Scattering of waves by subterranean structures via the boundary element method

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The boundary element method (BEM) is used to study the two-dimensional wave field generated when buried structures of arbitrary shape (i.e. inclusions) in an elastic medium are illuminated (or insonified) by dynamic line sources. Both steady-state responses and time-domain transients are presented. The problem is formulated in the frequency domain by means of appropriate Green's functions. The evaluation of the singular integrals is achieved (and to the best of the writers' knowledge, for the first time in the technical literature) in analytical form, which results in improvements in computational efficiency and accuracy. Closed-form solutions for regular geometries are then used to validate the method. The interaction of two cavities, the formation of shadow zones by inclusions and the complexity of the scattered field from bodies with irregular shapes are used as examples to demonstrate the versatility of the method. The responses computed in the time domain were invariably found to be causal, even for non-convex domains, which belies a recent assertion by some researchers that the application of boundary element methods to concave domains is associated with non-causal effects. Copyright © 1996 Elsevier Science Limited

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INTRODUCTION

Many studies in the past two decades have addressed the phenomenon of wave propagation in the vicinity of geological and topographical irregularities. While most previous efforts have been devoted within the field of engineering seismology, some recent researches have also focused on the development of dynamic testing procedures for the detection of subsurface anomalies in the ground; such methods are generically referred to as seismic tomography. The principal objective in seismic tomography is the location and identification of subterranean structures and cavities as well as variations in soil properties by means of acoustic (or even electromagnetic) sources and receivers placed on the ground's surface or inside holes. Such testing methods have been aided by the enormous progress made in recent years in computer-based data acquisition and storage capabilities as well as in signal processing. This has in turn stimulated research aimed at developing appropriate inversion algorithms for the extraction of accurate images of the subterranean structures from the collected data. Clearly, such inverse problems pose great theoretical difficulties and challenges, because one
cannot establish a priori that their solutions are unique or even exist, in contrast to direct problems, for which such attributes are usually taken for granted.

Some of the first analytical studies on wave diffraction and scattering were concerned with the problem of wave motion and reverberations in alluvial basins of regular shape, and with the issue of wave scattering induced by cavities. More recently, semi-analytical methods have been used also for the analysis of wave diffraction caused by geological irregularities of arbitrary shape within globally homogeneous media. By contrast, the application of purely numerical methods (i.e., finite elements or differences combined with transmitting boundaries) has been restricted, for the most part, to situations where the response is required only within localized irregular domains, as is the case in soil-structure interaction problems. Discrete methods have occasionally been used also for the modeling of large alluvial basins, but only in plane-strain. Finally, hybrid methods involving combination of finite elements to model the interior domain containing the inhomogeneities and semi-analytical representations for the exterior domain have been used as well. A detailed review on the above methods was presented by Sanchez-Sesma.

Perhaps the best tool for the analysis of wave propagation problems in unbounded, infinitely large media is the boundary element method (BEM); the reason can be found in its inherent ability in satisfying the far-field radiation conditions and its intrinsic virtue allowing description of the medium merely in terms of nodes at material discontinuities. Depending on how the algorithm is formulated, it is customary to distinguish between direct and indirect boundary element methods. In the direct method, the relationship between displacements and forces on the physical boundary result from consideration of virtual point sources and singular integrals on the boundary, while in the latter they are obtained by means of fictitious source densities with no immediate physical significance. The majority of applications is based on the direct formulation. Whatever the choice, boundary element methods require the availability of appropriate fundamental solutions, or Green's functions, relating the field variables (stresses, displacements) in a homogeneous medium to sources placed at some location in the medium. The fundamental solution most often used is that of an infinite homogeneous space, because it is known in closed-form and has a relatively simple structure.

