SEISMIC ANALYSIS OF FLUID STORAGE TANKS

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ABSTRACT: Refinements are presented on a model proposed originally by Wozniak for the seismic analysis of fluid storage tanks, which forms the basis of the provisions in the American Petroleum Institute design code API 650. It is found that use of the rules in API 650 may lead in some cases to underestimations of the maximum compressive stresses in the shell, and to overestimations of the tensile stresses in the straps of anchored tanks. The stiffness of these straps relative to that of the shell is shown to play an important role in the maximum compressive stresses that can be expected in the tank wall. In the case of unanchored tanks, the larger compressive shell forces result from the limited mass of fluid that can be mobilized before the buckling load in the shell is reached.

INTRODUCTION

Research on the problem of fluid-filled tanks subjected to dynamic loads developed along parallel avenues in the aeronautical, ocean, mechanical and civil engineering communities. Examples are the studies on the sloshing of fuel in rocket tanks or oil in container ships, and the investigations on the behavior of large industrial tanks during earthquakes. It is not surprising, then, that the literature on the subject, and particularly the number of technical papers is rather extensive (1–56). A thorough review would provide sufficient material for a lengthy state-of-the-art document, even if the emphasis were only on the seismic analysis of ground-based liquid storage tanks; thus, such a task will not be attempted here. Instead, the purpose of this paper is to present some improvements in the approximate models proposed by Wozniak (50) for the seismic analysis of anchored and unanchored fluid storage tanks, and which form the basis of the provisions in the American Petroleum Institute design code API 650. Appendix E. In particular, it will be shown that use of the rules in API 650 may lead in some cases to significant underestimations of the maximum compressive stresses in the shell under conditions of incipient or moderate shell uplift. In the case of anchored tanks, this effect is due to the relatively smaller stiffness of the anchoring straps in comparison to that of the shell. For unanchored tanks, on the other hand, use of the rules may result in overestimations of the contribution of the fluid weight in resisting lift-off, since they are based on the assumption that sufficient separation develops to induce plastic hinges in the tank bottom. Frequently, however, the amount of lift-off may be insufficient to fully mobilize the fluid weight, or local shell buckling may take place before such weight can be activated.

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CONTRIBUTION OF FLUID TO RESIST LIFT-OFF

When the edge of the tank separates from the ground, the lift-off displacement is not constant but changes with distance to the rotation axis. In the case of a cylindrical tank, the contributing weight of the fluid acts.
over a crescent-moon-shaped region (Fig. 1). Within this region, the fluid weight per unit arc length is proportional to its width and decreases to zero towards the two vertices of the crescent.

Consider a portion of unit width and length $L$ at the tank wall-bottom plate junction (Fig. 2). Neglecting the curvature of the edge (and thus eventual membrane effects), this portion can be analyzed as a uniformly loaded beam of variable length $L$. Let $R_A$, $R_B$, $M_A$, $M_B$ be the reaction forces and moments (per unit length) at the two ends. For the fully elastic case, it can be assumed that the rotation at the shell junction is zero, since the shell is usually much stiffer than the bottom. However, once the elastic limit is reached, a plastic hinge develops, and the rotation at $A$ is no longer zero. Instead, the moment remains constant and equal to the plastic moment, i.e., $M_A = f_y t^2 / 4$, with $f_y$ being the yield stress of the bottom plate, and $t$ being the corresponding thickness. Both in the elastic and inelastic cases, the moment at $B$ must be taken equal to zero ($M_B = 0$), since otherwise equilibrium could not be satisfied beyond $B$. In addition, the rotation at $B$ must be taken as zero. Using standard methods in beam analysis (as, for example, the conjugate beam method), one can solve for the various unknowns in this problem. Omitting details, the results are as follows ($E =$ Young’s modulus for bottom plate; $q =$ weight of fluid per unit area; and $u_A =$ lift-off):

a. Elastic case:

\[ u_A = \frac{27}{32 q} \frac{R_A^4}{q^3 E t^3} \] \quad (1a)

