IMPEDANCES OF UNDERWATER RIGID SQUARE FOUNDATIONS

Amir M. Kaynia\textsuperscript{1}
Eduardo Kauser\textsuperscript{2}, M. ASCE
Christian Madshus\textsuperscript{1}

ABSTRACT
A frequency-wave number domain formulation is used in this paper to derive the Green's functions for underwater layered soil media. These functions are used to calculate the impedances of rigid square foundations resting on the seafloor. Four soil profiles with different mechanical properties are considered and the effect of increasing the water depth on the impedances of the foundation is investigated. The results show that the presence of water is essentially similar to that of an added mass vibrating with the foundation. Further, it is shown that the water has practically no additional influence on the foundation impedances beyond a depth of the order of the foundation size.

INTRODUCTION
Impedances of foundations are traditionally derived for soil-structure interaction formulations which are based on substructuring scheme. These functions define the stiffness property and the response attenuation characteristic of the soil-foundation systems and, as such, play a central role in soil-structure interaction analyses. This is reflected in the extensive research devoted in the past three decades to the understanding the behaviour of these functions and their derivations. These studies have covered a wide variety of soil-foundation types, including surface versus embedded foundations, 2-D versus 3-D solutions, homogeneous versus layered soil, and finite stratum versus semi-infinite half space. However, for practical reasons, these studies have almost exclusively dealt with foundations on land, leaving out offshore foundations from consideration. This paper aims at addressing this problem.

\textsuperscript{1} Analysis Division, Norwegian Geotechnical Institute, P.O. Box 3930 Ullevaal Hageby, N-0806 Oslo, Norway.
\textsuperscript{2} Professor, Civil and Environmental Engineering Department, Massachusetts Institute of Technology, Cambridge, MA 02139.
The mathematical formalism of wave equation separation in underwater acoustics was developed by Pekeris (1949) and extended later by Ewing et al. (1957) to seismic wave propagation in soil sites with acoustic layers. The solution technique is based on the application of Fourier and Hankel transforms to the wave equation in a solid/fluid layer to reduce it to a series of ordinary differential equations. These equations are then solved by imposing the appropriate stress and compatibility boundary conditions at layer interfaces and the free surface. The response quantities are finally determined by evaluating the pertinent inverse integral transforms. A systematic implementation of this solution technique for viscoelastic media was developed by Thomson (1950) and Haskell (1953) using the so-called propagator matrix method. Other variants of this technique include the stiffness matrix approach advanced by Kausel and Roesset (1981) and the finite-wave-element approach proposed by Schmit and Jensen (1985). The former approach was later extended to seismoacoustic (underwater) applications by Stokoe et al. (1990) in their experiments with SASW offshore.

The objective of the present paper is to use the stiffness matrix approach of Kausel and Roesset (1981) to derive the Green's functions for loads acting on the surface of a layered half-space under water. These functions are subsequently used to derive the impedances of rigid foundations on seafloor. The analyses include all modes of vibration (translational and rotational). The seafloor in this study is treated as a nearly-incompressible viscoelastic (nonporous) medium. A poroelastic treatment of the half-space has been presented by Bougacha et al. (1991), among others.

**PROBLEM STATEMENT**

The problem under consideration is shown schematically in Fig. 1. A rigid square footing of dimension $2ax2a$ is bonded to a subsea layered half-space. Each soil layer is characterized by its thickness, $h$, mass density, $\rho$, shear wave velocity, $V_s$, pressure wave velocity, $V_p$ (or alternatively, Poisson's ratio, $\nu$), and hysteretic damping ratio, $\xi$. The water is assumed to be an ideal fluid and is thus represented by its pressure wave velocity, $C$, and mass density, $\rho_w$. The last two parameters were fixed at 1500 m/s and 1000 kg/m$^3$, respectively, throughout the analyses. The depth of the water, $H$, was varied from 0.0 to large values.

The technique used here to calculate the impedances is based on the one proposed by Wong and Luco (1976). According to this technique, the contact surface between the

![Figure 1- Representation of Problem](image-url)
soil and foundation is discretized into a regular grid of rectangular elements. By applying uniformly distributed unit loads on each element in the three directions and calculating the displacements at all element centers (nodes), one can derive the flexibility matrix of the soil/foundation interface. Inversion of this matrix, and imposition of the kinematic conditions for the various vibration modes of the rigid foundation, leads to the determination of the impedances. This formulation hinges on the use of a dynamic Green's function for the medium in order to establish the soil flexibility matrix. In the present application, the stiffness matrix approach proposed by Kausel and Roesset (1981) was used. Implementation of this method is achieved by assembling layer stiffness matrices and solving the resulting equations for the specified forces. The calculations are carried out under steady-state harmonic vibration at discrete frequencies $\omega$.

**LAYER STIFFNESS AND GREEN'S FUNCTIONS**

The stiffness matrices for elastic solid layers, derived by Kausel and Roesset (1981), are incorporated here for completeness.

