instead, approximate nonlinear solutions are obtained using an iterative scheme originally suggested by Seed and Idriss (20). In this procedure, each iteration (in the frequency domain) involves a linear solution, but the soil properties are chosen at the beginning of each cycle so as to be consistent with the levels of strain computed in the previous iteration. This level is usually measured by a characteristic value, which is typically a fraction of the peak principal shear strain. In (10), the characteristic strain is defined as:

\[
\gamma_{\text{char}} = \frac{2}{3} \left( \frac{\text{peak input acceleration}}{\text{RMS principal shear strain}} \right)
\]

The main advantage of this approach is that the RMS levels can be derived directly from the frequency spectrum of the strains, without having to compute time histories by means of inverse Fourier transformations:

\[
\text{RMS}(\gamma) = \sqrt{\frac{1}{2\pi T} \int_{-\infty}^{\infty} \left[ |\Gamma_{xy}|^2 + |E_{xx} - E_{yy}|^2 \right] |F|^2 d\omega}
\]

where \( T \) = duration of excitation; \( F \) = Fourier transform of input earthquake; \( \Gamma_{xy}, E_{xx}, E_{yy} \) = transfer functions of the components of strain; \( \gamma \) = principal shearing strain, and \( \omega \) = frequency.

As of this writing there are no studies that justify the applicability of the iterative algorithm in a two or three dimensional situation. Nevertheless, the validity of this scheme is being extrapolated to the multidimensional wave propagation situation, with the principal shearing strain taken as the typical measure of strain. In some cases, the iteration is carried out by reducing proportionally both the shear modulus and Young modulus. A better alternative is to carry out the iteration on the shear modulus only, maintaining either the bulk modulus or the constrained modulus constant. This ensures that as the material softens, it does not become at the same time more compressible.

In connection with the application of the iterative method to the finite element formulation, it is useful to distinguish between primary and secondary nonlinearities. The former are the result of the earthquake waves propagating vertically through the layered subgrade and take place even when no structure is present; the latter are the result
of the disturbances produced by the soil structure interaction effects and are generally restricted to the immediate neighborhood of the structural foundation.

The primary nonlinearities can be estimated economically with the iterative analysis of the one-dimensional amplification problem. These results are then used as initial values of the soil properties in the total solution with the finite element discretization of the structure and the soil. It is shown in (10) that the relative effect of the secondary nonlinearities is not important, even though local strain levels may be altered. Because of the uncertainties in knowledge of the soil properties, the refinement obtained by incorporating the secondary nonlinearities in the finite element analysis does not seem worth the expense.

2. Seismic excitation (Free-field ground motion)

It has become accepted practice in the seismic analysis of nuclear power plants to define design earthquakes on the basis of smooth response spectra, which will supposedly envelop over the frequency range of interest the response of single degree of freedom oscillators to any credible ground motion. These spectra constitute therefore a safe basis for the design of 1-DOF systems.

Sets of these responses spectra, and rules to construct them, have been derived from the normalized records of actual ground motions using statistical analyses. Most of the motions were recorded on firm ground, particularly in the West Coast of the United States. The question arises, therefore, as to the validity of using such rules in other parts of the world where seismic motions may have very different characteristics (focal depth, attenuation laws, etc.)

The use of an envelope spectrum makes sense for multi-degree of freedom systems when applied directly to estimate maximum model responses, which are then superimposed in an approximate way (square root of the sum of the squares). It is customary, however, to generate artificial earthquakes (stochastic processes) whose spectra will envelope everywhere the design spectra. Additional questions are introduced by this step since the synthetic accelerograms have little physical significance and their features can be arbitrarily changed while satisfying the envelope condition. Small adjustments in the motions can produce noticeable differences in the final results.

The most important point in this phase of the design is, however, the definition of where the control motion is supposed to take place. There are at least five different possibilities, illustrated in Fig. 1-a:

a. The motion is specified at the free surface of the soil deposit, without any structure (Point A in Fig. 1-a). If the particular soil profile under consideration can be classified as “firm ground” (i.e. it has somewhat similar characteristics to the soil at the sites where the available motions were recorded), this assumption would make sense. It implies basically that amplification effects by the soil are already included in the selected spectra.
b. The motion is specified at the hypothetical outcropping of rock, without any structure (Point B in Fig. 1-a). Some questions arise then as to the exact definition of rock. In order to be consistent, this material should have the same elastic properties as the assumed “bedrock” underlying the soil deposit. Selection of the design earthquake or spectrum should then be made on the basis of these properties.

c. The motion is specified at “bedrock” (Point C). The same question arises here as to the definition of rock, which is somewhat arbitrary. Both elastic properties and depth are normally considered in the selection of the rock level. Physically, this option is not as logical as the previous two, since the motion that occurs at the base of the soil stratum will be dependent on the characteristics of the stratum itself. Results will not be very different from assumption (b) if there is a very marked difference in elastic properties between the soil and the rock, or if there is a considerable amount of internal damping in the soil.