\[ M_A = \frac{3}{8} \frac{R_A^2}{q} \] \quad (1b)

\[ L = \frac{R_A}{q} + \sqrt{\left( \frac{R_A}{q} \right)^2 - 2 \frac{M_A}{q}} \] \quad (1c)

b. Elastic limit

\[ u_A = \frac{3}{8} \frac{f_y t}{E q} \] \quad (2a)

\[ M_A = \frac{1}{4} f_y t^2 \] \quad (2b)

\[ R_A = t \sqrt{\frac{2}{3}} q f_y \] \quad (2c)

\[ L = t \sqrt{\frac{3 f_y}{2 q}} \] \quad (2d)

c. Postyielding, one hinge at $A$

\[ u_A = \frac{L^2}{Et^3} \left( R_A L - \frac{3}{4} f_y t^2 \right) \] \quad (3a)
FIG. 3.—Normalized Lift-Off Force versus Normalized Lift-Off Displacement

\[ L = \frac{R_A}{q} \sqrt{\left(\frac{R_A}{q}\right)^2 - \frac{1}{2} \frac{f_y t^2}{q}} \]  \hspace{5em} (3b)

d. Upper limit (2 hinges)

\[ u_A = \frac{11 + 8\sqrt{2} f_y t}{8 Eq} \] \hspace{5em} (4a)

\[ R_A = t \sqrt{f_y q} \] \hspace{5em} (4b)

\[ L = t \left(1 + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{f_y}{q}} \] \hspace{5em} (4c)

Once the displacement, \( u_A \), exceeds the limit given by Eq. 4a, the external force, \( R_A \), cannot be increased further, but remains constant.

It is convenient at this time to normalize Eqs. 1a, 2a and 3a by the limiting values, Eqs. 4a–b; defining then the dimensionless force, \( \rho \), and displacement, \( \xi \), and plotting the resulting expressions, one obtains the force-displacement curve shown in Fig. 3. This figure shows that if the system were interpreted as a spring, the corresponding secant stiffness would be strongly nonlinear. Of course, if the lift-off displacement is large enough, one could assume an elasto-plastic spring, and the corresponding lift-off force would be constant; an example is the case of an oil storage tank of dimensions \( \phi = 48 \text{ m, } H = 14 \text{ m} \) (160 ft \( \phi \times 48 \text{ ft} \), with \( q = 12 \text{ N/cm}^2 \) (17.4 psi); \( t = 0.6 \text{ cm} \) (1/4 in.); \( f_y = 30,000 \text{ N/cm}^2 \) (43,531 psi); \( E = 2 \times 10^7 \text{ N/cm}^2 \) (2.9 \( \times 10^7 \) psi). Hence, two hinges would form when the lift-off displacement is (Eq. 4a) \( u_A = 6.3 \text{ cm} \) (2.5 in.). Such a separation may realistically take place during an actual earth-
quake. On the other hand, in some cases the elastic limit may involve rather large displacements, implying that an elastoplastic approximation is no longer appropriate (i.e., lift-off may not be large enough to form plastic hinges). Consider, for example, a liquefied natural gas tank made of a high strength steel with the following characteristics: \( E = 2 \times 10^7 \) N/cm\(^2\); \( f_s = 0.5 \) cm; \( f_e = 8 \times 10^3 \) N/cm\(^2\); and \( q = 15 \) N/cm\(^2\). The elastic limit would then correspond to a lift-off displacement (Eq. 2a) of \( u_A = 4 \) cm, and two hinges would form when the separation is (Eq. 4a) \( u_A = 29.8 \) cm (approx 1 ft)! While indirect observations after past earthquakes indicate that such large up-lifts are not impossible, one would have to consider all circumstances surrounding the events to properly interpret those observations (i.e., full or empty tank, foundation flexibility, damage to the tanks, etc.). At any rate, this much displacement (with an associated time variation similar to that of the ground acceleration) would require mobilizing the fluid to unrealistic levels. As will be shown, the compression load would initiate buckling of the tank well before such amplitude could be reached; hence the nonlinear characteristics of the fluid resistance would have to be taken into consideration in such a case. This does not imply, however, that the tank would necessarily fail, since stable (and damage free) configurations may conceivably develop after the onset of buckling (→ postbuckling behavior). On the other hand, a more accurate analysis of the formation of plastic hinges would have to consider also the bending stresses due to the initial hydrostatic pressure; however, their effect is expected to be small, since the lateral pressure is resisted mainly by hoop stresses, and not by bending stresses.

