Volume II

NUMERICAL METHODS IN GEOMECHANICS

Edited by C. S. Desai


Sponsor: Engineering Foundation Conferences
Partial Support: National Science Foundation
Technical Co-sponsors:
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Published by
American Society of Civil Engineers
345 East 47th Street, New York, NY 10017
DYNAMIC STIFFNESS OF PILES

by

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INTRODUCTION

The response of piles to dynamic excitations has been the subject of considerable interest and research in recent years. Present knowledge and understanding of the dynamic behavior of pile foundations is, however, less complete than for mat foundations, and the number of approximate formulae available for preliminary design estimates is still limited.

Analytical solutions have been obtained along two principal lines: using a continuous model and theory of elasticity (7,12), and using a discrete model with lumped masses, springs and dashpots (8,9). A detailed comparison of both approaches and the solutions they provide has been presented by Flores (3). The continuous model has the advantage that it incorporates automatically in the formulation the mass densities of soil and pile, as well as the effect of radiation damping. It is, however, restricted to linear soil behavior. The discrete model, on the other hand, encounters the difficulty of defining equivalent soil mass and fictitious dashpots to simulate radiation, but it allows to consider nonlinear soil behavior by using arbitrary force deformation characteristics for the spring. Whatever the method used for the analysis, the reasonableness of the results will depend on the ability to select, at least approximately, the soil parameters. Hence the importance of experimental studies where soil characteristics and accuracy of numerical solutions are evaluated (2,6,11,14).

In this paper a discrete model based on the use of finite elements to reproduce the soil around the pile and a consistent boundary matrix to simulate the effect of the far field is described. The procedure can be used to study the response of a pile embedded in a horizontally stratified soil deposit, where the properties vary with depth, but remain constant in horizontal planes in the far field. It can be also extended to simulate nonlinear soil behavior. The present work is limited, however, to the consideration of a linearly elastic homogeneous stratum resting on much stiffer material, which can be assumed rigid for practical purposes.

From this model approximate values of the lateral stiffness of the soil surrounding the pile are first obtained and compared with the values suggested by Penzien et al (9) for the static case, and by Novak (7) for a steady state harmonic motion with various frequencies. Dynamic stiffnesses of the pile under a horizontal force or a moment applied at the top are determined next and compared to the results of Novak (7). Approximate formulae are then suggested for

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these stiffnesses. Finally the case of a base motion (representing an earthquake excitation) is studied, and the motion at the top of the pile is compared to the motion which would occur at the free surface of the soil if there were no pile.

**MATHEMATICAL MODEL**

The mathematical model used in this study is an extension of the work done by Kausel (4,5) for the dynamic analysis of circular foundations. The soil surrounding the pile is discretized by means of toroidal finite elements of arbitrary expansion order having three degrees of freedom per nodal ring (fig. 1). The far field is represented by a semi-analytic energy transmitting boundary, based on the exact displacement functions in the horizontal direction and an expansion in the vertical direction consistent with that used for the finite elements. The effect of the far field is thus reproduced through a boundary matrix, which relates forces applied on the cylindrical lateral boundary to the displacements of the points of this boundary. It should be noticed that this is a full matrix, coupling all nodes along the boundary, and not a diagonal matrix as would correspond to the assumption of a Winkler foundation. Determination of this matrix requires the solution of a quadratic eigenvalue problem, but it is a special kind of problem for which efficient numerical solutions are available. Its terms will be complex quantities (the imaginary part representing the dissipation of energy through lateral radiation), function of frequency, of the radius of the boundary, and of the order of the Fourier expansion in the circumferential direction (n=0 for vertical or torsional vibrations, n=1 for lateral or rocking motion). Complete details of the derivation can be found in refs. 4, 5.

While it is possible to reproduce the pile equally with finite elements, this solution is unnecessarily expensive. A simpler approach is to condense the degrees of freedom of the nodal rings at the internal boundary, relating them to the degrees of freedom of the pile nodes through the assumption of a rigid cross section, and modeling the pile as a series of regular beam segments (fig. 1). Complete compatibility between the pile and the surrounding soil would require a cubic expansion for the finite elements. In this work, however, the element expansion was linear. This will introduce a small error, which is compensated for by taking a larger number of beam segments (and soil layers). Preliminary studies showed that the results obtained taking 10 and 20 segments were identical for all practical purposes.