d. The motion is specified at the level of the foundation but far away from the structure, so that any effects of structure or excavation may be neglected (Point D). This option is the same as (a) for a surface foundation, but differs considerably from it for an embedded structure. Its implications are hard to interpret physically, since the motion at any level within the soil mass has to depend on the soil characteristics. Its meaning is further confused if several adjacent structures are founded at different levels.

e. The motion is directly specified under the foundation, with the assumption of a rigid, massless slab, and before any structure has been built (Point E). It would be again the same as (a) or (d) for a surface foundation, but it is substantially different for an embedded structure. This option has some practical advantages when implementing it in the substructure method, since it bypasses entirely the first step.

A unique relationship between the motions at points A, B, C, D, and E can be obtained for an elastic material and any specified class of waves. In general, however, the relationship is obtained by assuming shear waves propagating vertically through the soil. This constitutes the one dimensional amplification theory, which has been extensively studied. The theory has obvious shortcomings: It assumes only one type of waves and does include, therefore, only the shear modes of the soil stratum. In addition, if the material is assumed linearly elastic, it would produce a unique amplification function irrespective of the magnitude of the earthquake, and if it is applied with the assumption of rigid bedrock, as often done in lumped mass or finite element models, it may result in unrealistically high amplifications at the fundamental frequency of the stratum, unless a very large amount of internal damping is assumed. Some of these shortcomings can be overcome by including the radiation effect in the underlying rock and by accounting for nonlinearity of the soil through non-linear time integration or approximate iterative linear analyses.

On the other hand, two dimensional wave propagation analyses with SV and P waves at arbitrary angles show amplification functions not very different from those predicted by the one dimensional theory, except for unusual combinations of amplitudes of both types
of waves or angles of incidence close to the horizontal. An extensive number of studies have shown in fact surprisingly good agreement between results obtained from the one-dimensional theory and actual recordings of an earthquake motion at different depths or different locations, and the theory has explained successfully the difference in motions in different parts of the Caracas’ Valley during the 1967 earthquake, the frequency content of motion in the Valley of Mexico, etc. Hence, a judicious application of the one-dimensional amplification theory provides a good qualitative picture of the effect of local soil conditions on the characteristics of the ground motion. More recently, however, doubt has been cast on the theory and even on the existence of any soil amplification from a failure to observe clear soil effects in the Los Angeles earthquake. It is important to recognize the limitations of the one-dimensional theory and the need to exercise engineering judgment in its application. Outright negation of its validity (and physical evidence) is on the other hand unwise.

Both the direct solution and the first phase of the substructure approach start normally with the specification of the uniform motion along the bottom boundary. They assume, therefore, that the motion is known at bedrock. Furthermore, since the motion is uniform along the rock-soil interface, the validity of one-dimensional theory is implied as a fundamental assumption, although this fact may not be always recognized. It would be possible, of course, to specify a motion that travels along this boundary, but this is equivalent to specifying the angle incidence of the seismic waves, introducing another arbitrary decision without proper justification. Thus, unless option (c) is adopted, with the design motion applied at bedrock, some preliminary computations are necessary to determine the motion at the base of the stratum which would produce the design earthquake at the level where it is specified, and one-dimensional amplification theory, in spite of its shortcomings, is the most appropriate way to perform this conversion consistently with the other phases of the analysis.

The determination of a bedrock motion compatible with the design earthquake is not needed for option (c) and is avoided with option (e) if the substructure approach is used and the first step is ignored. It is important to notice, however, that these two situations are not consistent and will yield different results. The computation is minimal with option (b) and not needed in fact if the other parts of the analysis include an elastic half-space at the base of the soil stratum. Option (a) will require a deconvolution of the motion from the surface to bedrock. The computation, if based on one-dimensional theory, is straightforward and consistent for a linearly elastic soil. It may present some problems if nonlinearity of the soil is included, since there is no guarantee then of the unicity of the solution. Base motions with rather different characteristics may create very similar surface motions with nonlinear soil behavior. This situation arises in particular for very deep strata. These problems become even more serious for option (d) with an embedded foundation. Then one-dimensional theory will then predict at some frequencies the existence of nodes at the foundation level, and the transfer function from this level to bedrock will have very large peaks at these frequencies (infinite peaks if there is no damping in the system). To avoid these peaks, gross manipulations are sometimes performed: the mass of the soil above the foundation level is lumped into a rigid block or removed completely; the soil properties are changed around these
frequencies; or the stratum depth is slightly changed. All these modifications have no physical significance and constitute arbitrary practices, which defeat the purpose of using sophisticated methods of analysis. Their net effect is to replace a zero term by a very small number in the denominator of a fraction, without any logical justification for the magnitude of this number.