While it may be possible, at least in principle, to account “accurately” (in the sense of the model) for the dependence of the fluid weight with the azimuth, it is neither wise nor practical to do so here. On the one hand, the model contains too many simplifications to warrant the refinement, and on the other, the formulation would become too involved. For practical purposes, it is sufficient to assume that the forces vary linearly with distance to the rotation axis, i.e., the fluid is assimilated into a spring, \( k_r = k_r(u_A) \), whose nonlinear characteristics (Fig. 3) are governed by the amount of lift-off at zero azimuth (Fig. 1). On the other hand, in order for lift-off to occur, it is necessary to overcome not only the gravitational force in the fluid, but its mass must be accelerated upwards as well. In addition, one should consider simultaneously the effect on lift-off of an eventual vertical component of the earthquake, since superposition no longer holds. Nevertheless, both of these effects—even if not negligible—will be disregarded in this analysis, in order to keep the formulation reasonably simple.

**Lift-Off Analysis**

In the case of anchored tanks, the anchor straps (or anchor bolts) may not fully prevent lift-off. On the one hand, their stiffness may be in sufficient to accomplish full fixity, while on the other, their initial (or intended) slackness may provide room for lift-off to occur. Since the stiffness, \( k \), of these straps will enter explicitly in the following analysis, it will not be necessary to distinguish between anchored and unanchored tanks at this time.
With reference to Fig. 4, it will be assumed that the annular tank section, at the elevation at which the straps are attached, remains plane during lift-off. The straps will be assumed to have a slackness, \( \delta \), which must be exceeded in order to activate their resistance. This may occur within the interval, \( (-\theta_0, \theta_0) \). The fluid resistance, on the other hand, is mobilized in the interval \( (-\theta_1, \theta_1) \), i.e., over the full angle of lift-off. If the straps had no slackness, i.e., \( \delta = 0 \), then \( \theta_0 = \theta_1 \).

From the plane-section-remains-plane assumption, it follows for the displacement of the strap-attachment section (Fig. 4)

\[ \rho - \alpha R \cos \theta - \omega_0 \]  
\( \rho \) \( \text{..........................} \) (5)

By definition, the slackness \( \delta \) is then

\[ \delta = \alpha R \cos \theta_0 - \omega_0 \]  
\( \delta \) \( \text{..........................} \) (6)

and at the vertices of lift-off
0 = αR \cos θ_1 - \omega_0 \quad \cdots \quad (7)

It follows from the aforementioned

\[ αR = \frac{\bar{δ} + \omega_0}{\cos θ_0} - \frac{\omega_0}{\cos θ_1} \quad \cdots \quad (8) \]

\[ \frac{\bar{δ}}{\omega_0} = \frac{\cos θ_0 - \cos θ_1}{\cos θ_1} \quad \cdots \quad (9) \]

The strap tension is then

\[ T_k = \begin{cases} 
  k (αR \cos θ - \omega_0 - \bar{δ}) & \theta \leq θ_0 \\
  0 & \text{else}
\end{cases} \quad \cdots \quad (10) \]

and the tank shell compression is

\[ T_s = \begin{cases} 
  K (ω_0 - αR \cos θ) & \theta \geq θ_1 \\
  0 & \text{else}
\end{cases} \quad \cdots \quad (11) \]

with \( K = \frac{Et}{h} \) = tank shell stiffness/length (may also include a foundation spring in series, if deemed necessary)

and \( k = \frac{NEA}{2πRh'} \) = strap stiffness/length

in which \( N = \) no. of straps; \( A = \) area of straps; \( h = \) length of annular shell section; and \( h' = \) length of straps. Also, the fluid weight is

\[ T_f = \begin{cases} 
  k_f (αR \cos θ - \omega_0) & 0 \leq θ_1 \\
  0 & \text{else}
\end{cases} \quad \cdots \quad (12) \]

with \( k_f \) being a function of the total lift-off. The total vertical resultant \( W (= \) shell weight) is then (Fig. 5)