The geometry shown in fig. 1 corresponds to the more general case, with the pile, a cylindrical finite element region and the consistent boundary to reproduce the far field. It allows to consider soil properties in the neighborhood of the pile varying with horizontal distance and different from those of the far field (where there is no horizontal variation of properties), and could be used to simulate nonlinear soil behavior around the pile. For the particular application described in this paper, the soil was assumed to be homogeneous and linearly elastic. In this case the intermediate finite element region is unnecessary. The boundary matrix can be obtained directly for the cylindrical cavity filled by the pile, and added to the stiffness matrix of the pile segments, after condensation, with the corresponding reduction in the number of degrees of freedom.

Internal dissipation of energy in the soil, in the form of hysteretic
(frequency independent) damping is reproduced in this formulation through the use of complex moduli \( E(1+2iD) \) and \( G(1+2iD) \) where \( D \) is the desired damping ratio.

**LATERAL SOIL STIFFNESS**

For the dynamic analysis of the pile the condensed boundary matrix is added directly to the stiffness matrix of the pile divided into segments. It is interesting, however, to look at the boundary matrix by itself, and derive from it an equivalent lateral stiffness of the soil. To obtain the spring constant of a Winkler type foundation from a matrix which is actually full, the approach followed by Penzien (9) was used. A uniformly distributed horizontal load was applied along the cylindrical cavity, assuming, however, a rigid cross section. The corresponding displacements were then inverted to obtain approximate stiffnesses. Fig. 2 shows the resulting variation of stiffness with depth for different frequencies, in terms of the dimensionless frequency \( \omega_0 = kR/v_S \), where \( \omega \) is the circular frequency of the steady state motion, \( R \) is the radius of the cavity and \( v_S \) is the shear wave velocity of the soil. As could be expected, the equivalent soil stiffness changes with depth and increases at the bottom where a zero displacement is assumed (condition of a rigid base). The variation is in addition strongly dependent on the frequency.

An average stiffness was then computed. Two different procedures were used for this purpose: averaging the computed soil stiffnesses over depth and averaging the soil displacements and computing the stiffness from the inverse of the mean displacement. For both schemes the averaging procedure was applied over the whole depth of the stratum and over the top half only. The results were similar in all cases with the small variations in the computed values which could be logically expected. The same procedure was applied to the stiffness values obtained from Penzien's analysis for zero frequency (static case).

Fig. 3 shows the variation of the average static soil stiffness with the slenderness ratio \( H/R \) (\( H \) depth of the stratum, \( R \) radius of the cavity) from both methods. The results follow the same trend, although the values of the stiffness derived from the present approach are about 20% higher. It should be remembered that here a finite stratum resting on rigid rock is considered, while Penzien's formulæ apply to a half space. In addition the condition of a rigid cross section was automatically enforced in this analysis, while in Penzien's solution the horizontal displacement varies over the circumference and an average value must be obtained.

Fig. 4 shows the variation of the average dynamic stiffness as a function of the dimensionless frequency \( \omega_0 \). In this case the stiffness is a complex number of the form \( k+i\omega \), where the imaginary part represents the radiation of energy into the far field. If no internal damping exists in the soil, the value of \( k \) will zero at the fundamental shear frequency of the stratum, \( \omega = 2\pi v_S/4R \) or \( \omega_0 = R/H \cdot \pi/2 \). The imaginary part will be zero below this frequency, since no lateral radiation can physically occur in this range. A more realistic situation is obtained when a certain amount of internal damping is included in the soil. If \( D \) is the assumed damping ratio, it is convenient to express the stiffness in the form

\[(k+i\omega)(1+2iD)\]  \hspace{1cm} (1)

where \( k \) and \( c \) would be, approximately, the values corresponding to zero damping. The curves shown in Fig. 4 for \( k \) and \( c \) were obtained using 5% internal damping.
and dividing the results by \((1+0.1i)\). Results obtained with 10% internal damping (dividing by \(1+0.2i\)) are essentially identical. In this case, however, \(k\) does not become zero at the fundamental frequency of the stratum, although a valley in the curve can be observed at this frequency.