Of the five options available, the most meaningful ones from the physical standpoint are:

- to specify the design motion at the free surface of the soil if the stratum can be classified as firm ground, since amplification effects would then be already included in the prescribed earthquake. The determination of a consistent base motion using one dimensional theory should not present problems in this case.

- to specify the design motion at a hypothetical outcropping of rock, if the soil profile cannot be classified as a standard firm ground and clear amplification effects are to be expected. The specified motion should then have characteristics consistent with the properties of the rock (it will be generally somewhat smaller than the firm ground motion). If there is substantial internal damping in the soil (high strain level), or the soil properties are markedly different from those of the underlying rock, specifying this motion at bedrock would be essentially equivalent. An argument often presented against the specification of the motion at the free surface is that the motion predicted at the nodal frequencies above-mentioned (for a homogeneous soil deposit above the foundation level, these frequencies are \( f = (2n-1)C_s / 4h \), where \( h \) is the depth of embedment). It is argued that accounting for this deamplification may be unconservative since the nodal points may not occur for other types of waves. The basic concern is thus motivated by a lack of confidence on the one dimensional amplification theory. (It is somewhat ironic that on the other hand the theory is explicitly or implicitly accepted in all remaining phases of the analysis). If this concern prevails, the logical option from a practical standpoint is to specify the motion directly under the foundation (option e) using the substructure approach. While this solution does not have a physical justification (especially if there are several adjacent structures founded at different levels), it has the practical merit of avoiding some of the artificial and arbitrary adjustments used at present in figuring back the bedrock motion.

3. Solution Methods

3.1 Direct Approach

In this approach, the structure (or structures) and the surrounding soil are analyzed together. The excitation is in the form of a base motion (prescribed directly or computed as discussed in the previous section), or in the form of equivalent lateral forces applied on the structures. Finite elements and regular linear members are normally used to model the different components of the system. Finite difference schemes may also be used to model the soil, although this procedure is less frequently used in practice.

The resulting equations of motion can be solved by two general methods:
a. Solution in the **time domain**, in which the system of differential equations is solved directly through a step-by-step integration with respect to time.

b. Solution in the **frequency domain**, in which the transfer function of any desired effect is obtained by solving at each frequency a system of linear algebraic equations, and the time history of response is then computed through the use of Fourier transforms. (Finding the direct transform of the excitation, multiplying it by the transfer function and obtaining the inverse-transform of the product.)

The first method allows for treatment of nonlinear behavior if the material properties are adjusted at each time step according to the instantaneous strain level. This type of solution is generally expensive since it involves a large number of degrees of freedom to keep the size of each element appropriately small, and the time step of integration to obtain reliable results must be correspondingly short. Special care must be exercised in using a numerical integration procedure and a time step which guarantee not only stability of the solution, but more importantly, its accuracy. In order to decrease the computational effort and the need in some well established methods to reproduce all the modes of the system, even if they do not contribute significantly to the solution, there has been a tendency to use algorithms which are unconditionally stable, that is, methods in which the solution will not blow up whatever the time step of integration. Unfortunately, mathematical stability is not a guarantee of accurate results and many of these methods will not only filter out the unnecessary modes but will also distort the significant ones. It makes little sense to use a sophisticated and expensive mathematical model with a poor solution algorithm. These problems can be avoided for a **linear solution** (or an iterative linear solution where nonlinear behavior is simulated by adjusting modules and damping in each cycle as a function of the strain level) by using modal analysis. It is then possible to determine the dynamic response with the necessary accuracy only for the limited number of modes which contribute significantly to the response.

A point of concern is the reproduction of the internal dissipation on energy in the soil when a true nonlinear analysis in not performed. The main source of this energy loss is the hysteretic behavior of the material, and there is considerable evidence that the loss per cycle is independent of frequency. This constitutes what has been called structural damping or, in more general terms, linear hysteretic damping. The best form to reproduce it, in the frequency domain, is through the use of complex moduli. Its reproduction in the time domain is somewhat harder unless the natural frequencies and mode shapes of the systems are known (the natural frequencies are at least necessary); Damping matrices proportional to the mass matrix, the stiffness matrix, or a combination of both (the so-called Rayleigh damping) are often used because of their simplicity. They produce energy losses that vary with frequency and are thus liable to excessively filter out low frequencies (damping matrix proportional to the mass matrix), high frequencies (damping matrix proportional to the stiffness matrix), or both (Rayleigh damping). Care must be exercised, therefore, when using these damping matrices, to ensure that the complete frequency range of interest has reasonable damping.
On the other hand, if is always possible to generate a damping matrix which ensures reasonable damping values throughout the frequency range of interest. This damping matrix is simply obtained from the equation:

\[ \mathbf{C} = 2 \mathbf{\Phi}^T \mathbf{B} \mathbf{\Lambda} \mathbf{\Phi}^{-1} \]