![Diagram](image)

**FIG. 5.—**Tank Equilibrium

\[ W + W_f = C - T \]
\[ W = 2 \left\{ - \int_0^{\theta_1} k_i (\alpha R \cos \theta - \omega_0) R d \theta + \int_{\theta_1}^{\pi} K (\omega_0 - \alpha R \cos \theta) R d \theta \right\} \] 

\[ - \int_0^{\theta_0} k (\alpha R \cos \theta - \omega_0 - \delta) R d \theta \} \] 

\[ \] 

(13)

Performing the preceding integrations, and defining the stiffness ratios

\[ \kappa = \frac{k}{K} = \left( \frac{\text{strap stiffness}}{\text{shell stiffness}} \right) \] 

(14a)

\[ \kappa_j = \frac{k_j}{K} = \left( \frac{\text{fluid "stiffness"}}{\text{shell stiffness}} \right) \] 

(14b)

we obtain (in combination with Eqs. 8–9)

\[ W = 2 \pi R K \alpha R \left[ \cos \theta_1 + \frac{1 - \kappa}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) \right. \] 

\[ \left. + \frac{\kappa}{\pi} (\theta_0 \cos \theta_0 - \sin \theta_0) \right] \] 

(15)

On the other hand, the overturning moment is (Fig. 5)

\[ M = 2 \left[ \int_0^{\theta_1} k_i (\alpha R \cos \theta - \omega_0) R^2 \cos \theta d \theta - \int_{\theta_1}^{\pi} K (\omega_0 - \alpha R \cos \theta) R^2 \cos \theta d \theta \right. \] 

\[ \left. + \int_0^{\theta_0} k (\alpha R \cos \theta - \omega_0 - \delta) R^2 \cos \theta d \theta \right] \] 

(16)

which upon integration and consideration of Eqs. 8–9 yields

\[ M = \pi R^2 K \alpha R \left[ 1 - \frac{(1 - \kappa_j)}{\pi} (\theta_1 - \sin \theta_1 \cos \theta_1) \right. \] 

\[ \left. + \frac{\kappa}{\pi} (\theta_0 - \sin \theta_0 \cos \theta_0) \right] \] 

(17)

Define now the dimensionless ratios (moment and slackness)

\[ \mu = \frac{2M}{WR} \] 

(18)

\[ \epsilon = \frac{\delta}{W} - \frac{\delta K}{2\pi RK} \] 

(19)

in which \( W_t \) = weight of shell/length. With these definitions, Eqs. 15 and 17 can be written as

\[ \mu = \frac{1 - \frac{1 - \kappa_j}{\pi} (\theta_1 - \sin \theta_1 \cos \theta_1) + \frac{\kappa}{\pi} (\theta_0 - \sin \theta_0 \cos \theta_0)}{\cos \theta_1 + \frac{1 - \kappa_j}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) - \frac{\kappa}{\pi} (\sin \theta_0 - \theta_0 \cos \theta_0)} \] 

(20)
\[ \epsilon \geq \frac{\cos \theta_0 - \cos \theta_1}{\cos \theta_1 + \frac{1 - \kappa_f}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) - \frac{\kappa}{\pi} (\sin \theta_0 - \theta_0 \cos \theta_0)} \quad \text{...... (21)} \]

The last inequality establishes the fact that the slack may exceed the deflection at \( \theta_0 \) if \( \theta_0 = 0 \).

Eqs. 20–21 constitute a system of implicit equations in \( \theta_0, \theta_1 \). For given values of \( \mu, \epsilon \) (external moment and slackness), they must be solved by trial and error. In addition, iterations must be made on the solutions to adjust for the dependence of \( \kappa_f \) on the lift-off at \( \theta = 0 \).