Shown in fig. 4 are also the values of \(k\) and \(c\) as a function of frequency, used by Novak (7). Good agreement exists between both solutions for values of \(a_0\) greater than 0.4, which corresponds to a frequency of 54 Hz for the case studied. For frequencies below this value, which include the range of interest in earthquake engineering, there are, however, substantial differences. Novak's stiffness function \(k\) decreases to zero at zero frequency, while the curves obtained in this study dip in the region of the soil resonance but return to the static value at zero frequency. The damping coefficient \(c\) from Novak's analysis decreases almost linearly with frequency, while the corresponding curve from the present solution remains zero at frequencies below the soil resonance (this last effect being due to the existence of a rigid bottom).

**PILE STIFFNESSES**

Using the representation of the pile as a series of beam segments, and adding to its stiffness matrix the full consistent boundary matrix, the response to a unit horizontal force and a unit rocking moment applied at the top of the pile was determined. The resulting displacements and rotations of the pile top provide the flexibility coefficients (compliance functions). Inverting the flexibility matrix, the stiffness matrix of the pile is obtained in the form

\[
K = \begin{bmatrix}
K_{xx} & K_{x\phi}
\end{bmatrix}
\]

The stiffness coefficients \(K_{xx}, K_{x\phi}, K_{\phi\phi}\) can be expressed as

\[
K_{ij} = (k_{ij} + ia_0c_{ij})(1 + 2iD)^b
\]

\(a_0\) is the dimensionless frequency \(\omega R/v_s\) and \(D\) is the amount of internal soil damping, of a hysteretic nature.

This expression would suggest that the coefficients \(k_{ij}, c_{ij}\) are independent of \(D\). As was pointed out before, this is not rigorously true. Since in this case the pile does not have the same amount of damping as the soil (for this study no internal damping was assumed in the pile) the exponent \(b\) should be less than 1. The approximate formulae obtained in this study and discussed later would suggest values of \(b\) of 0.75 for the \(xx\) term, 0.50 for the \(x\phi\) (or \(\phi x\)) term and 0.25 for the \(\phi\phi\) term. It should be noticed finally that in this expression the imaginary term is written in the form \(ia_0c_{ij}\) instead of simply \(ic\) as before. This form is more convenient because the imaginary terms have a variation nearly proportional to \(a_0\), which makes \(c_{ij}\) practically constant over an extended range of frequencies.

Fig. 5 shows the variation of \(K_{xx}\) with respect to frequency for a particular case studied by Novak (7). Included in this figure are the results for \(k_{xx}\) and \(c_{xx}\) from the present study, the results of Novak's solution and a "rigorous" solution based on Tajimi's work (12), as reported by Novak. The values obtained with the consistent boundary matrix are close to the values found by
Novak for the higher frequency range. The overall shapes follow well the results of the "rigorous" solution with the same dips in the values of k at the resonant frequencies of the stratum. They are, however, approximately 20% smaller.

These results apply to the case when there is no internal damping in the soil (D=0). Results for case D=0.05 are shown in fig. 6a. It can be seen that the values of k<sub>XX</sub> c<sub>XX</sub> are very similar to those obtained before, but the oscillations are smoothed out. k<sub>XX</sub> is practically constant with frequency and so is c<sub>XX</sub> above the fundamental shear frequency of the stratum (below this frequency it is zero). Fig. 6b and 6c show plots of k<sub>XX</sub>, k<sub>FF</sub> vs. a<sub>0</sub> for the same case. It is interesting to notice that while c<sub>XX</sub> is actually larger than k<sub>XX</sub> (above the resonant frequency), and c<sub>FF</sub> is of the same order of magnitude as k<sub>FF</sub> and also larger, c<sub>FF</sub> is substantially smaller than k<sub>FF</sub>.

Following Novak (7) it is convenient to express the stiffness coefficients in the form k<sub>XX R<sup>3</sup>/EI</sub>, k<sub>FF R<sup>2</sup>/EI</sub>, k<sub>FF R/EI</sub> and to use as basic parameters the slenderness ratio H/R, the dimensionless frequency a<sub>0</sub> = ωR/ν<sub>s</sub>, the velocity ratio ν<sub>s</sub>/ν<sub>c</sub>, the mass density ratio ρ<sub>S</sub>/ρ<sub>P</sub>, the value of Poisson's ratio for the soil ν. In these expressions G<sub>S</sub> is the shear modulus of the soil, and ρ<sub>S</sub> its mass density and I its bending moment of inertia.

\[
\begin{align*}
S_{b} &= \frac{G_{s} 0.5}{\rho_{s}} \\
S_{c} &= \frac{E_{b} 0.5}{\rho_{p}}
\end{align*}
\]  

(4)

