SUMMARY

The paper presents simplified rules to account for embedment and soil layering in the soil-structure interaction problem, to be used in dynamic analyses. The relationship between the spring method, and a direct solution (in which both soil and structure are modeled with finite elements and linear members) is first presented. It is shown that for consistency of the results obtained with the two solution methods, the spring method should be performed in the following three steps:

1. Determination of the motion of the massless foundation (having the same shape as the actual one) when subjected to the same input motion as the direct solution. For an embeded foundation it will yield, in general, both translations and rotations.
2. Determination of the frequency dependent subgrade stiffness for the relevant degrees of freedom. This step yields the so-called "soil springs".
3. Computations of the response of the real structure supported on frequency dependent soil springs and subjected at the base of these springs to the motion computed in step 1.

The first two steps require, in general, finite element methods, which would make the procedure not attractive. It is shown in the paper, however, that excellent approximations can be obtained, on the basis of 1-dimensional wave propagation theory for the solution of step 1, and correction factors modifying for embedment the corresponding springs of a surface footing on a layered stratum, for the solution of step 2. Use of these rules not only provides remarkable agreement with the results obtained from a full finite element analysis, but results in substantial savings of computer execution and storage requirements. This frees the engineer to perform extensive studies, varying the input properties over a wide range to account for uncertainties, in particular with respect to the soil properties.

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1. Introduction

Significant efforts have been directed in recent years to formulate engineering solutions to the problems of vibration of foundations and seismic response of buildings. The problem is of special interest in the seismic analysis and design of massive buildings such as nuclear containment structures.

Building foundations, and nuclear reactors in particular, are usually buried to some extent beneath the surface of the ground. This embedment has in many cases considerable effect on the dynamic response of the structure, both in terms of relative frequency contents and amplitude of the resulting motions. Because of the complicated boundary conditions that must be satisfied in a theoretical formulation, rigorous analytical solutions for embedded foundations are nonexistent at present. Hence, numerical (finite element), experimental and approximate analytical techniques are currently being used to provide a solution to the problem at hand for these complicated geometries. An awareness of the effects associated with embedment, coupled with the availability of numerical solutions and the lack of rigorous solutions, has been the basis in recent times for discrediting the spring method as a tool for analyses, particularly in the nuclear power industry. The detractor of the spring method has been argued by some researchers on the basis of comparisons between the classical half space method, and more involved finite element solutions. Many of these comparisons are not meaningful, since they were based on inappropriate values for both the "spring constant" and the "support motion." In fact, the spring method and finite element solutions can be shown to be mathematically equivalent; if they are classified as different, it is because of inconsistencies in their implementation.

It is the purpose of this paper to show the relationship between a more general spring method, and the solutions provided by direct finite element procedures, and to present practical rules for use in dynamic analysis.

2. The basic superposition theorem

Referring to Fig. 1, assume that the general equations of motion of a structure-foundation system are given by the matrix equation

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = \mathbf{0}$$

where \(\mathbf{M}, \mathbf{C}, \mathbf{K}\), are the system mass, damping and stiffness matrices; \(\mathbf{U}\) and \(\mathbf{Y}\) are the absolute and relative displacements, with respect to some general ground reference system. The solution of this equation is equivalent to the solution of the two matrix equations

$$\mathbf{M}_1 \ddot{\mathbf{U}}_1 + \mathbf{C}_1 \dot{\mathbf{Y}}_1 + \mathbf{K}_1 \mathbf{Y}_1 = \mathbf{0}$$

$$\mathbf{M}_2 \ddot{\mathbf{U}}_2 + \mathbf{C}_2 \dot{\mathbf{Y}}_2 + \mathbf{K}_2 \mathbf{Y}_2 = -\mathbf{M}_2 \ddot{\mathbf{U}}_1$$

where \(\mathbf{U}_1 = \mathbf{Y}_1 + \mathbf{U}_g\), \(\mathbf{U}_2 = \mathbf{Y}_2\), and \(\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2\). \(\mathbf{M}_1\) excludes the mass of the structure, while \(\mathbf{M}_2\) excludes the mass of the soil. \(\mathbf{U}_g\) is some generalized ground motion vector. The equivalence of 2) with 1) is demonstrated by simple addition. In Eq. 2, the response of the massless structure is found first, and will be referred to as the kinematic interaction. The results of this step are then used in Eq. 3) which shall define the inertial interaction, and which is solved by application of fictitious inertia forces applied to the structure alone.