To compute the maximum moment that can be resisted when no straps are present, one must set \( \kappa = 0 \) in Eq. 20. (Eq. 21 is then unnecessary.) However, because of the factor \( \kappa_f \), which is equivalent to a strap without slack, the implied moment may be large. This is meaningless, since large moments are associated with large separations, and the equivalent secant stiffness of the fluid breaks down.

The minimum condition for lift-off is \( \theta_0 = \theta_1 = 0 \), implying \( \mu = 1 \). If \( \mu < 1 \), one has compression only. On the other hand, if \( \mu \geq 2 \) (and \( \kappa_f = 0 \)), then the straps must take tension, since otherwise the tank turns over. Physically, the condition \( \mu = 2 \) is realized in the case without straps when the force resultant passes through the edge of the tank, i.e., when \( \theta = \pi \).

**Maximum Strap Force and Shell Compression**

The maximum strap force occurs at \( \theta = 0 \), and its value is

\[ \tilde{T}_k = k(aR - \omega_0 - \delta) \]

\[ = k aR (1 - \cos \theta_0) \quad \text{.............. (22)} \]

Substituting \( aR \) from Eq. 8, we obtain

\[ \tilde{T}_k = \kappa(1 - \cos \theta_0) \left( \frac{W}{2\pi R} \right) \]

\[ \cos \theta_1 + \frac{1 - \kappa_f}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) - \frac{\kappa}{\pi} (\sin \theta_0 - \theta_0 \cos \theta_0) \quad \text{...... (23)} \]

If the straps had been analyzed according to API 650 ("conventional method"), the resulting maximum strap force would have been given by a formula similar to the preceding one, but with \( \kappa' = 1, \theta'_i = \theta'_0, \kappa' = 0 \). This would have implied (from Eq. 20)

\[ \mu = \frac{1}{\cos \theta'_1} \rightarrow \cos \theta'_1 = \frac{1}{\mu} = \cos \theta'_0 \quad \text{.............. (24)} \]

so that

\[ \tilde{T}'_k = \frac{\mu}{\pi} \left( 1 - \frac{1}{\mu} \right) \frac{W}{2R} \]

\[ = \frac{W}{2\pi R} (\mu - 1) \quad \text{.............. (25)} \]
The ratio between the actual strap force and the conventional strap force is then (dividing Eq. 23 by Eq. 25)

\[
\sigma = \frac{T_k}{T_k'} = \frac{\kappa (1 - \cos \theta_0)}{(\mu - 1) \left[ \cos \theta_1 + \frac{1 - \kappa_f}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) - \frac{\kappa}{\pi} (\sin \theta_0 - \theta_0 \cos \theta_0) \right]}
\]  

(26)

On the other hand, the maximum compression force is

\[T_k = K(\omega_0 + \alpha R) = K \alpha R (1 + \cos \theta_1) \]  

(27)

so that

\[T_k = \frac{(1 + \cos \theta_1) \left( \frac{W}{2\pi R} \right)}{\cos \theta_1 + \frac{1 - \kappa_f}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) - \frac{\kappa}{\pi} (\sin \theta_0 - \theta_0 \cos \theta_0)} \]

(28)

Again, the conventional shell compression would have been (with \(\theta'_0 = \theta_0'\), \(\kappa' = 1\), \(\kappa'_f = 0\), \(\cos \theta_1' = 1/\mu\)).

\[T_k' = (1 + \mu) \frac{W}{2\pi R} \]  

(29)

Thus, the shell compression ratio is

\[\tau = \frac{T_k}{T_k'} = \frac{1 + \cos \theta_1}{(1 + \mu) \left[ \cos \theta_1 + \frac{1 - \kappa_f}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) - \frac{\kappa}{\pi} (\sin \theta_0 - \theta_0 \cos \theta_0) \right]} \]

(30)

The factors \(\sigma\), \(\tau\) can be used to correct the values obtained by a conventional analysis according to API 650. Since it can be shown that \(\sigma < 1\) and \(\tau > 1\), it follows that the strap forces are overestimated, and the compression forces underestimated when computed according to the API 650 Code.