where \( \mathbf{C} \) = damping matrix; \( \mathbf{\Phi} \) = modal matrix of the system, normalized with respect to the mass matrix \((\mathbf{\Phi}' \mathbf{M} \mathbf{\Phi} = \mathbf{I})\); \( \mathbf{B} = \text{diag}(\beta_i) \) = diagonal matrix with fractions of critical modal damping \( \beta_i \); \( \mathbf{\Lambda} = \text{diag}(\omega_i) \) = diagonal matrix with modal frequencies \( \omega_i \); and the exponent \(-T\) identifies the inverse transposed. This procedure has the disadvantage that modal shapes and frequencies have to be found first; also, once available, it is more advantageous to use a modal superposition analysis with only the significant modes (modal synthesis).

The frequency solution on the other hand, is limited to linear problems since it is based on the applicability of the principle of superposition. Nonlinear soil behavior must then be simulated in an approximate way by performing an iterative analysis and adjusting in each cycle modulus and damping according to some characteristic measure of strain resulting from the previous cycle. The validity of this procedure has been verified for the one dimensional amplification problem when the initial period of the stratum (for low levels of strain) is not too long and the motions are of small or moderate intensity. The approximation deteriorates, however, for deep or soft profiles and strong shaking. A proper verification of its validity for a two-dimensional case is still lacking. The procedure makes sense, however, from a physical point of view, and it may be acceptable when considering the uncertainties involved in estimating soil properties and their variation with strain. A frequency solution has some inherent advantages: it allows controlling the accuracy of the solution within different ranges of frequencies and once the transfer functions have been completed, it permits to change the control motion, or its location, without having to repeat the complete procedure.

An interesting alternative when applying a frequency domain solution is the use of Hermitian interpolation to compute the transfer functions. To illustrate this point, consider the matrix equation of motion of the soil-structure system:

\[ (\mathbf{K} + i\omega \mathbf{C} - \omega^2 \mathbf{M}) \mathbf{u} = 0 \]

where \( \mathbf{M}, \mathbf{C}, \mathbf{K} \) are the mass, damping and stiffness matrices; \( \mathbf{u} \) is the vector of absolute displacements (includes boundary points where motions are prescribed); and \( \omega \) is the driving frequency. The solution \( \mathbf{u} \), as usual, follows after application of specified displacements at given points (typically the rock-soil interface, that is, the bottom boundary). For linear hysteretic damping, \( \mathbf{C} = \frac{\xi}{\omega} \mathbf{D} \), and the equation can be written as

\[ (\mathbf{K} + i\mathbf{D} - \omega^2 \mathbf{M}) \mathbf{u} = 0 \]
Taking derivatives of this equation w/r to frequency yields

\[
\left( K + iD - \omega^2 M \right) u' - 2\omega M u = 0 \\
\left( K + iD - \omega^2 M \right) u'' = 2\omega M u
\]

Notice that the solutions of both \( u \) and \( u' \) involve the same dynamic stiffness matrix \( K_d = K + iD - \omega^2 M \). The difference is only in the loading condition. It follows that once \( K_d \) has been brought to its triangular form, it requires very little additional effort to compute \( u' \), particularly if \( M \) has a diagonal form. The procedure would be then to triangularize the dynamic stiffness matrix, \( K_d \), to solve for \( u \) (actually, for \( y = u - u_g \)), to form the new loading vector \( p = 2\omega M u \), and to solve for \( u' \). Fewer points are thus needed in the frequency domain to define properly the transfer functions, since both the ordinates and the slopes at each frequency are known. Similar procedures can also be employed for other dynamic loadings. However, the method cannot easily be extended to the case where consistent transmitting boundaries are used, because their stiffness matrices are functions of frequency. While it is possible to find the derivatives of these matrices w/r to frequency, the process is cumbersome and probably not worth the effort. In essence, the process would require the solution of a linear system of equations on the form

\[
R' A^{-1} R + R A^{-1} R' = 2\omega M
\]

where \( A, M \) are tri-diagonal matrices, \( R \) is the stiffness (rigidity) matrix of the far field, and \( R' \) is the derivative of \( R \) with respect to \( \omega \).

The direct approach discussed in this section has two main advantages:

a. It allows to solve a true nonlinear dynamic problem, where superposition is no longer valid, accounting both for the nonlinear effects in the soil amplification problem (variations of properties with depth) and in the interaction problem (variations of properties both in depth and with horizontal distance). This type of solution is, however, rarely used. It needs, in addition, further research.

b. It allows including the effect of the flexibility of the mat, and its exact connection to the structure in the three dimensional or cylindrical models. This is not so pertinent when using the two-dimensional approximation, since the structure is then normally modeled as a series of members connected at the center of the mat.