Fig. 7 shows the variation of the stiffness coefficients k<sub>FF</sub> versus the slenderness ratio for three different values of ν<sub>s</sub>/ν<sub>c</sub> at the frequencies a<sub>0</sub>=0 (static case) and a<sub>0</sub>=0.314. Values of c<sub>XX</sub>, c<sub>FF</sub> would be zero at a<sub>0</sub>=0. For a<sub>0</sub>=0.314 they follow the same trend of the real terms. It can be seen from these plots that for values of H/R larger than 20 or 25 the effect of the slenderness ratio is not significant, particularly for low and moderate values of ν<sub>s</sub>/ν<sub>c</sub>. For ν<sub>s</sub>/ν<sub>c</sub>=0.05 there is a slight decrease of k<sub>FF</sub> and particularly the coupling term k<sub>FF</sub> with increasing H/R. Results for the two frequencies are again very similar. The small variation of the stiffness coefficients with frequency was already illustrated in fig. 6.

These results apply to the case of a pile hinged at the bottom. Fig. 8 shows a comparison of the horizontal stiffness for piles with a hinged and a fixed tip. For values of H/R larger than 20 or 25 both solutions are practically identical. For smaller values of the slenderness ratio the fixed pile is of course stiffer. The same observation applies to the other two stiffness terms.

All these results are in good agreement with those reported by Novak (7) at a frequency a<sub>0</sub>=0.3. For piles with a slenderness ratio H/R larger than 25 one can assume approximately that the stiffness coefficients are independent of frequency, of the slenderness ratio and of the boundary conditions at the tip (fixed or hinged pile). The significant parameters are then the velocity ratio ν<sub>s</sub>/ν<sub>c</sub>, the density ratio ρ<sub>S</sub>/ρ<sub>P</sub> and the value of Poisson's ratio for the soil ν. Figs. 9, 10 and 11 show the variation of the stiffness coefficients versus ν<sub>s</sub>/ν<sub>c</sub> for a case with ρ<sub>S</sub>/ρ<sub>P</sub> = 0.625 and ν = 0.4.

The parameter ν<sub>s</sub>/ν<sub>c</sub> had been selected for these studies following Novak's work. Further studies indicated, however, that E<sub>S</sub>/E<sub>P</sub> is probably a more convenient parameter. The results expressed in terms of E<sub>S</sub>/E<sub>P</sub> are insensitive to variations in the density ratio ρ<sub>S</sub>/ρ<sub>P</sub> and Poisson's ratio for the soil ν.
The variation of the stiffness coefficients with $v_s/v_p$ or $E_s/E_p$ follows closely a straight line on log log paper. Approximate formulae, which could be used for preliminary design purposes, can be fitted to the numerical results obtained in this study. For slenderness ratios larger than 25 and values of $E_s/E_p$ smaller than $5 \times 10^{-3}$

\[
\begin{align*}
    k_{xx} & = 2 \frac{E_I}{R^3} \left( \frac{E_s}{E_p} \right)^{0.75} \\
    k_{x\phi} & = -1.2 \frac{E_I}{R^2} \left( \frac{E_s}{E_p} \right)^{0.5} \\
    k_{\phi\phi} & = 1.6 \frac{E_I}{R} \left( \frac{E_s}{E_p} \right)^{0.25}
\end{align*}
\]

(5)

The values of $c_{xx}$, $c_{x\phi}$ would be zero for $a_o < \pi/2 \cdot R/H$ if there is physically a much stiffer stratum on which the pile rests. Below this frequency there is no radiation damping. Above this frequency, and over the whole range if the soil deposit can be considered as a half space with constant properties,

\[
\begin{align*}
    c_{xx} & = 2k_{xx} \\
    c_{x\phi} & = 1.5 k_{x\phi}
\end{align*}
\]

(6)

The value of $c_{\phi\phi}$ would be zero for $a_o < \pi/2 \cdot R/H \cdot \sqrt{(1-v)/(1-2v)}$, if again the soil deposit has a finite depth and rests on much stiffer material which can be assumed rigid. Above this frequency

\[
    c_{\phi\phi} = 0.5 k
\]

(7)