**Anchored Tanks: Both Slackness and Fluid Contribution Negligible**

Consider the particular case of an anchored tank with zero slackness of the straps (\(\delta = 0\)) and negligible fluid contribution (\(\kappa_f = 0\)). The relevant equations are then (with \(\theta_1 = \theta_0\))
\[
\begin{align*}
\Delta &= \cos \theta_0 + \frac{1}{\pi} \kappa (\sin \theta_0 - \theta_0 \cos \theta_0) \\
1 - \frac{1 - \kappa}{\pi} (\theta_0 - \sin \theta_0 \cos \theta_0) \\
\mu &= \frac{\kappa (1 - \cos \theta_0)}{(\mu - 1) \Delta} \\
\sigma &= \frac{1 + \cos \theta_1}{(1 + \mu) \Delta}
\end{align*}
\]

Solving these equations by trial and error for a range of values of \( \mu \), one obtains the results plotted in Figs. 6–8. An indication on the order of magnitude of the effects being described here can be visualized by examining a typical example. Consider a cylindrical tank of radius \( R = 25 \) m and shell thickness (bottom course) \( t_s = 2.5 \) cm; the bottom plate has a thickness \( t = 0.5 \) cm. The tank is anchored with \( n = 100 \) strap of cross section \( 10 \) cm \( \times 1 \) cm; hence, (assume \( h = h' = \) length of strap)

\[
\kappa = \frac{k}{K} = \frac{h}{2 \times \pi \times 2.5 \times 10 \times 1 \times E} = 0.025
\]

The stiffness ratio is very small indeed. Hence, the shell compression

\[\text{FIG. 6.—Angle of Lift-Off versus Strap Stiffness and Overturning Moment}\]
FIG. 7.—Strap Stress Reduction Factor

factor, \( \tau \), will increase rapidly with moment ratios \( \mu > 2 \), which is a likely situation. Otherwise, the tank would not have been anchored in the first place. Thus, from Fig. 8, \( \tau > 1.5 \) (for \( \kappa = 0.025 \)).

The implication of the preceding is that an anchored tank designed according to API 650 may be subjected to buckling if the straps are not stiff enough in comparison to the tank itself. Thus, stiffness, and not only allowable stress, should normally be considered in the design of hold-down straps (or anchor bolts), since the most important factor in practice is the buckling load of the tank, and not the strength of the straps.

**Unanchored Tanks**

In principle, the equations developed earlier can be specialized for the
case of unanchored tanks by merely considering straps of vanishing stiffness. Such a procedure is, however, a significant oversimplification of the actual problem of unanchored tanks, since many of the assumptions underlying the analysis may not hold anymore; in particular, one would probably have to consider also membrane effects ("catenary forces") in the determination of the uplift resistance. Nevertheless, it can be shown that the simplified equations obtained by setting $k = 0$ agree with those in the Wozniak model when the separation is large; for small separations, on the other hand, these equations provide for a reduction in the effective fluid weight that can be transmitted to the bottom plate. Thus, they may be helpful in explaining how buckling of the shell may be initiated before all of the fluid weight is mobilized.

Consider then the case of zero strap stiffness ($k = 0, \kappa = 0$). In this situation, the angle $\theta_0$ is irrelevant. The equations are now
\[
\Delta = \cos \theta_1 + \frac{1 - \kappa_f}{\pi} (\sin \theta_1 - \theta_1 \cos \theta_1) \\
\mu = \frac{1 - \frac{1}{\pi} \kappa_f (\theta_1 - \sin \theta_1 \cos \theta_1)}{\Delta} \\
\tau = \frac{1 + \cos \theta_1}{(1 + \mu) \Delta}
\]

which are identical to Eq. 31 if one replaces \( \theta_0 \) and \( \kappa \) by \( \theta_1 \) and \( \kappa_f \). The secant fluid stiffness, however, is now an implicit quantity that depends on the lift-off at \( \theta = 0 \). This separation is (Eqs. 5 and 7)

\[ u = \alpha R - \omega_0 = \alpha R (1 - \cos \theta_1) \] ................................. (33)

and from Eqs. 15 and 32

\[ u = \frac{W}{2 \pi R K} \frac{1 - \cos \theta_1}{\Delta} = \frac{W_f}{K} \frac{1 - \cos \theta_1}{\Delta} \] ................................. (34)