The main advantage of the direct solution is its relative cost, since a larger number of degrees of freedom are treated simultaneously (those representing the soil and those of the structures). Fewer parametric studies are normally conducted with this approach, since variations in the properties of any part of the system require a complete new analysis.
**3.2 Substructure method**

The general equations of motion for the soil-structure system considered in the direct approach can be written in matrix form as:

\[ \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{0} \]  \hspace{1cm} (1)

where \( \mathbf{y} \) is a vector of relative displacements, \( \mathbf{u} \) the vector of absolute accelerations, and \( \mathbf{y} = \mathbf{u} - \mathbf{u}_g \), where \( \mathbf{u}_g \) is a generalized ground acceleration vector. It is possible, alternatively, to write this equation in the form of 2 equations

\[ \mathbf{M}_1\ddot{\mathbf{u}}_1 + \mathbf{C}\dot{\mathbf{y}}_1 + \mathbf{K}\mathbf{y}_1 = \mathbf{0} \]  \hspace{1cm} (2)

\[ \mathbf{M}\ddot{\mathbf{y}}_2 + \mathbf{C}\dot{\mathbf{y}}_2 + \mathbf{K}\mathbf{y}_2 = -\mathbf{M}_2\ddot{\mathbf{u}}_1 \]  \hspace{1cm} (3)

where \( \mathbf{u}_1 = \mathbf{y}_1 + \mathbf{u}_g \), \( \mathbf{u} = \mathbf{u}_1 + \mathbf{y}_2 \), \( \mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2 \), and \( \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 \). \( \mathbf{M}_1 \) represents the mass of the system excluding the mass of the structure, while \( \mathbf{M}_2 \) represents exclusively the mass of the structure. (\( \mathbf{M}_1, \mathbf{M}_2 \) are conveniently filled with zeros to match the dimensions of \( \mathbf{M} \)). The equivalence of eq. 2) and 3) with 1) is demonstrated by simple addition.

In eq. 2, the response of the massless structure is found first, and can be referred to as the *kinematic interaction*. The results are then used in eq. 3, which defines the *dynamic interaction*, and which is solved by application of fictitious inertia forces applied to the structure alone (Fig. 2).

In the solution of the second step, it is irrelevant whether the soil is modeled with finite elements or with equivalent stiffness functions. Only for the particular case of structures with no embedment, and under the additional assumption of vertically propagating earthquake waves, the kinematic interaction phase can be omitted.

For this case, phase 2 can equivalently be solved by specifying a uniform support motion under the “springs” modeling the subgrade. On the other hand, kinematic interaction does take place for surface foundation if the stress waves in the subgrade don not propagate vertically; also, it is fundamental for embedded foundations, even when the stress waves propagate vertically. This means that the “support” motion to be applied under the “springs” is different from the motion occurring at the location of the foundation when the structure is not present.

For this particular situation where the combination foundation – structure is very rigid (as is the case for nuclear containment structures), it is possible to remove the structure altogether from equation (2) and replace it by an infinitely rigid, massless foundation. This is legitimate, since the structure in this step is acting as a rigid body without mass. Equation (2) describes then the solution of a massless rigid foundation subjected to the specified ground motion.
The vector $\mathbf{y}_2$ can then be regarded as the displacements relative to a fictitious support, while $\mathbf{u}_1$ is the equivalent support motion. For a rigid foundation (slab and lateral walls if the structure is embedded) it is therefore valid to break the solution into three steps (Fig. 3):

a. Determination of the motion of the massless rigid foundation when subjected to the same input motion as the total solution. This is the solution of equation (2). For an embedded foundation it will yield in general both translations and rotations.

b. Determination of the frequency-dependent subgrade stiffness for the relevant degrees of freedom. This step corresponds formally to a dynamic condensation of the degrees of freedom of the soil. It yields the so called soil “springs”.

c. Computation of the response of the real structure supported on frequency dependent soil “springs” and subjected at the base of these “springs” to the motion computed in a). This analysis can again be performed in the frequency domain or in the time domain. Some problems arise in the last case because of the frequency dependence of the soil stiffness, and various approximations are introduced to account for this effect and for the lack of normal modes.

The only approximation involved in this approach concerns the deformability of the structural foundation. If this foundation were rigid, the solution of this procedure would be identical to that of the direct approach (assuming of course consistent definitions of the motion and same numerical procedures).

**Step a (Kinematic Interaction)**

The first step is particularly simple for a surface foundation, since it requires only the solution of the one-dimensional amplification problem to produce results consistent with those of the direct approach. Since this is a one-dimensional problem, it can be solved economically with continuous solutions or discrete models. For an embedded foundation, however, it requires the use of the same numerical techniques described for the direct solution (finite elements or finite differences). This fact would make the procedure not attractive since one could equally well include the structure at little extra cost. However, it is possible to obtain excellent approximations to the kinematic interaction problem on the basis of one-dimensional wave propagation theory. The method is described in detail in ref. (16). A simplified version is given below.