If $k$ denotes the wave number, $\mu$ is the shear modulus, and $q$ and $s$ are defined as

$$q = \sqrt{1 - (\omega / k V_p)^2}$$

$$s = \sqrt{1 - (\omega / k V_s)^2}$$

then the symmetric layer stiffness matrix for the "SV-P wave" case is given by

$$K = 2k\mu \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

where

$$K_{11} = \frac{1-s^2}{2D} \begin{bmatrix} \frac{1}{s} \left( C^g S^s - qs C^s S^g \right) & -\left( 1 - C^g C^s + qs S^g S^s \right) \\ -\left( 1 - C^g C^s + qs S^g S^s \right) & \frac{1}{q} \left( C^s S^g - qs C^g S^s \right) \end{bmatrix} - \frac{1+s^2}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$K_{22} =$ same as $K_{11}$, with off-diagonal signs changed

$$K_{12} = \frac{1-s^2}{2D} \begin{bmatrix} \frac{1}{s} \left( qs S^g - S^s \right) & -\left( C^g - C^s \right) \\ C^s - C^g & \frac{1}{q} \left( qs C^g - S^s \right) \end{bmatrix}$$
and the stiffness matrix of the half-space is:

\[ K = 2k\mu \left[ \frac{1-\nu^2}{2(1-\nu\mu)} \begin{pmatrix} q & 1 \\ 1 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right] \]  

\hspace{1cm} (4)

where

\[ C' = \cosh kqh \quad S' = \sinh kqh \]
\[ C^* = \cosh ksh \quad S^* = \sinh ksh \]
\[ D = 2(1-C'^*C^*) + \left( \frac{1}{qS} + qS \right)S^*S^* \]

The stiffness matrices for layer and half-space for the "SH-wave" case are

\[ K = \frac{k_s\mu}{\sinh ksh} \begin{pmatrix} \cosh ksh & -1 \\ -1 & \cosh ksh \end{pmatrix} \]  

\hspace{1cm} (5)

\[ K = k_s\mu \]  

\hspace{1cm} (6)

Finally, the stiffness matrix of a water layer with free surface is given by (Stokoe et al., 1990)

\[ K_w = -\frac{\rho\omega^2}{\beta} \begin{pmatrix} \cosh \beta h & \beta g \omega^2 & -1 \\ \sinh \beta h & \omega^2 & \sinh \beta h \\ 1 & \sinh \beta h & \cosh \beta h \end{pmatrix} \]  

\hspace{1cm} (7)

where \( \beta \) is defined as

\[ \beta = k \sqrt{1 - (\omega / kC)^2} \]  

\hspace{1cm} (8)

The stiffness matrices are assembled in a finite element sense and the displacements in the transformed domain are obtained for the desired forces. The steady-state responses (Green's functions) are then evaluated by applying the appropriate inverse Hankel transforms. For instance, it can be shown that the vertical response due to a disk load with radius \( R \) at a distance \( r \) is expressed as

\[ u_z(r, z) = \frac{1}{\pi} \int_0^\infty u_z(kr) \frac{J_1(kR)}{kR} kdk \]  

\hspace{1cm} (9)

where \( J_0 \) and \( J_1 \) are the zeroth order and first order Bessel functions of the 1st kind, respectively, and \( u_z \) is the vertical component of Hankel transformed displacement.
NUMERICAL INTEGRATION SCHEME

The integrals derived above for the various components of Green's functions have to be evaluated numerically. For this purpose, one needs to use a robust procedure to make sure that the sharp peaks in the variation of the integrands are captured within the wavenumber step Δk. A rigorous scheme has been proposed by Apsel and Luco (1983). It establishes the size of Δk by considering the location of the sharp peaks in the integrand, and uses asymptotic expansions of the Bessel functions for large wavenumbers. However, one could also adopt a simple integration procedure and obtain fairly accurate results by carefully selecting Δk and taking advantage of the exponentially decaying properties of the integrands. As a general rule, the upper end of the integration, \( k_{\text{max}} \) should be sufficiently larger than \( 2.0\omega V_{\text{s,min}} \), where \( V_{\text{s,min}} \) is the smallest shear wave velocity in the soil profile. The reason for this restriction is that the integrands are highly wavy below \( k_{\text{max}} \) while above this value they decay either uniformly or exponentially with wavenumber, depending on the elevation distance between the observation and source points.

Discretization of the integrals introduces an artificial spatial periodicity of the load which is another potential source of inaccuracy. For a selected Δk, the spatial period of the load is \( 2\pi/\Delta k \). Therefore, Δk has to be small enough to ensure an accurate representation of the load. These guidelines and rules have formed the basis for the numerical integration implemented in the present work.