While the Green's functions for a half-space satisfy automatically the zero stress conditions at the free surface so that no boundary elements are required here, their disadvantage is that they can only be obtained by semi-analytical means. Full-space fundamental solutions, on the other hand, are much simpler and easier to implement. Boundary element methods based on the Green's functions for either an infinite space or a half-space have been used in the past to solve a variety of problems involving diffraction of waves by surface irregularities of arbitrary shape. Boundary elements have also been used in studies on plane harmonic or transient body and surface wave diffraction by cavities and buried structures. For a recent review of this subject, the reader is referred to Beskos.

A major difficulty associated with the application of the direct boundary element method lies in the evaluation of the singular integrals, a topic which has received a great deal of attention in the technical literature. One strategy used to overcome this difficulty consists in decomposing the integrand into two parts, namely a singular and a nonsingular component. The singular part is determined analytically, while the non-singular part is evaluated numerically by means of a standard Gauss quadrature. An alternative approach consists in combining the direct boundary element method with discrete wave-number representations of the Green's functions; while this strategy allows evaluating the element integrals in analytical form for each wave-number, this advantage comes at the expense of having to carry out cumbersome numerical integrations over wave-number.

In the sections that follow, the direct boundary element method is used together with the two-dimensional full-space Green's functions in an evaluation of the displacement field associated with waves illuminating a cavity or inclusion within a homogeneous elastic medium (i.e., a full-space, a half-space and a homogeneous slab). The problem is formulated in the frequency domain, and the response in the time domain is obtained by means of the fast Fourier transform. Particular attention is given to the derivation of analytical expressions for the singular integrals. After validation, the boundary element algorithm is used to study four wave diffraction problems involving cavities and wave-number.

BOUNDARY ELEMENT FORMULATION

Considering the voluminous literature currently available on the boundary element method, it does not appear necessary to repeat yet again the details of the formulation required for the type of scattering problems presented herein (see for example, Ref. 42). It suffices to state that the BEM requires evaluation of the two integrals

\[ G_{ij}^{kl} = \int_{C_l} G_{ij}(x_k, x_l) dC_l \]  

\[ H_{ij}^{kl} = \int_{C_l} H_{ij}(x_k, x_l, n_l) dC_l \]

in which \( G_{ij}(x_k, x_l) \) and \( H_{ij}(x_k, x_l, n_l) \) are, respectively, the components of the Green's tensor for displacement and traction components at \( x_k \) in direction \( i \) due to a
concentrated load at \( x_1 \) in direction \( j \), and \( n_i \) is the unit outward normal for the \( l \)th boundary segment \( C_l \).

In the case of a medium under plane-strain conditions subjected to in-plane and anti-plane line loads, the required Green’s functions are as follows:\(^{43}\)

(a) Anti-plane unit line load \((i,j = 3)\):

\[
G_{33}(x, x_0) = \frac{i}{4\mu} H_0^{(2)}(kr) \tag{3}
\]

(b) In-plane unit line load \((i,j = 1, 2)\):

\[
G_{ij}(x, x_0) = \frac{i}{4\mu} \left\{-\delta_{ij} H_0^{(2)}(kr) + \frac{\rho \omega}{k_3} \left[ H_1^{(2)}(kr) - \gamma H_2^{(2)}(kr) \right] \right\}
- \frac{i}{2\mu} \left\{ \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \left[ H_2^{(2)}(kr) - \gamma^2 H_2^{(2)}(kr) \right] \right\} \tag{4}
\]