In the solution of the second step, it is irrelevant whether the soil is modelled with finite elements, or equivalently, with a (far-coupled) matrix of stiffness functions modelling the subgrade, and defined at the soil-structure interface. These stiffness functions can be
regarded as resulting from a dynamic condensation of all the degrees of freedom in the soil
(a frequency domain solution is implied).

For the particular situation where the combination foundation-structure is very rigid, it
becomes legitimate to replace the matrix of stiffness functions by the overall vertical, tor-
sional, rocking and swaying stiffness functions, i.e., by frequency dependent "springs and
dashpots." For this case, the nodal displacements at the foundation-soil interface are
linearly dependent and, at most, six degrees of freedom are needed to describe the motion of
the foundation. It also follows that the solution of the kinematic interaction phase is com-
pletely defined by the rotations and translations of the massless structure, which moves as a
rigid body. Hence, one can replace the massless structure in Eq. 2) by a rigid massless
foundation, subjected to the same ground excitation as the original system.

Also, a more careful examination of Eq. 3 will show that the solution \( Y_2 \) can be regarded
as a vector of displacements relative to a fictitious support, while the rigid body transla-
tions and rotations of the massless foundation in Eq. 2 are the equivalent support motion.

Provided that the assumption of rigid foundation is pertinent, it is, therefore, valid to
break the solution into three steps: (also see Fig. 2)

1. determination of the motion of the massless rigid foundation, when subjected to the
same input motion as the total solution. This is the solution of Eq. 2). For an
embedded foundation, it will yield, in general, both translations and rotations.

2. determination of the frequency dependent subgrade stiffnesses for the relevant
degrees of freedom. This step yields the so-called soil "springs."

3. computation of the response of the real structure supported on frequency dependent
soil springs, and subjected at the base of these springs to the motion computed in 2).

Notice that the only approximation involved in this approach concerns the deformability
of the structural foundation. If this foundation were rigid, the solution of this procedure
should be identical to that of the direct (or one-pass) approach (assuming, of course,
consistent definitions of the motion and the same numerical procedures).

The superposition principle is valid only for a linear system. While the modulus and
damping of the soil are strain-dependent, studies (6) have shown that most of the nonlinearity
occurs as a result of the earthquake motion, and not as a result of soil-structure interaction.
Thus, the soil properties consistent with the levels of strain in the free field (i.e., before
the structure has been built) may be used in steps 1 and 2 without further modification to
account for the additional strains imposed by the structure.

The first two steps require, in general, finite element methods, and thus it might appear
that the 3-step method has no advantage as compared to considering both kinematic and inertial
interaction together in a single step. However, reasonable approximations can be obtained on
the basis of one-dimensional wave propagation theory for the solution of step 1, and correc-
tion factors modifying for embedment the corresponding springs of a surface footing on a
layered stratum for the solution of step 2. Use of these rules not only provides remarkable
agreement with the results obtained from a full finite element analysis, but results in
substantial savings of computer execution and storage requirements. This frees the engineer
to perform extensive studies, varying the input properties over a wide range to account for
uncertainties, in particular with respect to the soil properties. Also, deviations from axial
symmetry may be introduced into the model of the structure in step 3 (which means that a
torsional "spring" and "dashpot" must be evaluated in step 2) and the effect of changes in the
mass and stiffness of the structure be evaluated without having to rerun an entire analysis.
3. Approximate Solution for Circular, Embedded Foundations