Let $F(\omega)$ be the Fourier transform of the acceleration at the free surface in the free field (in most cases, the design earthquake). The translation and rotation of the massless cylindrical foundation of radius $R$ are then given approximately by

$$
\ddot{u} = \text{IFT} \begin{cases} 
F(\omega) \cos \frac{\pi f}{f_n} & f = \frac{\omega}{2\pi} \leq 0.7 f_n \\
0.453 F(\omega) & f > 0.7 f_n
\end{cases}
$$

$$
\dot{\phi} = \text{IFT} \begin{cases} 
F(\omega) \times 0.257 \left(1 - \cos \frac{\pi f}{f_n}\right) / R & f \leq 0.7 f_n \\
0.141 F(\omega) / R & f > 0.7 f_n
\end{cases}
$$
where IFT stands for Inverse Fourier Transformation; \( f_n \) is the fundamental shear beam frequency of the embedment region (for uniform soil properties, this value is given by \( f_n = C_s / 4 / E \), with \( C_s \) being the shear wave velocity, and \( E \) the depth of embedment); the expression in brackets describes an approximation to the transfer functions for the translation and rotation of the massless foundation. For surface footings, \( E = 0, f_n = \infty \) and \( \phi = 0 \). It follows that no kinematic interaction takes place for this case. A comparison between the transfer functions and response spectra using both the approximation and the true functions is shown in figures 4, 5, 6, 7. As can be seen, the approximation yields satisfactory results, particularly for the translation.

**Step b (Stiffness Functions)**

The frequency dependent soil stiffness would again be computed for an embedded foundation by methods similar to those used in the direct approach. The procedure is to subject the base of the foundation slab (Fig. 2), assumed infinitely rigid and massless, to unit steady state harmonic displacements and rotations, and determine the corresponding reactions (terms of the stiffness matrix). Alternatively, one could specify unit harmonic forces and moments and determine the steady state displacements and rotations, obtaining flexibility coefficients (compliance functions). The stiffnesses are then obtained by inversion of the flexibility matrix. Each stiffness coefficient is of the form

\[
k_i = k_0 (1 + 2i\beta)(k + ia_0c),
\]

where \( k_0 \) is the static stiffness, \( \beta \) is a measure of the internal damping in the soil (of a hysteric nature) and \( a_0 \) is the dimensionless frequency \( \omega R / C_s \) (\( \omega \) is the circular frequency of the motion and excitation, \( R \) is the radius of the foundation slab, and \( C_s \) is a reference shear wave velocity). The functions \( k \) and \( c \) are frequency dependent coefficients, normalized with respect to the static stiffness. The coefficient \( c \) is related to the energy loss by radiation.

There are several particular cases for which the solution of step b) can be performed without finite elements. Solutions for the stiffness coefficients are available in the literature for circular footings on the surface of a uniform half space (12), (22) and layered half space (13). Solutions can also be obtained for surface strip footings (2-dimensional case) or rectangular foundations (3-dimensional case) on a layered medium with elastic or rigid rock at the base (5). All of these solutions can be mathematically exact, although sometimes small approximations are introduced (smooth versus rough footing). While these solutions apply only to surface foundations, studies with the finite element method (8) seem to indicate that the frequency variation of these functions is not very different for embedded foundations. The main effect of embedment seems to be in the static stiffness \( k_0 \). This would allow using finite element techniques just for the static case, with considerable savings. It would also seem that good approximate relationships can be obtained to account for the increase in stiffness in terms of two embedment ratios: depth of embedment to depth of stratum, and depth of embedment to radius of foundation. It is in this step where a considerable amount of research has been done, and where a large number of different approximations are often used. Different possibilities encountered in practice are:
• to determine by finite elements the static stiffness of the embedded foundation and to assume the same frequency variation as for a surface footing on the actual layered medium.
• to use the frequency-dependent stiffnesses corresponding to a surface footing on the layered medium, scaled throughout the frequency range by factors derived from approximate relationships to account for embedment. More work is necessary to justify these relationships and extend their range of application.
• to use the static stiffnesses corresponding to the actual situation and assume that the frequency variation is the same as for an elastic half-space.
• to use frequency-independent stiffness functions with the correct static value (obtained by finite elements).
• to use frequency-independent stiffness functions with approximate static values, derived from the approximate relationships and the half-space static value.
• to use outright the frequency-dependent stiffnesses for a half-space without consideration of layering or embedment.
• to use frequency-independent stiffnesses with the static values corresponding to a half-space, without consideration of layering or embedment.