In these equations \( \gamma = \beta/\alpha \), where \( \alpha = \sqrt{(\lambda + 2\mu)/\rho} \) and \( \beta = \sqrt{\mu/\rho} \) are the velocities of compressional and shear waves, respectively; \( \lambda \) and \( \mu \) are the Lamé constants; \( \rho \) is the mass density; \( k_\alpha = \omega/\alpha \) and \( k_\beta = \omega/\beta \) are the wave-numbers; \( \omega \) is the circular frequency; \( r = |x - x_0| \) is the source-receiver distance; \( \delta_{ij} \) is Kronecker’s delta; the leading factor \( i = -1 \) is the imaginary unit; \( H_n^{(2)}(\cdot) \) are Hankel functions of the second kind and \( n \)-th order. These Hankel functions, together with an implicit exponential factor \( e^{i\omega t} \) represent waves in the far-field that travel outwardly toward infinity. (Note: some researchers instead formulate the Green’s functions using a factor \( e^{-i\omega t} \) and Hankel functions of the first kind \( H_n^{(1)}(\cdot) \) in eqns (3) and (4); such choice requires a reversal of the usual positive sign in the Fourier transformation from the frequency domain into the time domain.)

The corresponding expressions for the tractions \( H_{ij} \), which may be obtained from \( G_{ij} \) by taking partial derivatives to deduce the strains and then applying Hooke’s law to obtain the stresses, are omitted here for the sake of brevity.

**Element integration**

When the element to be integrated in eqns (1) and (2) is not the loaded element, the integrands are non-singular and the integrations are best carried out using standard Gaussian quadrature. For the loaded element, however, the integrands exhibit a singularity, but it is then possible to carry out the integrations in closed form, as will be shown.

To demonstrate this assertion, consider the \( l \)-th singular element of length \( 2L \) shown in Fig. 1 for the anti-plane loading case. Since for this case \( \mathbf{r} \cdot \mathbf{n}_l = 0 \), the singular terms \( H_{33}^{(2)} \) vanish. On the other hand, the quantities \( G_{33}^{(2)} \) can be evaluated from the expressions (Ref. 44, p. 480)

\[
\int_{-L}^{+L} H_1^{(2)}(kr) ds = 2 \int_0^L \left[ J_0(kr) - iY_0(kr) \right] ds = 2L \left[ I_1(b) - iI_2(b) \right] \tag{5}
\]

with \( b = k_3 L \), and

\[
I_1(b) = J_0(b) + \frac{\pi}{2} \left[ S_0(b) J_1(b) - S_1(b) J_0(b) \right] \tag{6a}
\]

\[
I_2(b) = Y_0(b) + \frac{\pi}{2} \left[ S_0(b) Y_1(b) - S_1(b) Y_0(b) \right] \tag{6b}
\]

where \( S_0(\cdot) \) and \( S_1(\cdot) \) are Struve functions, and \( J_n(\cdot) \) and \( Y_n(\cdot) \) are \( n \)-th order Bessel functions of the first and second kind, respectively.

A similar procedure can be followed to evaluate the singular integrals for the in-plane case. In this case, it can first be observed that the terms \( H_{i1}^{(2)} \) for \( i,j = 1, 2 \) (which correspond to the loaded segment) vanish because the tractions are anti-symmetric with respect to the nodal point at the center of the element.

Consider next the integration of \( G_{ij}^{(2)} \) for \( i,j = 1, 2 \), which in view of eqn (4) may be written as:

\[
G_{ij}^{(2)} = \int_{-L}^{+L} G_{ij}(x, x_0) ds
= \frac{i}{2\mu} \left[ \delta_{ij}(-B_1 + B_2) - \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} B_3 \right] \tag{7}
\]

with

\[
B_1 = \int_0^L H_0^{(2)}(kr) dr \tag{8a}
\]

\[
B_2 = \int_0^L \frac{1}{kr} \left[ H_1^{(2)}(kr) - \gamma H_2^{(2)}(kr) \right] dr \tag{8b}
\]

\[
B_3 = \int_0^L \left[ H_2^{(2)}(kr) - \gamma^2 H_3^{(2)}(kr) \right] dr \tag{8c}
\]

The partial derivatives \( \partial/\partial x_i, \partial r/\partial x_j \) in eqn (7) represent the slopes of the line connecting the nodal point with the integration point. Since for the loaded
element they equal the slope of the segment, it follows
that they are constant and do not affect the integration.
This is the reason for their placement outside of the
integral B3 in eqn (7).