Taking into account the previous observations in relation to $c_{xx}$, $c_{x\phi}$, $c_{\phi\phi}$, these expressions can be combined to give

\[
\begin{align*}
    K_{xx} & = 2(1+2ia_o) \frac{E_I}{R^3} \left( \frac{E_s}{E_p} \right)^{0.75} (1+2iD)^{0.75} \\
    K_{x\phi} & = -1.2(1+1.5ia_o) \frac{E_I}{R^2} \left( \frac{E_s}{E_p} \right)^{0.50} (1+2iD)^{0.50} \\
    K_{\phi\phi} & = 1.6(1+0.5ia_o) \frac{E_I}{R} \left( \frac{E_s}{E_p} \right)^{0.25} (1+2iD)^{0.25}
\end{align*}
\]

(8)

The variation of the stiffness coefficients with $E_s/E_p$ is consistent with results of previous studies for static loads where the exponents are also 0.75, 0.50 and 0.25 if the elastic length of the pile is computed in terms of $E_s(1,10)$ and about 0.81, 0.54 and 0.27 if it is computed in terms of the modulus of subgrade reaction $\lambda_{10}$.

The values of the coefficients 2, 1.2 and 1.6 are clearly debatable, but could be adjusted for any specific situation on the basis of experimental data.

Results obtained with these formulae would be in good agreement with those tabulated by Novak (7) for a frequency $a_o = 0.3$, but 10 to 20% smaller. The
ratio of the imaginary to the real term for rocking obtained here is also smaller than that of Novak (about 0.7). The accuracy in the determination of $C_{qg}$ in this study was not as good as for the other terms, because of the small values below the vertical resonance frequency.

The selection of the exponents for the term $(1+2iD)$ was arbitrary, on the basis of the exponent of $C_{5}$. Further studies are necessary to refine these points. It is believed, however, that expressions 5 to 8 can be reasonably used for preliminary estimates.

In order to apply these results to the case of an actual pile foundation, the vertical stiffness of the pile should also be obtained. This determination was not undertaken in the present study, but results can be found in Novak's work [7]. A simple formula cannot be obtained, however, for this case, since the vertical stiffness would be more strongly dependent on the slenderness ratio R/H and on the properties of the underlying stratum.

**EARTHQUAKE MOTIONS**

As a final step, the effect of the pile on the motion at the free surface of the soil deposit, for a specified motion at the base, was investigated. The motion at the base was assumed to be uniform, corresponding to the case of shear waves propagating vertically through the soil (one-dimensional amplification). Fig. 12 shows the ratio $u_0/u_5$ of the displacement at the top of the pile to the displacement at the free surface of the soil without pile (or at a sufficient distance from the pile) as a function of the dimensionless frequency $b_0 = \omega H/V_S$.

In the small frequency range the pile displacement may be actually larger than the soil displacement without pile, but the difference is less than 25% for H/R = 20 (a caisson) and less than 15% for H/R = 40 (a stubby pile). For most piles considered in design the relative motion between the pile head and the soil surface is less than 10% in the low to moderate frequency range. For values of the dimensionless frequency $b_0$ greater than $\pi$ (twice the fundamental shear frequency of the stratum) the motion of caissons (H/R < 10) drops quickly to a small value, indicating that the caisson remains practically still while the soil vibrates around it. The motion of stubby piles continues to follow that of the soil stratum up to a dimensionless frequency of approximately 1.5, while a regular pile (H/R = 80) shows only a reduction of motion for frequencies higher than 3.

The above results seem to confirm Flores' conclusion [3] that relative motion between the pile and the soil is only significant for low values of H/R. The "intermediate piles" mentioned by Flores correspond to piles with H/R of approximately 40 in the present case, for which significant pile-soil relative motion occurs only at high frequencies. Piles with H/R of about 80 follow essentially the soil over the entire range of frequencies of interest in earthquake problems, as do the "slender piles" of Flores.

**CONCLUSIONS**

The work presented in this paper represents the first of a series of studies on the dynamic response of pile foundations. Results obtained to date would seem to indicate that:

1. The suggested procedure, with the use of finite elements surrounding the pile and a consistent boundary matrix to reproduce the far-field, shows a good


potential for the analysis of more complicated situations, where soil properties vary with depth, and even in the horizontal direction in the immediate neighborhood of the pile.

2. For the case of a homogeneous soil stratum on a much stiffer base considered here, the stiffnesses obtained are in reasonably good agreement with other solutions applied normally to a half space. For the static case the equivalent lateral stiffness of the soil is about 20% higher than that used by Penzien (9). Pile stiffnesses are however some 10 to 20% smaller than those reported by Novak.