The solutions presented in the following sections have been obtained with a three-dimen-
sional axisymmetric finite element formulation. A fundamental feature of the program used is
the exact representation of the model boundary which separates the finite element region from
the semi-infinite continuum (the free field). This consistent energy transmitting boundary
was developed for the plane strain case by Maas and Lysmer (11), (7), and was extended to the
three-dimensional case by the first author (2), (3), (5). In essence, it can be regarded as
a virtual extension of the finite element mesh to infinity, and can be placed without loss of
accuracy immediately next to the foundation.

In the following section, it will be assumed that the motion which the ground experiences
before any structure (or hypothetical massless foundation) has been built, can be described by
means of one-dimensional wave propagation theory. The control motion, i.e., the specified
earthquake record, will be assumed to take place at the free surface in the "free field." Mo-
tions at other points in the free field can be obtained by the so-called "deconvolution"
process, which makes use of one-dimensional wave-propagation theory.

3.1 Approximations to the Kinematic Interaction

The third author investigated the kinematic interaction problem in a parametric study
(8), using a suite of embedment ratios E/R and stratum ratios H/R, covering a range of values
typically found in nuclear reactor design, and proposed rules to approximate the kinematic
interaction.

Referring to Fig. 3, a unit harmonic displacement was specified at the free surface, and
deconvolved to bedrock. Using the finite element program, frequency dependent transfer
functions $u$, $\phi$ for the displacement and rotation of the massless foundation, relative to the
motion at the free surface, were determined. Similarly, the frequency dependent transfer
functions for the displacement in the free field at the elevation of the foundation, and for
the pseudorotation of the free field

$$\phi_B (\Omega) = \frac{u_A - u_R}{E} \left( 1 - \frac{u_B}{E} \right)$$

were computed. Typical results are found in Fig. 4. The transfer functions $u$ and $u_B$,
$\phi$ and $\phi_B$ were then compared for a range of embedment and stratum depth values, and rules to
approximate these functions were suggested (8). A simplified version of these rules is given
below.

Let $F (\Omega)$ be the Fourier transform of the acceleration at the free surface in the free
field (in most cases, the design earthquake). The translation and rotation of the massless,
rigid foundation are then given approximately by

$$\ddot{u} = \text{IFT} \left\{ \begin{array}{ll} F(\Omega) \cdot \left[ \cos \left( \frac{\Omega}{2} \right) \cdot \frac{\Omega}{f_n} \right] & \text{if } f < 0.7 f_n \\ F(\Omega) \cdot \left[ 0.453 \right] & \text{if } f > 0.7 f_n \end{array} \right.$$
\[ \hat{\varphi} = \text{IFT} \begin{cases} F(n) \cdot \left[ 0.257 \left(1 - \cos \frac{f}{f_n} \right) \right] & \text{if } f \leq f_n \\ F(n) \left[ 0.257/R \right] & \text{if } f > f_n \end{cases} \]

(\hat{\varphi} \text{ is positive clockwise})

where IFT stands for Inverse Fourier Transformation; \( f_n \) is the fundamental shear beam frequency of the embedment region (for uniform soil properties in the embedment region, this value is given by \( f_n = C_s/4E \) with \( C_s \) being the shear wave velocity, and \( E \) the depth of embedment. The expression in square brackets describes an approximation to the transfer functions for the translation and rotation of the massless foundation. For surface footings, \( E = 0 \), \( f_n = \infty \cos 0 = 1 \); it follows that no kinematic interaction takes place for this case.

The procedure described yields satisfactory results for a wide range of embedment ratios, see for instance Fig. 5. It should be noted, however, that the rotational component is sensitive to the lateral soil conditions, and particularly to the flexibility of the lateral walls. For flexible sidewalls, the actual rotation is significantly smaller, and in the extreme case of no sidewalls, the rotation even changes sign! Nevertheless, the contribution of the rotational component to the response of the structure is in most cases not very significant.