The first integral (B1) is obtained directly from eqn
(5), that is,

\[ B_1 = B_1(b) = L \left[ I_1(b) - iI_2(b) \right] \]  

To evaluate B2 and B3, it is necessary to use the well
known recurrence relations for the Bessel functions, and
to dispose of an apparent singularity at the lower limit
of the integrals by considering the ascending series for
the Bessel functions and observing that the terms
contributing to the singularity vanish exactly on account
of the fact that \( \gamma = \beta/\alpha = k_\alpha/k_\beta \).

For example,

\[ B_2 = \lim_{\epsilon \to 0} \int_0^L \left\{ \left[ H_0^{(2)}(k_\beta r) - \frac{1}{k_\beta} \frac{d}{dr} H_1^{(2)}(k_\beta r) \right] \right. \]

\[ - \left. \left( \frac{k_\alpha}{k_\beta} \right)^2 \left[ H_0^{(2)}(k_\alpha r) - \frac{1}{k_\alpha} \frac{d}{dr} H_1^{(2)}(k_\alpha r) \right] \right\} dr \]

\[ = \frac{1}{k_\beta} \lim_{\epsilon \to 0} \int_0^L \left[ \frac{d}{dr} H_1^{(2)}(k_\beta r) - \frac{k_\alpha}{k_\beta} \frac{d}{dr} H_1^{(2)}(k_\alpha r) \right] dr \]

\[ = \frac{1}{k_\beta} \lim_{\epsilon \to 0} \left[ H_1^{(2)}(k_\beta r) - \frac{k_\alpha}{k_\beta} H_1^{(2)}(k_\alpha r) \right] \]

\[ B_2 = B_1(b) - \gamma^2 B_1(a) - \frac{L}{b} \left[ H_1^{(2)}(b) - \gamma H_1^{(2)}(a) \right] \]  

(10)

with \( a = k_\alpha L \). By similar manipulations, one obtains

\[ B_3 = B_1(b) - \gamma^2 B_1(a) - \frac{2L}{b} \left[ H_1^{(2)}(b) - \gamma H_1^{(2)}(a) \right] \]  

(11)

Equations (5) and (9)–(11) provide the exact integrals
for the singular terms associated with line sources in a
two-dimensional homogeneous space (plane–strain). To
the best of the writers’ knowledge, this constitutes the
first such symbolic representation. In principle, similar
equations could also be written for a half-space, a
stratum, or a slab under anti-plane loads, because the
Green’s functions for such problems can be obtained
from those of the homogeneous full-space by recourse to
the method of images.

Evaluation of response in the time-domain

In the following sections, the method outlined pre-
viously is applied to the solution of four scattering
problems. In each case, the computations are first
performed in the frequency-domain, and then Fourier-
transformed into the time-domain by means of the well
known FFT algorithm. The variation with time of the
dynamic sources is assumed to be given by a Ricker
wavelet, as defined below. This wavelet is chosen not
only because it decays very rapidly in both time and
frequency, but because it facilitates the interpretation of
the computed time signatures, as they are then localized
in space–time. The Ricker wavelet function is given by

\[ u(\tau) = A (1 - \tau^2) e^{-\tau^2} \]  

(12a)

where \( A \) is the amplitude, \( \tau = (t - t_s)/t_0 \) and \( t \) denotes
time; \( t_s \) is the time at which the maximum occurs, while
\( t_0 \) is the characteristic (dominant) period of the
wavelet. Its Fourier transform is

\[ U(\omega) = A \left[ 2\sqrt{\pi} t_0 e^{-i\omega t_0} \right] \Omega^2 e^{-\Omega^2} \]  

(12b)

in which \( \Omega = \omega t_0/2 \).