Each one of these approximations represents a reduction in accuracy. A number of comparative studies indicate that it is more important to reproduce correctly the static stiffnesses than their complete frequency variation. It is also worth noticing that the increase in stiffness due to embedment is very sensitive to the properties of the lateral soil, which may be disturbed. Considering in addition the uncertainties in the soil properties and its nonlinear behavior, it is clear that engineering judgment is needed in the selection of the most appropriate model and that parametric studies in which the assumed conditions are varied within reasonable limits, are advisable (this would also apply to the direct approach).

Analytical or closed form solutions are not available for the case of deeply embedded foundations, although the effects of deep embedment have been studied using both numerical and experimental techniques. In essence, the embedment effects are expressed in larger values for the static stiffnesses. The rocking stiffness, in particular, may increase in value by a factor of 1.5 to 3, depending on the degree of adhesion of the lateral wall to the soil. It is therefore evident that embedment may result in significant shifts in the resonant rocking-swaying frequencies, even when the lateral walls are not welded to the backfill.

If the subgrade consist of a shallow stratum, or is composed by clearly distinct layers, then due consideration must be given to the reduction in the amount of radiation damping when using solutions for the homogeneous half-space in combination with correct factors to modify embedment. This problem is solved automatically when using analytical solutions for the actual layered condition (5), (13), as well as in (dynamic) finite element solutions.

Conversely, finite element idealizations of very deep soils in which modulus increases uniformly with depth may predict unrealistically low radiation damping coefficients
because of the finite boundaries placed on the model. Unless the internal dissipation of energy (material damping) is high, a significant portion of it will be reflected at the boundaries. These problems can be overcome using large finite element models, or alternatively, using consistent transmitting boundaries. If these requirements cannot be met, then more realistic results are obtained with a half-space solution modified by correction factors to account for embedment conditions. Excellent approximations to the stiffness functions of embedded circular foundations, provided that the subgrade properties change slowly and uniformly with depth, can be obtained as follows (4):

**Static Values**

\[
K_{xx}^0 = \frac{8GR}{2 - \nu} \left(1 + \frac{1}{2} \frac{R}{H}\right) \left(1 + \frac{2}{3} \frac{E}{R}\right) \left(1 + \frac{5}{4} \frac{E}{H}\right)
\]

\[
K_{x\phi}^0 = K_{xx}^0 \frac{2}{5} \frac{E}{R} - 0.03
\]

\[
K_{\phi\phi}^0 = \frac{8GR^3}{3(1 - \nu)} \left(1 + \frac{1}{2} \frac{R}{H}\right) \left(1 + \frac{2}{5} \frac{E}{R}\right) \left(1 + \frac{0.71}{E} \frac{E}{H}\right)
\]

\[\frac{E}{H} \leq 0.5\]

In these formulas, \(G\) = the shear modulus of the soil underneath the mat; \(R\) = the radius of the foundation; \(E\) = the depth of embedment; \(H\) = the depth to bedrock; and \(\nu\) = Poisson’s ratio.

**Dynamic Stiffnesses**

\[
K_{xx} = K_{xx}^0 \left(k_1 + i a_0 c_1\right) \left(1 + 2i D\right)
\]

\[
K_{x\phi} = K_{xx}^0 \left(k_1 + i a_0 c_1\right) \left(1 + 2i D\right)
\]

\[
K_{\phi\phi} = K_{\phi\phi}^0 \left(k_2 + i a_0 c_2\right) \left(1 + 2i D\right)
\]

where \(k_1, c_1, k_2, c_2\) are frequency dependent stiffness functions; \(a_0 = \omega R / C_s\) is the dimensionless frequency; and \(D\) = fraction of linear hysteretic damping. The stiffness coefficients can be obtained as follows:

\[
k_1, k_2 \text{ from reference (22)}
\]

\[
c_1 = \begin{cases} 
Da_0 & f \leq f_h \\
\text{ref (22)} & f > f_h
\end{cases}
\]
Here, $f_h, f_v$ are the first horizontal and vertical frequencies of the soil stratum (for uniform soil properties, they are given by $f_h = C_s / 4H$, $f_v = f_h \sqrt{2(1-v)/(1-2v)}$. As can be seen, the coefficients are approximated by those of the half-space, except for $c_1, c_2$ in the low frequency range.

For applications of these formulas to modal analysis, frequency independent values can be used, evaluating the coefficients $k_1, c_1, k_2, c_2$ at the fundamental frequency of the complete soil-structure system. This requires a trial and error procedure, which usually converges in a few iterations.

Simplified expressions for the half-space coefficients were suggested by Hall (19), which can be used in place of those reported by Veletsos and Wei for preliminary calculations. They are

$$k_1 = k_2 = 1$$

$$c_1 = 0.576$$

$$c_2 = \frac{0.3 a_0^2}{1 + a_0^2}$$

The $k_1, c_1, c_2$ given above are good approximations for the actual half-space functions, whereas $k_2$ is less satisfactory, particularly for high values of Poisson’s ratio.