Having \( u \), one can use the relationships between lift-off force and displacement (Eqs. 2–4) to determine the secant stiffness \( k_f \) and thus \( \kappa_f \). The procedure must then be repeated until convergence is established.

**Design Implications**

The formulas presented earlier in this paper seem to indicate that buckling of the tank shell may be initiated at lower seismic loads then predicted by API 650. This observation, however, does not necessarily imply that stable equilibrium configurations do not exist for greater loads. In fact, empirical and experimental evidence demonstrates that some tanks can resist substantially larger loads without apparent damage. While the reasons for this reserve capacity are controversial and not yet fully understood, there are several possible explanations for it. On the one hand, it is conceivable that substantial post-buckling elastic stress redistribution can take place in the compressive region, allowing for rapid increases in the amount of lift-off, which in turn implies that greater portions of fluid can be mobilized. On the other hand, the larger separation will be resisted to a growing extent by catenary (membrane) effects in the bottom plate, the modeling of which was not attempted here.

It should be noted, however, that any explanation for increases in the forces that resist lift-off (such as fluid weight, strap tension, catenary action of the tank floor, or some other phenomenon) must account also for necessary increases in the compressive forces, since overall equilibrium must be preserved (Fig. 5). Inasmuch as the elastic buckling formulas indicate that instabilities could be initiated before any such mechanisms can be activated, it appears that stable postbuckling configurations should exist involving redistribution of compressive stresses along the
circumference. Alternatively, it is possible that use of the elastic buckling formula may lead to excessively conservative values for the stability limit itself, since its derivation is based on neglecting edge effects near the tank bottom as well as internal pressure and variation of compressive stresses with height and azimuth. Such questions cannot be answered with a simple model such as the one presented here, but must be addressed by means of more powerful techniques. However, this method (and possible generalizations that account for foundation flexibility) could be used to establish safe and conservative bounds for the design loads, if the associated conservatism can be tolerated. The writers would also suggest that the stiffness of the straps, in the case of anchored tanks, be normally considered as a design parameter.

**Conclusions**

The refinement presented in this paper on a model originally proposed by Wozniak (50) for the seismic analysis of fluid storage tanks demonstrates that use of the rules incorporated in the API 650 code may lead in some cases to underestimations of the maximum compressive stresses that may develop in both anchored and unanchored tanks.

For anchored tanks, the stiffness of the straps relative to that of the shell plays an important role in the maximum compressive stresses that may be expected in the tank wall, since for given levels of tank rotation, compression stresses develop faster in the tank than tensile forces in the straps. While graphic results showing increases of these stresses (relative to computations by API 650) were presented only for the case of perfectly anchored straps, the equations developed here are valid also for the more general case of tanks with straps having some built-in or initial slackness. Additional effects similar to the ones illustrated could be expected also from consideration of the soil and foundation stiffness, or the lack of it.

In the case of anchored tanks, the larger values of the compressive stresses result from the reduced mass of fluid that can be mobilized before the buckling load in the shell is reached. In the recommendations of API 650, the resisting force contributed by the fluid is based on lift-off displacements of the tank which often may not be attained without initiating buckling of the shell. Inasmuch as tanks have been observed to experience large separations without sustaining damage, one wonders whether stable damage-free configurations can develop after buckling. More sophisticated numerical models as well as experimental and empirical evidence will be needed to elucidate these points.

**Appendix I.—References**


56. “Seismic Design of Storage Tanks,” American Petroleum Institute, Welded Steel Tanks for Oil Storage, API Standard 650, Appendix E.