As is well known, the frequency increment \( \Delta f = \Delta \omega/2\pi \) determines the total time duration for the
numerical analysis, namely \( T = 1/\Delta f \). If \( T \) is smaller
than the actual duration of motion, the response beyond
\( T \) wraps-around (i.e. is aliased) in the time window and
may be seen as a non-causal event occurring before the
first true wave arrivals, as detailed in textbooks on signal
analysis. One way to overcome this problem is to reduce
the frequency increment \( \Delta f \), but in most cases this
alternative may result in excessive computing time. A
more effective strategy is to use the complex frequency
method proposed by Phinney;\(^{45}\) in this method, a
fictitious attenuation is introduced by adding an
imaginary component to the frequency, i.e. \( \tilde{\omega} = \omega - i\lambda \),
the effect of which is an exponential attenuation of the
response in the time domain. This fictitious damping is
then removed by multiplying the computed response by
the exponential window \( e^{i\lambda} \). Clearly, \( \lambda \) cannot be made
too large, since doing so would lead to substantial
numerical errors that would show up toward the end of
the time window.\(^{46}\) The numerical calculations presented
herein were made with an imaginary component of
frequency \( \lambda = 0.7\Delta \omega \), which implies a \( 1/80 \) attenuation
at the end of the time window \( t = T \) (i.e. the wrap-around
is attenuated by two orders of magnitude).

Validation of the algorithm

The method and expressions described previously were
implemented and validated by applying them to simple
problems with known analytical solution, namely a
cylindrical inclusion in an infinite, homogeneous space
subjected to SH waves (particle motion parallel to the
axis of the inclusion) and SV-P waves (particle motion
perpendicular to the axis). These problems were
modeled by means of boundary elements placed at the
interface between the inclusion with the surrounding medium, as shown in Fig. 2.

As a first test, the inclusion was assigned the same material properties as the surrounding medium. Clearly, this test should reproduce the free-field conditions, and in fact it did. The program was subsequently tested by considering an elastic inclusion with material properties as listed in Fig. 2, and which is subjected to SV-P waves caused by a blast (dilatational) source at point O. Such a source elicits compressional waves with concentric wave-fronts having a center at O, which in turn induce particle motions in the radial directions. Upon impingement of this incident field on the inclusion, waves are scattered in all directions. The motion components are then computed and recorded at the indicated receivers, both with the implemented boundary elements and with the analytical solution reported by Pao & Mow.47

The results are computed at 64 frequencies in the range from 2 to 128 Hz, that is, for compressional (P) waves whose wavelength are, respectively, between 33.33 and 0.5208 times the radius of the structure. Figure 3 displays the real and imaginary parts of the response for the numerical and closed-form solutions and reveals the very good accuracy of the BEM approach. The comparison shows an excellent agreement for low frequencies, and only slight differences at high frequencies. This behavior was expected, because the accuracy of the BEM solution depends on the ratio...
between the length of the incident waves and the length of the boundary elements; in our example, this ratio decreases from 265 to 4.1 in the range of frequencies considered.

APPLICATIONS

Two cylindrical cavities in unbounded medium

Next, we consider the interaction of two cylindrical cavities in a homogeneous medium when they are both illuminated by SH (anti-plane) line sources placed first at one position and then at another, as shown in Fig. 4. The computations are performed in the range from 1 to 256 Hz with an increment of 1 Hz. Thus, the total time duration of the numerical analysis is $T = 1\, \text{s}$. A total of 220 boundary elements is used to model both cavities. Each line source elicits an incident wave field with cylindrical wavefronts and center at the source, which consists of a Ricker wavelet with a characteristic frequency of 100 Hz. The motion is computed and recorded at 21 receivers lying in a plane above the cavities but below the sources, as indicated.

Figure 5 displays, for each source, the response recorded at the receivers. The first wave arrivals observed in this figure correspond to wave trains that are directly reflected by each of the cavities. These are followed by pulses having progressively lower amplitudes, which are the result of reverberations between the cavities. As these pulses reflect back and forth between the cavities, they scatter and lose energy to the surrounding medium, and eventually dissipate. By measuring the arrival times of the pulses and echoes, one can infer the relative location and size of the cavities.