For nuclear containment structures, the effect of the rotational component on the structural response is of the order of 15-20 percent.

3.2 Approximations for the Stiffness Functions

As with the kinematic interaction, the values of the subgrade stiffness functions (impedance functions) depend only on the geometric configuration of the foundation and on the properties of the founding soil. These functions can be evaluated using analytical, experimental, or numerical methods.

The results presented in this paper are based on parametric studies performed by the fourth writer to determine approximate expressions for stiffness functions of circular, embedded foundations (1). For each particular geometry, the static values were evaluated with two or three meshes (a fine, a standard, and a coarse mesh), and the results were corrected for mesh size error in a manner similar to that described in Ref. 4. The study was limited to the coupled horizontal translation and rotation (rocking-swaying) of the rigid, circular, embedded foundation in a homogenous stratum. For this particular case, the force displacement relationship can be written

\[ \{ F \} = \begin{bmatrix} K_{XX} & K_{X\theta} \\ K_{\theta X} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix} \]

where \( F \) = the horizontal force; \( \theta \) = the rocking moment; and \( u, \theta \), are the corresponding displacements (rotations). The elements \( K_{XX}, K_{X\theta}, K_{\theta\theta} \) of the stiffness matrix depend on the frequency of excitation \( \Omega \) of the forces (moments). Since these forces and the resulting displacements are generally not in phase with each other, these elements are complex functions of frequency. Each stiffness function is of the form \( K^0 (1 + 2i\beta) \) \((k + im\omega)\), where \( K^0 \) is the static stiffness, \( \beta \) a measure of the internal damping in the soil (of a hysteretic nature), \( i = \sqrt{-1} \), and \( m\omega \) is the dimensionless frequency \( \Omega R/C_s \) \((\Omega \) is the circular frequency of the motion and excitation, \( R \) the radius of the foundation, \( \omega \) a reference shear wave velocity.)
k and c are frequency dependent coefficients normalized with respect to the static stiffness. The coefficient c is related to the energy loss by radiation.

Using the program described earlier, the dynamic stiffness functions were computed for a range of embedment and stratum depth ratios, and written as described above as:

\[
\begin{align*}
K_{xx} &= K_0^{xx} \left( k_{11} + i a_0 \cdot c_{11} \right) \left( 1 + 2i\beta \right) \\
K_{xd} &= K_0^{xd} \left( k_{12} + i a_0 \cdot c_{12} \right) \left( 1 + 2i\beta \right) \\
K_{dd} &= K_0^{dd} \left( k_{22} + i a_0 \cdot c_{22} \right) \left( 1 + 2i\beta \right)
\end{align*}
\]

Analysis of the results obtained provided the following approximations to the static values \(K_0^{xx}, K_0^{xd}, K_0^{dd}\), and to the dynamic stiffness coefficients \(k_{11}, k_{12}, c_{11}, c_{12}, c_{22}\):

**Static Values:**

\[
\begin{align*}
K_0^{xx} &= \frac{8GR}{2(1-\nu)} \left( 1 + \frac{1}{2} \frac{R}{H} \right) \left( 1 + \frac{2}{3} \frac{E}{R} \right) \left( 1 + \frac{5}{4} \frac{E}{R} \right) \\
K_0^{xd} &= K_0^{xx} \left( \frac{2}{3} \frac{E}{R} - 0.03 \right) \\
K_0^{dd} &= \frac{8GR^3}{3(1-\nu)} \left( 1 + \frac{1}{2} \frac{R}{H} \right) \left( 1 + \frac{E}{R} \right) \left( 1 + 0.71 \frac{E}{R} \right)
\end{align*}
\]

In these formulas, \(G\) is the shear modulus of soil underneath the mat; \(R\) is the radius of the foundation, \(E\) is the depth of embedment, \(H\) is the depth to bedrock, and \(\nu\) is Poisson's ratio.