**Step c (Dynamic Interaction)**

Once the input motion and the base stiffness are known, the last step is reduced to a simple dynamic analysis of a relatively small multi-degree of freedom system. The solution can be carried out in the time domain or in the frequency domain. A frequency solution is particularly convenient, since the stiffnesses are frequency dependent. It is simple and allows controlling the accuracy in different ranges of frequency. It allows furthermore to perform probabilistic type analyses, since spectral densities are directly related to the transfer functions (relatively little work has been done along the lines, perhaps because this type of consideration is still mainly of academic interest). The main complaint about this kind of solution has been the lack of familiarity with these techniques and a more difficult interpretation of the solution.

A time domain solution, and a modal solution in particular, offer a simpler physical interpretation of the results. They require, however, frequency-independent stiffnesses, and for the latter in addition the existence of normal modes. Several approximations are then introduced:
• The soil stiffness are assumed frequency independent. The constant value chosen can be a) the static value; or b) the values at a frequency corresponding to the fundamental frequency of the combined soil-structure system. This last option requires a trial and error procedure.

• The existence of normal modes is assumed. To compute the values of effective damping in each mode, weighted averages of the internal damping in the different components are computed. Several methods have been suggested to obtain a weighted modal damping. A procedure based on energy considerations (17) is outlined below. This method is actually equivalent to several others, is particularly simple and has been shown to give excellent results. It is important, however, in the application of these formulas to account properly for the type of energy dissipation present in the different components. The structural damping and the internal soil damping will be generally of a hysteretic nature. Radiation damping in the soil, on the other hand, is of a viscous form (with energy dissipation per cycle being function of frequency). Since this radiation damping can be very large (for a half-space or for a layer above its natural frequency), improper representation of its nature (frequency dependence) can lead to erroneous results. An alternative often used in practice, because of design regulations, is to limit damping in any mode to a maximum amount arbitrarily imposed on the basis of structural damping (of the order of 10 percent). This limitation will produce in many cases quite unreasonable results and is often the source of large discrepancies between the different approaches.

Roësset et al (17) have shown how viscous and linear hysteretic damping values can be combined into modal damping values by the Biggs-Roësset equation for weighted modal damping:

$$\beta_{eq} = \frac{\sum_j \beta_j \omega_i}{\omega_j} + D_j k_j \Delta_j$$

where

$$\beta_{eq} = \text{equivalent damping ratio in mode } i$$

$$\beta_j = \text{viscous damping ratio in component } j$$

$$\omega_i = \text{undamped resonant frequency of mode } i$$

$$\omega_j = \text{undamped resonant frequency of component } j \text{ as a single DOF. system.}$$

$$D_j = \text{linear hysteretic damping ration in component } j$$

$$k_j = \text{spring constant in component } j$$

$$\Delta_j = \text{strain of component } j \text{ in mode } i$$

Thus, a critical damping ratio can be assigned to each mode in such a way as to account for the variation of damping among the components and for the differences between viscous and hysteretic damping.
The substructure method has the advantage of being less time consuming when approximations are used for the kinematic interaction problem. It allows, therefore, conducting more parametric studies, and the accuracy of each step is subject to better control. Of particular importance is the possibility in this method to make use of symmetry or cylindrical conditions if the foundation meets these requirements, even if the structure does not (which is a frequent situation). The coupling between the corresponding terms will come in naturally in the third step. From a practical standpoint, the procedure has an additional advantage if the kinematic interaction phase is used to define the seismic motion components (translation, rotations and torsion), whatever the physical arguments against this type of specification. In this manner, undesirable deamplification of certain frequency components following the use of one-dimensional amplification theory can be avoided, at the same time that a full three-dimensional representation of the structure can be achieved. It is important to mention again that a solution consistent with the direct approach will involve both rotations and translations in an embedded massless rigid foundation. Furthermore, the translation is not, in general, equal to the control motion, nor is equal to the translation of the subgrade in the free field at the foundation level.

The main shortcomings of the substructure method are the inability in reproducing a flexible mat and in performing a true nonlinear analysis in the time domain. The first one does not seem to be important, although its effects have been studied only for a limited number of cases. The importance of the second is dependent on the relative weight of the secondary nonlinearities vs. the primary nonlinearities. From the results found in ref. 10 it appears that this shortcoming is also not important.
References


FIGURE 2: SUPERPOSITION THEOREM

FIGURE 3: THE 3-STEP SOLUTION
FIGURE 4: MASSLESS FOUNDATION
FIGURE 5: TRANSFER FUNCTIONS MASSLESS FOUNDATION
FIGURE 6: RESPONSE SPECTRUM FOR 1% DAMPING, TRANSLATION MASSLESS FOUNDATION
FIGURE 7: RESPONSE SPECTRUM FOR 1% DAMPING
ROTATION OF MASSLESS FOUNDATION