Another interesting aspect of this problem is the creation of shadow zones behind the cavities. Figure 6 illustrates this phenomenon: it shows the amplitude of the displacements in the surrounding medium that is produced by a harmonically vibrating source in position 2. The total and scattered responses are shown as contours in Fig. 6 for three values of the excitation frequency, namely, 2, 50 and 100 Hz, which correspond to incident waves with wavelengths of 50, 2 and 1 m, respectively. The results for 2 Hz do not show any shadow zone, while those for 50 and 100 Hz depict distinct shadows behind the cavities. This behavior was, of course, very much expected since it is very well known that waves whose wavelength is large compared to the obstacles they encounter suffer little scattering (i.e. they do not see the inclusions). These drawings illustrate, however, the virtues and advantages of the boundary element formulation for the study of complicated wave diffraction phenomena such as those being considered here. They also clearly demonstrate the intense degree of interaction between the cavities, which are the result of the previously cited reverberations.

Irregularly shaped cavity in an unbounded medium

One important aspect in the study of waves impinging on obstacles is the effect of the shape of the inclusion on the scattered wave-field. Basically, the question is whether or not one can make inferences about the shape of the inclusion from measurements of the scattered field at selected recording stations. As an initial exploration of this subject, we consider a kidney-shaped cavity in an unbounded elastic medium which is illuminated, as before, by anti-plane (SH) line sources placed at three different positions, as shown in Fig. 7. The response is recorded at 25 receivers placed between the source and the cavity. Computations are once more performed in the frequency range from 1 to 256 Hz, with a frequency increment of 1 Hz, and the time variation of the sources is a Ricker wavelet with a characteristic frequency of 100 Hz. A total of 400 boundary elements are used to model the inclusion. The computed responses are displayed in Fig. 8.

Analysis of the results obtained reveals complicated wave patterns which originate in reverberations within the concave section of the cavity. The primary reflections, coming from the left and right sides of the inclusion (i.e.
Fig. 6. Steady state response for two cylindrical cavities in an unbounded medium: total field (left) and scattered field (right).

the convex parts), are identified with the labels SL and SD, respectively. The time-of-flight of each of these pulses corresponds to the travel path of a pulse from the source to the inclusion and back to the receivers. Hence, the position of the sources defines whether SL or SD pulses are the first events to arrive at a given receiver location, and ray theory can be used to verify the accuracy of the numerical prediction, at least as far as arrival times of pulses is concerned. Even then, a ray analysis of the concave part can be cumbersome indeed.
Nonetheless, a careful analysis of the earliest wave arrivals which are consistent with ray analysis, demonstrated that all computed responses were causal. In other words, the signals did not arrive at the receivers earlier than the shortest possible ray paths would have allowed, even though the inclusion considered included a concave region. This observation is in contradiction with some recently expressed opinions that the use of boundary element techniques to non-convex domains may lead to violations in causality.

Cylindrical cavity in a stratum

The previous examples dealt with inclusions embedded in an unbounded homogeneous medium. Applications in the field of geomechanics, however, involve soils that have a free surface and perhaps also a rock interface. Thus, it is of interest to consider the problem of waves scattered by inclusions in a homogeneous stratum. The material discontinuities in this medium complicate the problem in at least two ways. First, the numerical solution becomes more involved because the Green’s functions must account for the presence of the bounding interfaces, unless they are also modeled with boundary elements (an option which increases the computational burden). Second, these interfaces reflect the incident as well as the scattered waves, which leads to complex wave patterns, as will be seen.

