DISCUSSIONS AND CLOSURES

Discussion of “Uniform Shear Buildings under the Effect of Gravity Loads” by M. Sahin and M. Ozturk

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The authors of the original paper present a rigorous solution to the problem of lateral vibrations of an upright cantilever shear beam subjected to the effect of self-weight. While some of the equations may indeed serve as a point of comparison for alternative formulations based on, say, discrete models, this is nonetheless a classical problem on which numerous reports have been published in the technical literature during the last three or four decades. Moreover, some of this material is often covered in graduate courses in structural dynamics in which the theory of the so-called $P$-delta effect for a cantilever beam is typically presented in the more general context of a Timoshenko beam that includes both shear and bending deformations, or in connection with space frames and walls braced by floor diaphragms. Thus, the effect is very well known. Before providing a very compact alternative solution to this problem, some preliminary observations on this paper are appropriate.

First, Eq. (1) of the original paper gives the moment contributed by the gravity forces yet ignores the moment provided by the dynamic inertia $m\dot{y}$, which happens to be equivalent to the peculiar term added to the 1960s. Hence, the differential element behaves as if it had an effective shear stiffness $k_{\text{equi}}$ that is consistent with Eq. (5) in the original paper. This equation is similar to that of an inhomogeneous shear beam in which the shear modulus changes linearly with height (the so-called Gibson solid), the solution to which has been known since at least the 1960s.

The same problem considered by the authors can also be solved with exquisite accuracy by means of a very simple model based on the well-known assumed modes method. In this alternative, one expresses the continuous displacement field in terms of assumed modes method and obtains the equations of motion for free vibrations of the shear beam with $P$-delta effects included is then simply

$$m\ddot{u} + \frac{\partial}{\partial y} \left( k - mg(H-y) \frac{\partial u}{\partial y} \right) = 0$$

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The same problem considered by the authors can also be solved with exquisite accuracy by means of a very simple model based on the well-known assumed modes method. In this alternative, one expresses the continuous displacement field in terms of trial functions $\psi_j(y)$ that must satisfy (at least) the essential— or geometric—boundary conditions and obtains the equations of motion for generalized coordinates $q_j(t)$ in the context of the following energy formulation:

Second, the origin of Eq. (2) in the original paper is rather obscure and is not consistent with the diagram of forces shown in Fig. 1. Instead, the normal force $W$, which is parallel to the axis, contributes an additional horizontal component $-Wu'$ to the shear, which happens to be equivalent to the peculiar term added to Eq. (2), so that the final dynamic equilibrium in Eq. (5) in the original paper is indeed correct. A simple alternative derivation of this equation based on energy concepts (using the authors’ notation) would be as follows:

The net external energy density $U$ needed to deform the element shown in Fig. 1 in the original paper is the strain energy needed to accomplish the shearing strain $\gamma = u'/\dot{y}$, minus the potential energy released by the gravity force $W$ as it descends slightly during the shearing of the element, that is, $U = 1/2k\gamma^2 - W(1 - \cos \gamma)$. But $\cos \gamma = 1 - 1/2\gamma^2 \cdots$, so $U = 1/2k\gamma^2 - 1/2W\gamma^2 = 1/2(k-W)\gamma^2$. Hence, the differential element behaves as if it had an effective shear stiffness $k_{\text{equi}} = k - W = k - mg(H-y)$. The equation of motion for free vibrations of the shear beam with $P$-delta effects included is then simply

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Applying this ansatz in the context of Lagrange’s equations, one ultimately arrives at a free vibration problem for an $N$-DOF system in generalized coordinates of the form

$$M\ddot{q} + (K - G)q = 0$$

with

$$M = \bar{m} \int_0^H \Psi^T \Psi \, dy; \quad K = k \int_0^H \left( \frac{\partial}{\partial y} \Psi^T \right) \left( \frac{\partial}{\partial y} \Psi \right) \, dy$$

and

$$G = \bar{m} g \int_0^H (H - y) \left( \frac{\partial}{\partial y} \Psi^T \right) \left( \frac{\partial}{\partial y} \Psi \right) \, dy$$

Fig. 1 shows a very compact Matlab program incorporating these equations. It provides both the frequencies and modal shapes for a system with $N$ trial functions, taken here as the modes of the shear beam without gravity effects included. The program displays the modes graphically and produces virtually instantaneous and highly accurate results, despite not taking advantage of any symmetry or narrow bandwidth considerations. Observe that the computational part of this program involves only nine statements. A hardly longer program can also be written for a Timoshenko beam.