**Dynamic Stiffness Coefficients:**

\[
k_{11}, k_{22} \text{ half space solution (i.e., Ref. 10)}
\]

\[
c_{11} = \begin{cases} 
0.88 \frac{a_0}{a_{01}} & \text{for } a_0 < \frac{8}{2} \frac{R}{H} = a_{01} \\
\text{Half space solution for } a_0 > a_{01} \end{cases}
\]

\[
c_{22} = \begin{cases} 
0.58 \frac{a_0}{a_{02}} & \text{for } a_0 < \frac{8}{2} \frac{R}{H} \frac{C_p}{C_s} = a_{02} \\
\text{Half space solution for } a_0 > a_{02} \end{cases}
\]

\[
k_{12} = k_{11}, c_{12} = c_{11}
\]

where \(\beta\) is the internal (hysteretic) damping in the soil; \(C_p\) and \(C_s\) are the dilatational wave velocities in the subgrade; \(a_{01}\) and \(a_{02}\) are the (non-dimensional) fundamental shear beam and dilatation frequencies of the stratum, as defined above.

Except for the stiffness coefficient \(k_{11}\), which displays a somewhat wavy nature, the suggested approximation for the stiffness and damping coefficients provide reasonable substitutes for the true functions. It can be observed that the radiation damping coefficients \(c_1, c_2\) are larger for the embedded case than for the surface footing; therefore, the suggested procedure should give conservative results for an embedded structure.
4. Soil-Structure Interaction Problem

Once the input motion and the base impedances are known, the last step is reduced to a simple dynamic analysis of a multidegree of freedom system. The "stiffness" and "damping" terms can be added directly to the corresponding terms for the structure in a solution in the frequency domain. A time domain solution, and a modal solution in particular (9), offer a simpler physical interpretation of the results. They require, however, frequency independent stiffness and damping coefficients, and for the latter, in addition the existence of normal modes.

A number of comparative studies indicate that it is more important to reproduce correctly the static stiffnesses than their complete frequency variation. It is also worth noticing that the increase in stiffness due to embedment is very sensitive to the properties of the lateral soil, which may be disturbed. Considering, in addition, the uncertainties in the soil properties and its nonlinear behavior, it is clear that engineering judgment is needed in the selection of the most appropriate model, and that parametric studies, varying the assumed conditions within reasonable limits, are advisable. This, of course, is equally true whether the analysis is carried out in a single step (one-pass method) or in three steps as suggested here.

The spring method has the advantage of being less time-consuming when approximations are used for the kinematic interaction problem. It allows, therefore, more parametric studies, and the accuracy of each step is subject to better control. Of particular importance is the possibility in this method to make use of symmetry or cylindrical conditions if the foundation meets these requirements even if the structure does not (which is a frequent situation). The coupling between the corresponding terms will come in naturally in the third step. From a practical standpoint, the procedure has an additional advantage when the design motion is specified by a broad band response spectrum not tailored to the soil conditions at the site. If the direct approach is applied to such a case, deamplification of certain frequency components with depth resulting from the use of one single wave pattern (i.e., vertically propagating waves) may lead to unconservative estimates for the motions of the structure. Under such conditions, it may be better to regard the design motion as an "average" motion in the vicinity of the structure, and to use it directly as input to Step 3.
References


FIGURE 1
SUPERPOSITION THEOREM
FIGURE 2
THE 3-STEP SOLUTION
FIGURE 3
KINEMATIC INTERACTION PROBLEM
Figure 4
Motion of Massless Foundation, Transfer Functions (Abs. Value)
FIG. 5
MOTION OF MASSLESS FOUNDATION, RESPONSE SPECTRA
1% OSCILLATOR DAMPING
H/R = 2, E/R = 1
FIGURE 6
DYNAMIC STIFFNESS COEFFICIENTS, ν = 1/3, β = 0.05