Figure 9 shows a cylindrical cavity in a homogeneous
soil layer underlain by rigid bedrock. The cavity is modeled with 140 boundary elements, and is illuminated from the free surface by two SH Ricker wavelet sources, one placed directly above the cavity, the other at a horizontal distance of 20 m. Computations are performed in the frequency range from 1 Hz to 128 Hz, with a frequency increment of 0.5 Hz. The surface and the interface between the stratum and the rigid bedrock were not discretized with boundary elements because the Green's function used were based on the method of virtual images (for details, see Ref. 48).

Figure 10 depicts the total and scattered displacement fields for this case. In contrast to the problems considered previously, the pulse trains at each receiver result now from the reflection and diffraction of waves at the boundary of the cavity, at the free surface and at the rigid interface. The incident and the first two reflected pulses from bedrock are tagged with labels I, P, and PP, respectively, while those that reflect once or twice at the surface are identified with labels S and SS. It is interesting to observe that each time a pulse hits bedrock the reflected pulse changes phase. The primary reflection, which results as the incident pulse strikes the boundary of the cavity and returns to the surface, is very clear in the plots.

When the source is above the cavity (source 1), the total response at the receivers located close to it do not provide information about the rigid interface: in this case the cavity acts as a barrier to the waves and creates a clear shadow zone underneath. As the distance of the receivers from the source increases, the reflections returning from bedrock start to be visible. When the source is located at position 2, however, the reflections from the rigid interface are clearly evident, although now some shadowing becomes evident at receivers 4 to 8 on the side opposite to the source.

**Cylindrical cavity in a half-space**

As a final application example, consider a cylindrical cavity in a half-space illuminated by a dilatational line source (P-waves) placed between the surface and the cavity, as detailed in Fig. 11. The source generates a Ricker wavelet pressure pulse with a center frequency of 22 Hz. Calculated frequencies range from 1 to 64 Hz.
with an increment of 1 Hz. We compute the response at receivers placed on the surface, using boundary elements with Green's functions for the elastic half-space obtained via discrete wave-number summation (see Ref. 48 for details).

Figure 12 shows the vertical and horizontal displacements at the receivers, which in the near field result mainly from body waves on account of the depth of the source. Because of symmetry, the horizontal response is zero at receiver 1. As waves impinge on both the cavity and the free surface, they scatter back into the medium as P and SV waves (the latter as a result of P-SV conversion). The first set of pulses recorded at the receivers correspond to the incident field, second arrivals are due to the initial P waves reflected from the cavity, while third arrivals are SV mode converted waves resulting from the initial P wave incidence on the cavity. Higher-order reflections can also be seen and can be identified by computing the travel time for different ray paths and mode conversions.

CONCLUSIONS

A boundary element formulation for plane-strain elastodynamic problems was developed and used to efficiently evaluate problems of wave diffraction and scattering. The aim was to implement a tool that could be used to in the context of research on seismic tomography problems, the results of which will be reported in a companion paper. In the first phase presented here, this program was used to compute the wave-fields in the vicinity of obstacles (inclusions) embedded in homogeneous media when they are illuminated by acoustic (SH and SV-P) line sources in various relative positions. The cases considered included cavities and elastic media embedded in a full-space, in a half-space, and in a stratum. The results obtained were consistent with predictions by ray acoustics, and were used by the writers to elucidate the most important aspects of wave acoustics, with an eye on the development of non-destructive testing and imaging methods.

An important observation made in the course of this research is that the response signals computed in the time domain are invariably causal, even for non-convex domains. Hence, this result belies the controversial view held by some researchers that the boundary element method fails to satisfy causality for non-convex domains. It is conceivable, however, that computations made with coarse discretizations (i.e. few boundary elements) may lead to errors that could be misconstrued as violations of causality.

Of central importance to the work presented in this paper, and to the theory of boundary elements in general, is the evaluation of the singular integrals, which was here achieved in closed form. To the best of the writers’ knowledge, this is the first such evaluation, which holds the promise of improved accuracy and efficiency for elastodynamic problems.

REFERENCES