3. While Novak's approximate solution is not valid for small values of the dimensionless frequency \( a_0 \) (the range of practical interest for earthquake motions) use of his results for \( a_0 = 0.3 \) will provide reasonable estimates for the pile stiffnesses.

4. For regular piles, with slenderness ratios H/R larger than 25, the stiffnesses can be considered independent of the slenderness ratio, of frequency, and of the tip condition. The imaginary parts, representing the dissipation of energy through radiation, follow the same bend as the real parts. \( c_{xx} \) and \( c_{xy} \) will be, however, zero below the fundamental shear frequency of the stratum and \( c_{yy} \) will be zero below the fundamental dilatational frequency, if the soil deposit in which the pile is embedded rests on a much stiffer material.

5. For regular piles the presence of the pile will not affect significantly earthquake motions at the top of the soil deposit.

ACKNOWLEDGMENT

The work presented in this paper is based on research conducted at the Civil Engineering Department of the Massachusetts Institute of Technology, under a grant from the National Science Foundation, RANN Division.

APPENDIX I. REFERENCES


APPENDIX II - NOTATION
The following symbols are used in this paper:

\[ a_0 = \omega R/v_s, \quad b_0 = \omega H/v_s, \text{ dimensionless frequencies.} \]
\[ b = \text{exponent} \]
\[ c = \text{imaginary part of soil stiffness} \]
\[ c_{xx}, c_{x\phi}, c_{\phi \phi} = \text{imaginary part of stiffness coefficients} \]
\[ D = \text{damping ratio in the soil} \]
\[ E_p, E_s = \text{modulus of elasticity of the pile and soil.} \]
\[ G_s = \text{shear modulus of the soil} \]
\[ H = \text{depth of soil stratum} \]
\[ k = \gamma T \]
\[ K = \text{real part of soil stiffness} \]
\[ K_{xx}, K_{x\phi}, K_{\phi \phi} = \text{stiffness matrix for top of pile.} \]
\[ K_{xx}, K_{x\phi}, K_{\phi \phi} = \text{stiffness coefficients for pile} \]
\[ n = \text{order of Fourier expansion in circumferential direction} \]
\[ \nu = \text{Poisson's ratio of soil} \]
\[ R = \text{radius of pile} \]
\[ \rho_p, \rho_s = \text{mass densities of pile and soil} \]
\[ u_p, u_s = \text{horizontal displacements at top of pile and free surface of soil} \]
\[ V_s = \text{shear wave velocity of soil} \]
\[ V_c = \text{rod wave velocity for pile} \]
\[ \omega = \text{frequency of excitation} \]
Figure 1: Geometry of the Problem

Figure 2: Variation of soil stiffness with depth and dimensionless frequency

Figure 3: Average soil stiffness versus slenderness ratio H/R at zero frequency

Figure 4: Comparison of present study with Novak (1974) solution (5% damping)

Figure 5: Comparison of Novak (1974) and rigorous solution after Novak with present solution

Figure 6A: Stiffness $K_{xx}$ versus dimensionless frequency
FIGURE 6B: STIFFNESS $K_{pp}$ VERSUS DIMENSIONLESS FREQUENCY

FIGURE 6C: STIFFNESS $K_{p6}$ VERSUS DIMENSIONLESS FREQUENCY

FIGURE 7A: STIFFNESS $K_{xx}$ VERSUS $H/R$ FOR DIFFERENT VALUES OF $V_S/V_C$

FIGURE 7B: STIFFNESS $K_{xx}$ VERSUS $H/R$ FOR DIFFERENT VALUES OF $V_S/V_C$

FIGURE 7C: STIFFNESS $K_{xx}$ VERSUS $H/R$ FOR DIFFERENT VALUES OF $V_S/V_C$

FIGURE 8: VARIATION OF STIFFNESS $K_{xx}$ WITH $H/R$ FOR FIXED AND HINGED TIP
FIGURE 9: VARIATION OF STIFFNESS $K_{MK}$ WITH $V_s/V_c$

FIGURE 10: VARIATION OF STIFFNESS $K_{MB}$ WITH $V_s/V_c$

FIGURE 11: VARIATION OF STIFFNESS $K_{BB}$ WITH $V_s/V_c$

FIGURE 12: RATIO OF PILE DISPLACEMENT TO STRATUM DISPLACEMENT FOR DIFFERENT VALUES OF $H/R$