RESULTS

This section presents the impedances of a square foundation with side dimension \( 2a = 4m \) for several seabed soil profiles. The derived impedances are complex quantities which are represented by their real and imaginary parts. In order to make comparisons with other published results easier, the translational (horizontal and vertical) impedances and the rocking impedance have been normalized by \( \mu a \) and \( \mu a^3 \), respectively, and their variations with frequency are portrayed in terms of the nondimensional frequency \( a_0 = \omega a/V_{\text{s,1}} \), where \( \mu \) and \( V_{\text{s,1}} \) are the shear modulus and the shear wave velocity of the top layer, respectively. (As the torsional impedance for the underwater condition is the same as that for the no-water case, it is not reported here.) All the results presented in this section were obtained by taking the mass density of soil \( \rho = 2000 \text{ kg/m}^3 \) and its hysteretic damping ratio \( \xi = 0.05 \). The Green's functions were evaluated by discretizing the soil/foundation interface into an 8x8 grid.

Figure 2 shows the real and imaginary parts of the normalized vertical impedance of the foundation on a uniform half-space with \( V_{\text{s}} = 150 \text{ m/s} \) and \( V_{\text{p}} = 1500 \text{ m/s} \) (water-saturated with Poisson's ratio = 0.495). To illustrate the influence of water depth on the impedance, the results are shown for \( H = 0.0 \) (no water) to \( H = 10m \). No results are given for higher depths as the impedance practically reached its asymptotic value at \( H = 10m \). Examination of the plots in Figure 2 suggests that the presence of water can be viewed essentially as an added mass-dashpot effect. The added mass is approximately equivalent to the mass of a block of water directly over the foundation with a height of 0.8m.
Figure 3 shows the rocking impedance of the foundation for the same set of parameters. Similar features are discerned for the rocking impedance, except that the approach of the curves to the asymptotic one is even faster as $H$ increases. The horizontal impedance for the same foundation is displayed in Figure 4. As expected, the water has practically no effect on this impedance component.

Fig. 2 - Normalized vertical impedance : half-space with $V_S = 150$ m/s and $\nu \approx 0.5$

Fig. 3 - Normalized rocking impedance : half-space with $V_S = 150$ m/s and $\nu \approx 0.5$
Fig. 4 - Normalized horizontal impedance: half-space with $V_s = 150$ m/s and $\nu = 0.5$

To investigate the effect of the half-space soil stiffness, the vertical and rocking impedances are calculated for a half space with $V_s = 300$ m/s, with all other parameters unchanged. Figures 5 and 6 display the variation of these impedances. Comparisons with Figs. 2 and 3 suggest that the normalized impedances are fairly insensitive to the shear modulus of the half-space.

Fig. 5 - Normalized vertical impedance: half-space with $V_s = 300$ m/s and $\nu = 0.5$
Fig. 6 - Normalized rocking impedance: half-space with \( V_S = 300 \) m/s and \( v = 0.5 \)

Figures 7 and 8 portray the normalized impedances for a two-layered soil profile consisting of a 4m-thick stratum with \( V_S = 150 \) m/s over a half space with \( V_S = 300 \) m/s, \( (V_P = 1500 \) m/s for both layers, as before). These figures are directly comparable to their counterparts in Figs. 2 and 3, as they are normalized with respect to the same shear modulus. As expected, the two sets of results converge at high frequencies. At low frequencies, on the other hand, the two-layer model is stiffer but displays less damping. The general trend of the results and the influence of water depth on the variation of the impedances are essentially similar.

In order to investigate the effect of compressibility contrast between the half-space and water, the vertical and rocking impedances for a half-space with \( V_S = 300 \) m/s and \( V_P = 750 \) m/s (Poisson's ratio = 0.4) were calculated and are displayed in Figures 9 and 10. These figures can be directly compared with Figs. 5 and 6 \( (V_P = 1500 \) m/s). It is interesting to observe that, in a more compressible half-space, the water has a more pronounced effect on the impedances at high frequencies. This parameter, however, has minor influence at low frequencies.

CONCLUSIONS

A frequency-wave number domain formulation was used in this paper to derive the impedances of rigid square foundations resting on the seafloor. Four soil profiles with different mechanical properties were considered and the effect of increasing the water depth on the impedances of the foundation was investigated. It was shown that the presence of water is essentially similar to that of an added mass vibrating with the foundation. It was also shown that the water has practically no additional influence
on the foundation impedances beyond a depth of the order of the foundation size. Further, the results showed that the water has a more pronounced influence in less compressible soil media under water.

Fig. 7 - Normalized vertical impedance: two-layered half-space with $V_{S1} = 150$ m/s and $V_{S2} = 300$ m/s

Fig. 8 - Normalized rocking impedance: two-layered half-space with $V_{S1} = 150$ m/s and $V_{S2} = 300$ m/s
Fig. 9 - Normalized vertical impedance: half-space with $V_s = 300$ m/s and $v = 0.4$

Fig. 10 - Normalized rocking impedance: half-space with $V_s = 300$ m/s and $v = 0.4$

REFERENCES


